YEAR 12

IARTV TEST — OCTOBER 1996

MATHEMATICAL METHODS Units 3 and 4

CAT 2 — Facts and Skills Task

SECTION A — ANSWERS & SOLUTIONS

NAME: SOLUTIONS

Question 1	B	Question 12	A	Question 23	B
Question 2	C	Question 13	B	Question 24	A
Question 3	D	Question 14	C	Question 25	E
Question 4	E	Question 15	B	Question 26	E
Question 5	D	Question 16	C	Question 27	C
Question 6	C	Question 17	D	Question 28	B
Question 7	A	Question 18	D	Question 29	C
Question 8	E	Question 19	A	Question 30	C
Question 9	E	Question 20	D	Question 31	B
Question 10	B	Question 21	C	Question 32	D
Question 11	D	Question 22	D	Question 33	B

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MATHEMATICAL METHODS Units 3 and 4
CAT 2 — Facts and Skills Task — SECTION B — ANSWERS & SOLUTIONS

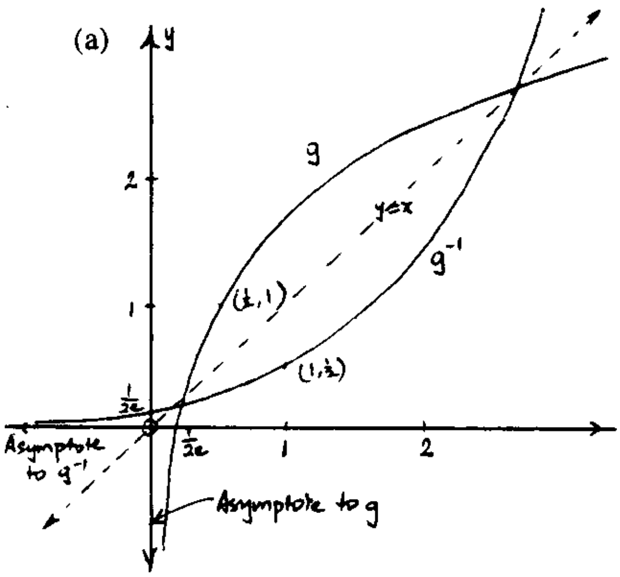
1 (a) $y = 2x + x^2 - x^3$
$= -x(x^2 - x - 2)$
$= -x(x - 2)(x + 1)$

Hence x-intercepts are $(0,0)$, $(2,0)$ and $(-1,0)$

(b) Area

$$= \int_0^2 2x + x^2 - x^3 \,.\, dx - \int_{-1}^0 2x + x^2 - x^3 \,.\, dx$$
$$= \left[x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 - \left[x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_{-1}^0$$
$$= \left(4 + \frac{8}{3} - \frac{16}{4}\right) + \left(1 - \frac{1}{3} - \frac{1}{4}\right)$$
$$= \frac{8}{3} + \frac{5}{12}$$
$$= \frac{37}{12} = 3\frac{1}{12}$$

2 (a)



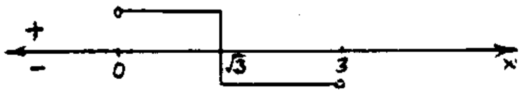
(b) For inverse, $x = 1 + \log_e(2y)$
$\Rightarrow \quad 2y = e^{x-1}$
$\Rightarrow \quad y = 0.5e^{x-1}$
Dom g^{-1} = Ran $g = \mathbb{R}$

Hence $g^{-1} : \mathbb{R} \to \mathbb{R}$,
$g^{-1}(x) = 0.5e^{x-1}$

3 (a) Domain $= \{x : 0 < x < 3\}$
$= (0,3)$

(b) $A(x) = 18x - 2x^3$
$A'(x) = 18 - 6x^2$
$= -6(x^2 - 3)$

3 For stationary points, $A'(x) = 0$
$\Rightarrow x^2 = 3$
In the domain $(0,3)$, $x = \sqrt{3}$
Sign diagram for $A'(x)$:



Hence maximum when $x = \sqrt{3}$.
Hence for maximum area ($12\sqrt{3}$ sq.units), rectangle is $2\sqrt{3}$ units wide and 6 units high.

4 Let X be number of successes in 5 trials. Hence $X \sim \text{Bi}(5,p)$
$\Pr(X > 0) = 0.99968$
$\Rightarrow \Pr(X = 0) = 1 - 0.99968$
$= 0.00032$
$\Pr(X = 0) = {}^5C_0\, p^0\, (1 - p)^5$
$\Rightarrow (1 - p)^5 = 0.00032$
$\Rightarrow (1 - p) = 0.2$
$\Rightarrow p = 0.8$
Hence probability of running under 55 seconds on one trial is 0.8

5 Let X be the study score within the school. Hence $X \sim N(\mu, 5^2)$
$\Pr(X < 45) = 0.95$
Using cdf table in reverse
$\Pr(Z < 1.645) \approx 0.95$

$\Rightarrow \frac{45 - \mu}{5} \approx 1.645$

$\Rightarrow 45 - \mu \approx 8.225$
$\Rightarrow \mu \approx 36.775$

6 It is suspected that $p = 0.5$

Hence $se(p) = \sqrt{\frac{0.5 \times 0.5}{n}}$

We require $2\sqrt{\frac{0.5^2}{n}} < 1\% = 0.01$
$\Rightarrow \sqrt{\frac{0.25}{n}} < 0.005$
$\Rightarrow \frac{0.25}{n} < 0.000025$
$\Rightarrow \frac{n}{0.25} > \frac{1}{0.000025} = 40000$
$\Rightarrow n > 0.25 \times 40000 = 10000$

Hence minimum sample size is 10000