# 1995
# VCE
# MATHEMATICAL METHODS
# CAT 2

## DETAILED SUGGESTED SOLUTIONS

**CHEMISTRY ASSOCIATES**
**P.O. BOX 2227**
**KEW, VIC., 3101**
**AUSTRALIA**
**TEL:(03) 9817 5374 FAX: (03) 9817 4334**

**email: chemas@vicnet.net.au**

**Internet: http://www.vicnet.net.au/~chemas/education.htm**

## PART 1: Multiple-Choice Booklet

**Question 1 B**

The graph shown is that of the linear function

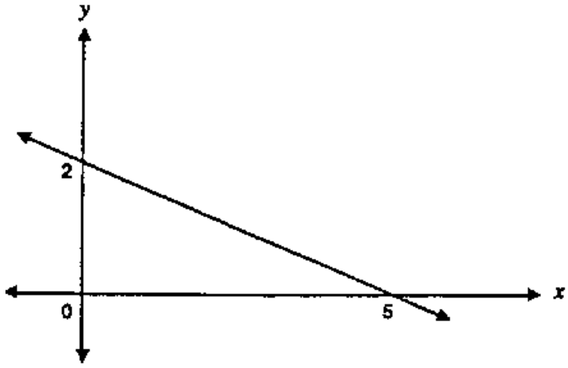
**A.** $f : R \to R,\ f(x) = 5x + 2$

**B.** $f : R \to R,\ f(x) = 2 - \frac{2x}{5}$

**C.** $f : R \to R,\ f(x) = \frac{2x}{5} + 2$

**D.** $f : R \to R,\ f(x) = -\frac{5x}{2} + 5$

**E.** $f : R \to R,\ f(x) = 2x + 5y$




Let $(x_1, y_1) = (0,2),\ (x_2, y_2) = (5,0)$ and the equation of the linear function be $f(x) = mx + c$

$m$ = gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{5-0} = -\frac{2}{5}$, $c = y$ − intercept $= 2$, $f(x) = -\frac{2x}{5} + 2$

The graph shows the linear function $f : R \to R,\ f(x) = 2 - \frac{2x}{5}$

**Question 2 E**

The graph of $f : R \to R$ crosses the $x$-axis exactly three times, which of the following rules could **not** be the rule for $f$?

**A.** $f(x) = x(x^2 - 4)$

**B.** $f(x) = x(x-2)(x+4)(x^2+1)$

**C.** $f(x) = (3-x)(x^4 - 16)$

**D.** $f(x) = (x^2 - x - 6)(x - 4)$

**E.** $f(x) = (x^2 - x - 6)(x^2 - x - 12)$

$f(x) = (x^2 - x - 6)(x^2 - x - 12) = (x-3)(x+2)(x-4)(x+3)$

$x$-intercepts at $x = 3, -2, 4, -3$

$f(x)$ crosses the $x$-axis four times so E could not be the rule for $f$.

**Question 3 B**

The range of the function with graph as shown is

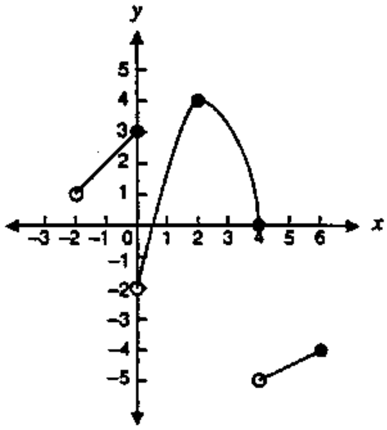
**A.** (−2,6]

**B.** (−5,−4] (−2,4]

**C.** (−5,4]

**D.** (−5,−4] (−2,3]

**E.** (−2,4] (5,6]




The range of the graph is given by the $y$-values.
Range = (−5,−4] (−2,4]

**Question 4 B**

Data about the relationship between quantities $x$ and $y$ is represented graphically as shown below:
If $a$ and $b$ are positive constants, the equation relating $x$ and $y$ is most likely to be of the form

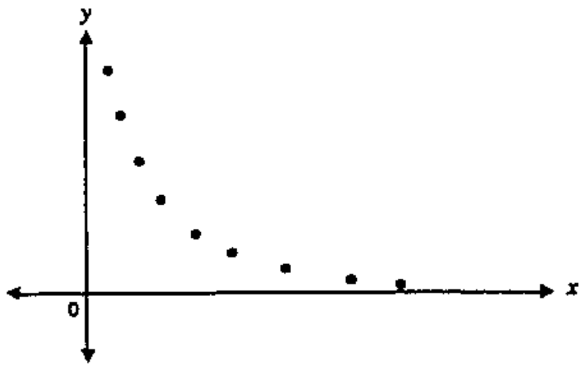
**A.** $y = ax^2$

**B.** $y = \frac{a}{x^2}$

**C.** $y = a\log_{10}(bx)$

**D.** $y = ae^{bx}$

**E.** $y = a\cos(bx)$




As $x \to \infty$, $y \to 0$ and $x \to 0$, $y \to \infty$.

$y = \frac{a}{x^2}$ is the most likely form of the equation relating $x$ and $y$.

**Question 5** **A**

The five graphs shown below are graphs of the relations

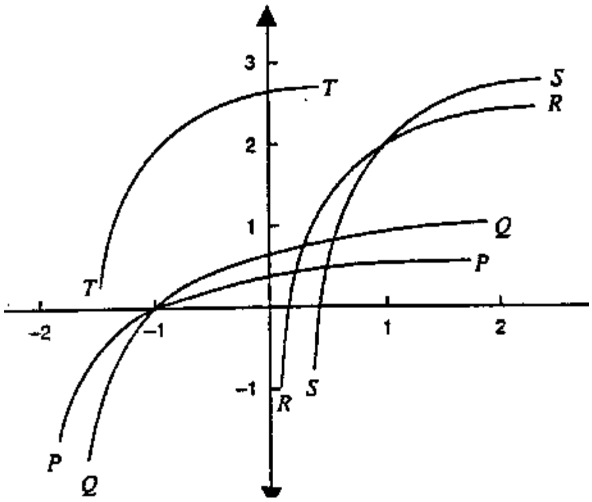
$y = \log_e(x+2)$

$y = \log_e x + 2$

$y = \log_e(x+2) + 2$

$y = \log_{10}(x+2)$

$y = \log_{10} x + 2$




Which one is the graph of the relation $y = \log_{10}(x+2)$?

**A.** *P* **B.** *Q* **C.** *R* **D.** *S* **E.** *T*

For $y = \log_{10}(x+2)$ locate the *x*-intercept.

Let $y = 0$, $0 = \log_{10}(x+2)$

$$10^0 = x + 2$$

$$x = -1$$

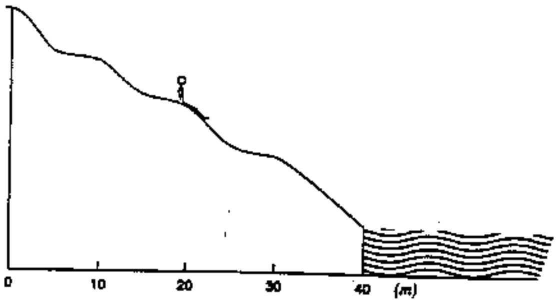
For $y = \log_{10}(x+2)$ locate the *y*-intercept.

Let $x = 0$, $y = \log_{10} 2 \approx 0.30 < 0.5$

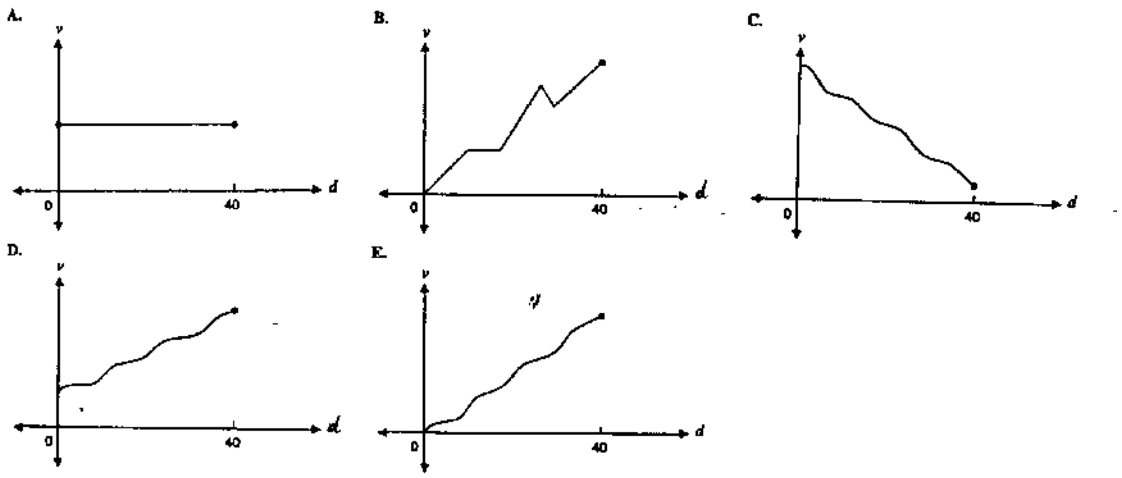
graph *P* represents $y = \log_{10}(x+2)$

**Question 6 E**

At the swimming pool, Vinh goes as fast as he can down a water slide. He starts from rest at the top of the slide.




Which one of the following best represents Vinh's speed ($v$) on the slide as a function of the horizontal distance travelled($d$)?



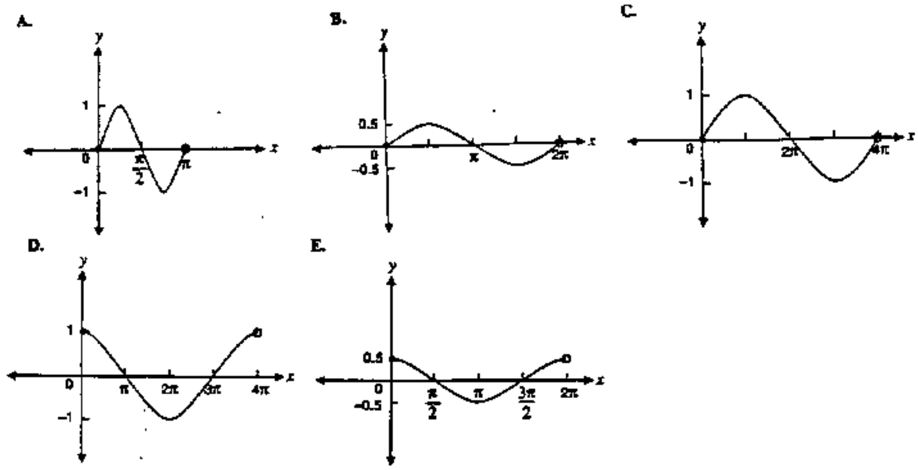

Vinh starts from rest. when $d = 0$, $v = 0$

Speed, represented by the gradient of the curve, will increase but at different rates over the horizontal distance travelled.

graph E represents Vinh's speed on the slide as a function of the horizontal distance.

**Question 7 C**

Which one of the following best represents one cycle of the graph with equation $y = \sin(0.5x)$?




For $y = \sin(0.5x)$:

period $= 2\pi \div 0.5 = 4\pi$

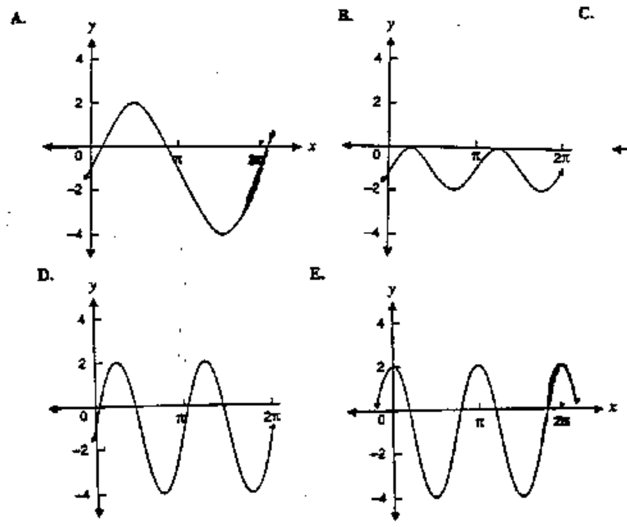
amplitude = 1

$y$-intercept $= \sin 0 = 0$

Graph C represents one cycle of the graph with equation $y = \sin(0.5x)$.

**Question 8 D**

Which one of the following best represents the graph with equation $y = 3\sin(2x) - 1$?




For $y = 3\sin(2x) - 1$:

period $= 2\pi \div 2 = \pi$

amplitude = 3

$y$-intercept $= 3\sin 0 - 1 = -1$

As the graph is translated 1 unit vertically downward range is $[-4, 2]$.

Graph D fits these criteria.

**Question 9 E**

The number of solutions of the equation $\cos(3x) = -0.5$ between $x = 0$ and $x = 2\pi$ is equal to

**A.** 0 **B.** 1 **C.** 2 **D.** 3 **E.** 6

Domain for $x$: $(0,2\pi)$
Domain for $3x$: $(0,6\pi)$
Solutions to the equation $\cos(3x) = -0.5$ are in the 2nd and 3rd quadrants.
Over the domain $(0,6\pi)$ the equation will have six solutions.

**Question 10 E**

A trigonometric function is given by $f : R \to R,\ f(x) = 3\cos\big(2(x-\pi)\big) + 1$.
The amplitude, period and range, respectively, of the function are

| | amplitude | period | range |
|---|---|---|---|
| **A.** | 3 | $\pi$ | $R$ |
| **B.** | 2 | $\frac{2\pi}{3}$ | $[-4,4]$ |
| **C.** | 2 | $\frac{2\pi}{3}$ | $R$ |
| **D.** | $\pi$ | 3 | $[-2,4]$ |
| **E.** | 3 | $\pi$ | $[-2,4]$ |

Amplitude = 3
Period = $2\pi \div 2 = \pi$
Basic shape is translated one unit vertically up, therefore the range is $[-2,4]$.

**Question 11 B**

If $f(x) = a\cos x + c$, where $a$ is a positive real number, then $f(x) < 0$ for all real values of $x$ if

**A.** $c > a$ **B.** $c < -a$ **C.** $c = 0$ **D.** $-a < c < a$ **E.** $c > -a$

Amplitude = $a$
Range = $[(-a+c),(a+c)]$

If $f(x) < 0$ for all real values of $x$, then the maximum value, $(a+c)$, must be negative.
i.e. $a + c < 0$

$c < -a$

**Question 12 C**

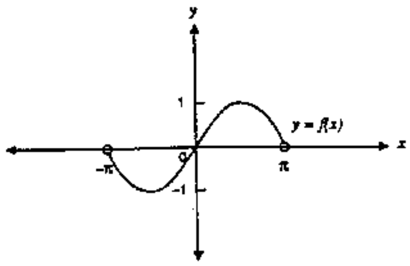
The derivative of $2\sqrt{x}$ is equal to

**A.** $\frac{1}{2\sqrt{x}}$ **B.** $\frac{1}{4\sqrt{x}}$ **C.** $\frac{1}{\sqrt{x}}$ **D.** $\frac{4x^{\frac{3}{2}}}{3}$ **E.** 2

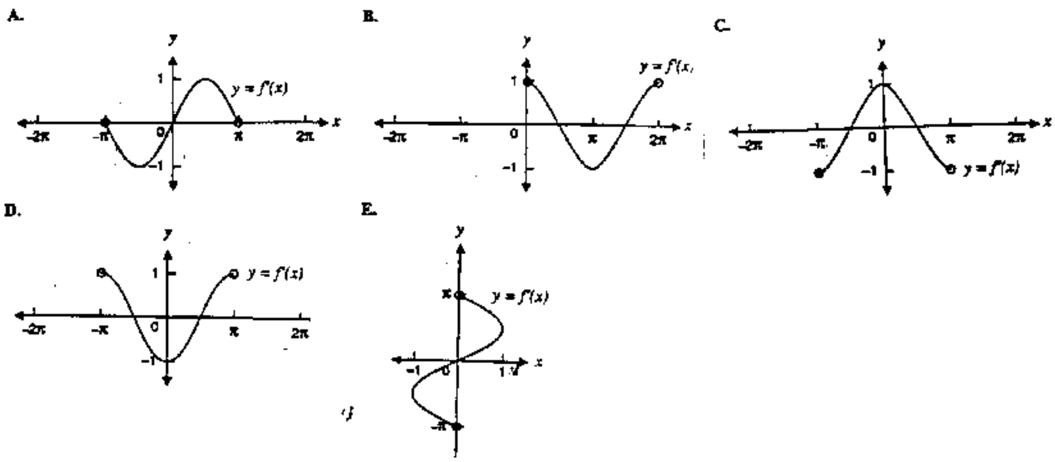
Let $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$ $\frac{dy}{dx} = 2 \times \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

**Question 13 C**

The graph of the function $f:(-\pi,\pi)\to R$ is shown below.




The graph of the derived function $f'$ is most likely to be




| $x$ | $(-\pi,-\frac{\pi}{2})$ | $-\frac{\pi}{2}$ | $(-\frac{\pi}{2},\frac{\pi}{2})$ | $\frac{\pi}{2}$ | $(\frac{\pi}{2},\pi)$ |
|---|---|---|---|---|---|
| $f'(x)$ | $<0$ | 0 | $>0$ | 0 | $<0$ |

Graph C represents the information in this gradient table.

**Question 14 A**

The derivative of $e^{\sin x}$ is equal to

**A.** $(\cos x).e^{\sin x}$ **B.** $e^{\sin x}$ **C.** $e^{\cos x}$ **D.** $(\cos x).e^{\cos x}$ **E.** $(\cos x).e^{x}$

If $y=e^{\sin x}$ and $u=\sin x$ then $y=e^{u}$

$$\frac{du}{dx}=\cos x \text{ and } \frac{dy}{du}=e^{u}=e^{\sin x}$$

Using the Chain Rule: $\frac{dy}{dx}=\cos x.e^{\sin x}$

**Question 15 A**

The gradient of the normal to the curve $y = \log_e x$ at the point where $x = 2$ is equal to

**A.** -2 **B.** $-\frac{1}{\log_e 2}$ **C.** -0.5 **D.** 0.5 **E.** 2

Gradient of tangent $= \frac{dy}{dx} = \frac{1}{x}$

At $x = 2$ the gradient of the tangent $= \frac{1}{2}$. Gradient of normal $= -1 \div \frac{1}{2} = -2$

**Question 16 C**

If $f'(x) = \frac{1}{x^2}$, then $f(x)$ could be

**A.** $\frac{1}{x}$ **B.** $\frac{-2}{x^3}$ **C.** $\frac{-1}{x}$ **D.** $\log_e(x^2)$ **E.** $\log_e x$

$f'(x) = \frac{1}{x^2} = x^{-2}$

$f(x) = -x^{-1} + c = \frac{-1}{x} + c$

$f(x)$ could be $\frac{-1}{x}$

**Question 17 E**

If $y = \frac{\sin x}{e^x}$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

**A.** -1 **B.** 0 **C.** $\frac{1}{e}$ **D.** $\frac{1}{2}$ **E.** 1

Using the Quotient Rule, $\frac{dy}{dx} = \frac{e^x(\cos x) - (\sin x)e^x}{(e^x)^2} = \frac{e^x(\cos x - \sin x)}{e^{2x}}$

At $x = 0$, $\frac{dy}{dx} = \frac{e^0(\cos 0 - \sin 0)}{e^0} = \frac{1(1-0)}{1} = 1$

**Question 18 B**

An antiderivative of $\frac{1}{(2x+5)^4}$ is

**A.** $4\log_e(2x+5)$ **B.** $\frac{-1}{6(2x+5)^3}$ **C.** $\frac{-6}{(2x+5)}$ **D.** $\frac{-1}{3(2x+5)^3}$ **E.** $\frac{10}{(2x+5)^5}$

$$\int \frac{1}{(2x+5)^4}\,dx = \int (2x+5)^{-4}\,dx$$

$$= \frac{(2x+5)^{-3}}{-3\times 2} + c$$

$$= \frac{1}{-6(2x+5)^3} + c$$

An antiderivative of $\frac{1}{(2x+5)^4}$ is $\frac{-1}{6(2x+5)^3}$.

**Question 19 A**

An antiderivative of $e^{3x} + \sin(3x)$ is

**A.** $\frac{e^{3x}}{3} - \frac{\cos(3x)}{3}$ **B.** $3e^{3x} + 3\cos(3x)$ **C.** $e^{3x} - \frac{\cos(3x)}{3}$

**D.** $\frac{e^{3x}}{3} - \cos(3x)$ **E.** $e^{3x} - \cos(3x)$

$\int e^{3x} + \sin(3x)\, dx = \frac{1}{3}e^{3x} - \frac{1}{3}\cos(3x) + c$

An antiderivative of $e^{3x} + \sin(3x)$ is $\frac{e^{3x}}{3} - \frac{\cos(3x)}{3}$.

**Question 20 E**

The total area of the shaded rectangles can be used as an approximation for the area between the curve with equation $y = \frac{1}{x}$, the $x$-axis, and the lines with equations $x = 1$ and $x = 5$. The value of this approximation is equal to

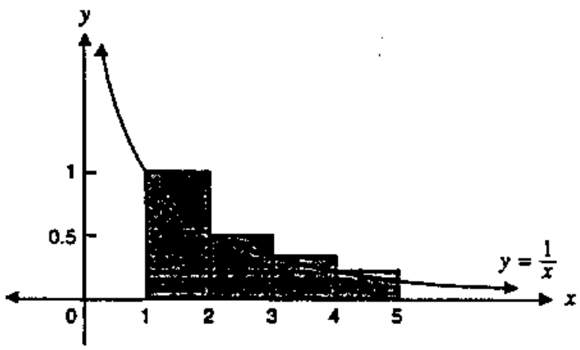
**A.** 1

**B.** 1.61

**C.** 2

**D.** 2.08

**E.** $2\frac{1}{12}$



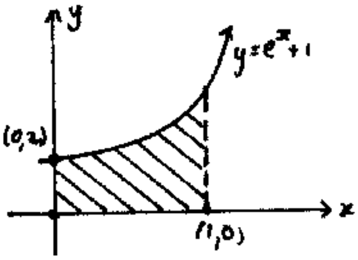

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y = \frac{1}{x}$ | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ |

Area shaded $= 1 \times 1 + 1 \times \frac{1}{2} + 1 \times \frac{1}{3} + 1 \times \frac{1}{4} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{12+6+4+3}{12} = \frac{25}{12} = 2\frac{1}{12}$ square units

**Question 21 C**

The area between the curve $y = e^x + 1$, the $x$-axis and the lines with equations $x = 0$ and $x = 1$ is equal to

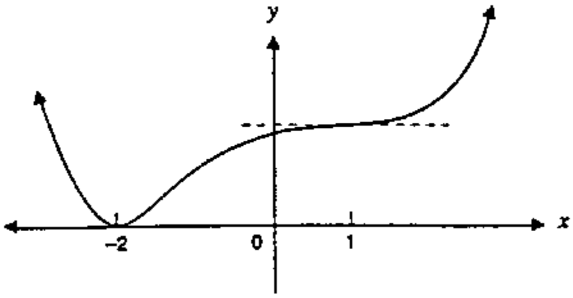
**A.** $e - 1$ **B.** $1$ **C.** $e$ **D.** $e + 1$ **E.** $e + 2$




$$
\begin{aligned}
\text{Area required} &= \int_0^1 (e^x + 1)\,dx \\
&= \left[e^x + x\right]_0^1 \\
&= e^1 + 1 - (e^0 + 0) \\
&= e + 1 - 1 \\
&= e
\end{aligned}
$$

**Question 22 D**

The graph shown is of the function with rule $y = (x+2)^2(x^2 - 4x + 6)$




Which one of the following is **not** true?

**A.** The gradient of the tangent to the graph at $x = 1$ is zero.
**B.** $\frac{dy}{dx} = 0$ when $x = 1$ and when $x = -2$ and at no other point.
**C.** There is only one turning point on the graph.
**D.** There is only one stationary point on the graph.
**E.** $y \geq 0$ for all values of $x$.

The graph has two stationary points, a minimum turning point at $x = -2$ and a stationary point of inflection at $x = 1$.

Therefore, statement D is not true.

For stationary points let $\frac{dy}{dx} = 0$

$$
\begin{aligned}
2(x+2)(x^2 - 4x + 6) + (2x - 4)(x+2)^2 &= 0 \\
(x+2)\left[2x^2 - 8x + 12 + 2(x-2)(x+2)\right] &= 0 \\
(x+2)(4x^2 - 8x + 4) &= 0 \\
4(x+2)(x-1)^2 &= 0 \\
x = -2 \text{ or } x &= 1
\end{aligned}
$$

**Question 23 C**

$\log_e(x^2)+2\log_e x+\log_e(4x)$ is equal to

**A.** $\log_e(2x^2+4x)$ **B.** $\log_e(x^2+6x)$ **C.** $\log_e(4x^5)$
**D.** $5\log_e(4x)$ **E.** $8\log_e x$

$$\log_e(x^2)+2\log_e x+\log_e(4x) = \log_e x^2+\log_e x^2+\log_e 4x$$
$$= \log_e(x^2\times x^2\times 4x)$$
$$= \log_e(4x^5)$$

**Question 24 D**

Let $a$ be the coefficient of $x^2$ and $b$ be the coefficient of $x$ in the expansion of the polynomial $(2x-1)^4$. Then

**A.** $a=b$ **B.** $a=3b$ **C.** $a=4b$ **D.** $a=-3b$ **E.** $a=-2b$

$$(2x-1)^4 = (2x)^4-4(2x)^3(1)+6(2x)^2(1)^2-4(2x)(1)^3+(1)^4$$
$$= 16x^4-32x^3+24x^2-8x+1$$

Coefficient of $x^2 = 24 = a$, coefficient of $x=-8=b$

$a=-3b$

**Question 25 E**

The inverse of the function $f:[0,\infty)\to R,\ f(x)=2\log_e(x+1)+1$ is

**A.** $f^{-1}:[0,\infty)\to R,\ f^{-1}(x)=2\log_e(x+1)+1$ **B.** $f^{-1}:[0,\infty)\to R,\ f^{-1}(x)=e^{\frac{x-1}{2}}-1$

**C.** $f^{-1}:R\to R,\ f^{-1}(x)=e^{\frac{x-1}{2}}-1$ **D.** $f^{-1}:[1,\infty)\to R,\ f^{-1}(x)=2\log_e(x+1)+1$

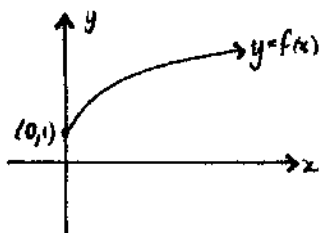
**E.** $f^{-1}:[1,\infty)\to R,\ f^{-1}(x)=e^{\frac{x-1}{2}}-1$

Let $y=2\log_e(x+1)+1$

Interchanging $x$ and $y$ for the inverse gives:

$$x=2\log_e(y+1)+1$$
$$\frac{x-1}{2}=\log_e(y+1)$$
$$e^{\frac{x-1}{2}}=y+1$$
$$y=e^{\frac{x-1}{2}}-1$$

When $x=0$, $f(0)=2\log_e 1+1=1$.




Domain of $f^{-1}$ = range of $f=[1,\infty]$

$f^{-1}:[1,\infty]\to R,\ \ f^{-1}(x)=e^{\frac{x-1}{2}}-1$

**Question 26 C**

Which one of the following random variables is **not** discrete?

**A.** The number of runs scored by a team in a one-day cricket match.
**B.** The time, to the nearest minute, that it takes a student to walk to school.
**C.** The volume of milk consumed by a family in one week.
**D.** The number of customers served in one day at a milk bar.
**E.** The number of baseball caps owned by a student.

The volume of milk consumed by a family in one week is a continuous variable and therefore is not discrete.

*The following information relates to questions 27 and 28.*

Over a twenty-five day period Police Sergeant Bob Cryer kept a record of the number of motorists per day that PC booked for speeding. His results are given in the table below.

| Number of motorists booked ($x$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Number of days on which $x$ motorists are booked ($f$) | 2 | 0 | 4 | 3 | 2 | 6 | 8 |

**Question 27 B**

During the twenty-five day period, the proportion of days on which PC Data booked fewer than 4 motorists for speeding is equal to

**A.** $\frac{2}{25}$ **B.** $\frac{9}{25}$ **C.** $\frac{11}{25}$ **D.** $\frac{14}{25}$ **E.** $\frac{16}{25}$

proportion of days where fewer than 4 motor cars booked $= \frac{2+0+4+3}{25} = \frac{9}{25}$

**Question 28 C**

During the twenty-five day period, the mean number of motorists booked per day for speeding by PC Data is equal to

**A.** 3.5 **B.** 4 **C.** 4.12 **D.** 5 **E.** 6

mean $= \frac{\sum xf}{\sum f} = \frac{0+0+8+9+8+30+48}{25} = \frac{103}{25} = 4.12$

**Question 29 D**

Angie notes that 2 out of 10 peaches on her peach tree are spoilt by birds pecking at them. If she randomly picks 30 peaches the probability that exactly 10 of them are spoilt is equal to

**A.** 0.2 **B.** $(0.2)^{10}(0.8)^{20}$ **C.** $(0.2)^{20}(0.8)^{10}$

**D.** ${}^{30}C_{10}(0.2)^{10}(0.8)^{20}$ **E.** ${}^{30}C_{20}(0.2)^{20}(0.8)^{10}$

Let $X$ denote the number of peaches which are spoilt

$X$ is Binomial with $n = 30$ and $p = \frac{2}{10} = 0.2$

$\Pr(X = 10) = {}^{30}C_{10}(0.2)^{10}(0.8)^{20}$

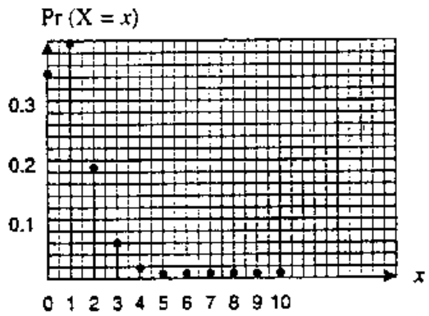
**Question 30 D**
A random variable X has the probability distribution as shown in the table below.

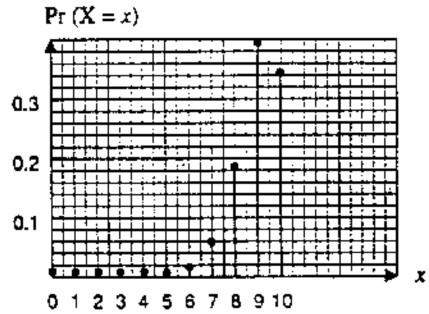
| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Pr(X = $x$) | 0.2 | 0.4 | 0.3 | 0.1 |

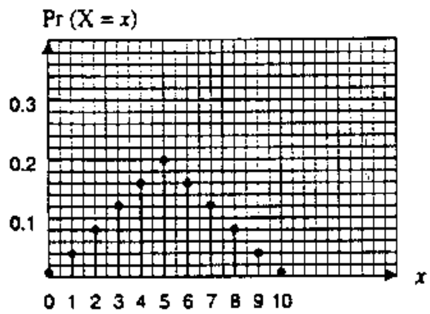
The expected value is equal to
**A.** 0 **B.** 0.25 **C.** 1.0 **D.** 1.3 **E.** 1.5

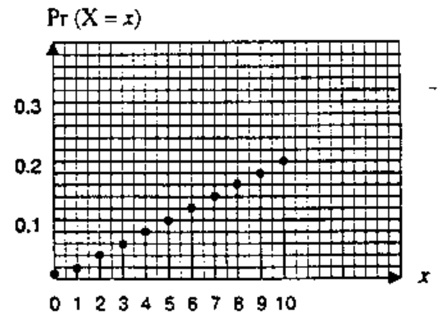
$$E(X) = \sum x.\Pr(X = x) = 0 + 0.4 + 0.6 + 0.3 = 1.3$$

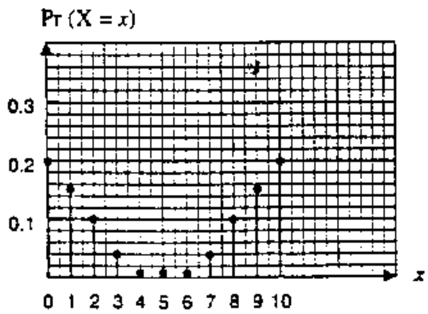
**Question 31 A**
Which one of the following graphs best represents the shape of a binomial distribution of the random variable X with 10 independent trials and probability of success for each trial equal to 0.1?

**A.**



**B.**



**C.**



**D.**



**E.**



If $p = 0.1$ then the shape will be skewed to the right. Therefore graph A is correct.

*The following information refers t questions 32 and 33*

A large group of students is given a fitness test rated on a scale from 0.0 ('very unfit') to 50.0 ('extremely fit').

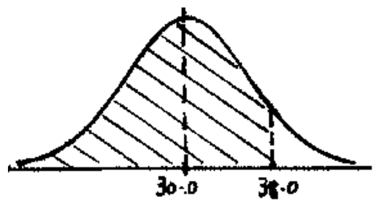
The test results follow a normal distribution with mean 30.0 and standard deviation 7.0.

**Question 32 D**

If a student has a score of 38 what proportion of the population will be less fit than this student?

**A.** 0.1265 **B.** 0.2800 **C.** 0.7200 **D.** 0.8735 **E.** 1.1430

$$\begin{aligned}\Pr(X < 38) &= \Pr\left(Z < \tfrac{38-30}{7}\right)\\ &= \Pr(Z < 1.143)\\ &= 0.8735\end{aligned}$$

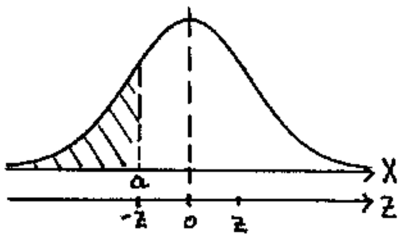



**Question 33 D**

From the test, the lower 40 per cent of the population are defined to be unfit. What is the minimum score for a fit person?

**A.** 20.0 **B.** 23.5 **C.** 25.9 **D.** 28.2 **E.** 31.8

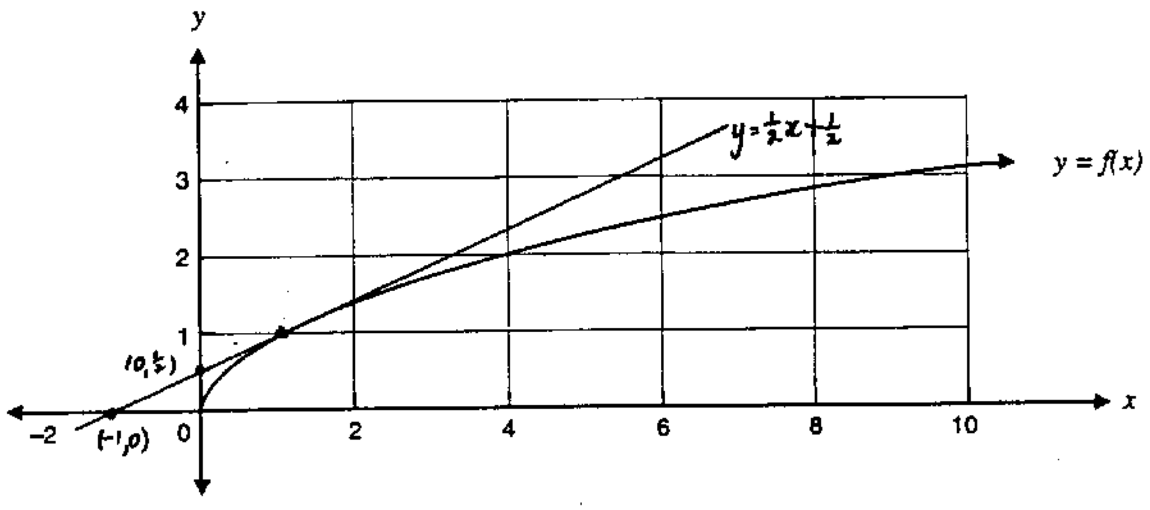
$$\begin{aligned}\Pr(X < a) &= 0.4\\ \Pr(Z < -z) &= 0.4\\ \Pr(Z > z) &= 0.4\\ \Pr(Z < z) &= 0.6\\ z &= 0.253\\ \tfrac{a-30}{7} &= -0.253\\ a &= 28.2\end{aligned}$$



## PART II: Question and Answer Booklet

**Question 1**

The graph of the function $f:[0,\infty) \to R,\ f(x) = \sqrt{x}$ is shown below.




**i.** Find the equation of the tangent to the graph of $f(x)$ at $x = 1$.

$y = \sqrt{x} = x^{\frac{1}{2}}$

Gradient of tangent = $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

At $x = 1$, $\frac{dy}{dx} = \frac{1}{2\sqrt{1}} = \frac{1}{2}$

Let equation of tangent be $y = \frac{1}{2}x + c$

When $x = 1$, $y = \sqrt{1} = 1$

sub (1,1) in $y = \frac{1}{2}x + c$

$1 = \frac{1}{2} + c$

$c = \frac{1}{2}$ and $y = \frac{1}{2}x + \frac{1}{2}$

**ii.** Sketch the tangent on the axes above and clearly **label** its point of intersection with the axes.

For $x$-intercept let $y = 0$

$0 = \frac{1}{2}x + \frac{1}{2}$

$-\frac{1}{2} = \frac{1}{2}x$

$x = -1$

For $y$-intercept let $x = 0$

$y = \frac{1}{2}(0) + \frac{1}{2} = \frac{1}{2}$

See diagram above for the sketch of the tangent.

**Question 2**

**i.** On the set of axes below, sketch the graph with equation $y = 2e^x - 4$

Graph translated vertically down 4 units, therefore horizontal asymptote is $y = -4$

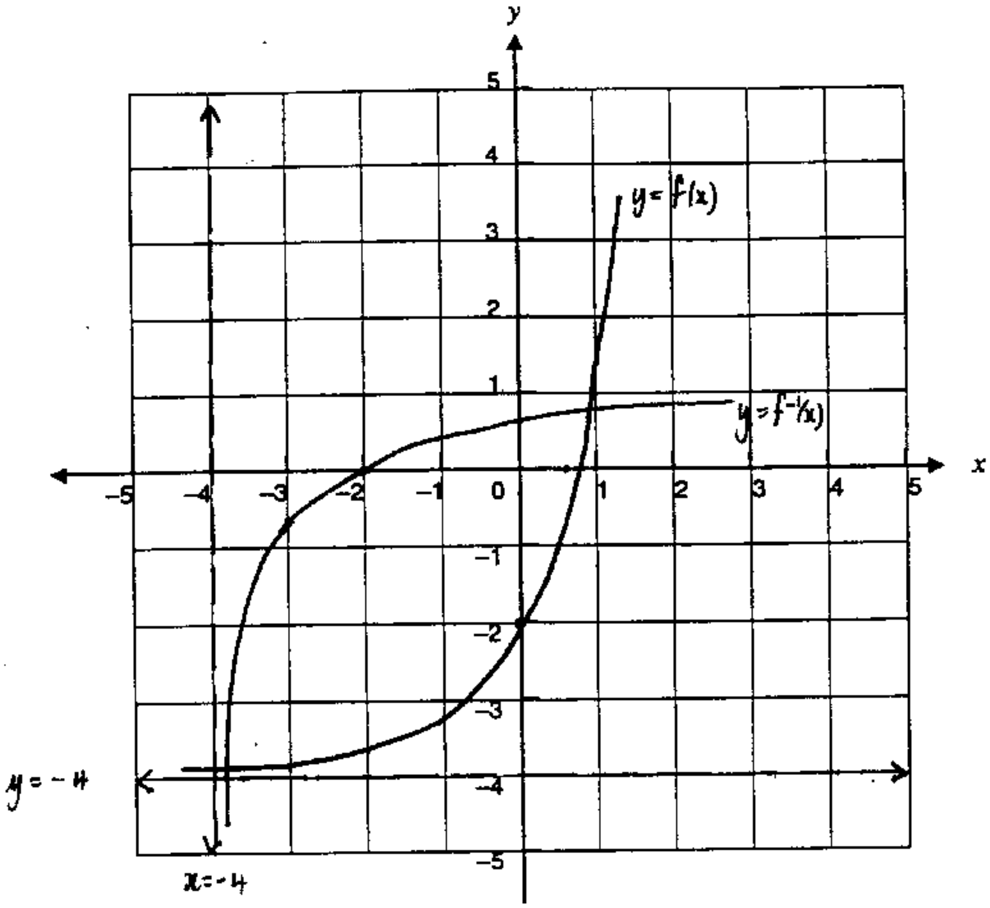
For $x-$intercept, let $y = 0$ : $0 = 2e^x - 4$

$e^x = 2$

$x = \log_e 2 \approx 0.69$

For $y$-intercept let $x = 0$

$y = 2e^0 - 4 = -2$




**ii.** Find the **rule** for the inverse of the function $f : R \to R,\ f(x) = 2e^x - 4$

Let $y = 2e^x - 4$ and interchange $x$ and $y$ for inverse

$x = 2e^y - 4$

$\frac{x+4}{2} = e^y$

$y = \log_e\left(\frac{x+4}{2}\right)$

Domain of $f^{-1}$ = range of $f = (-4, \infty)$

the rule for the inverse function is $f^{-1} : (-4, \infty) \to R,\ f^{-1}(x) = \log_e\left(\frac{x+4}{2}\right)$

**iii.** On the same set of axes, sketch and clearly **label** the graph of the inverse of $f$.

Interchanging $x$ and $y$ gives:

$x$-intercept at $(-2,0)$ , $y$-intercept at $(0, \log_e 2)$ and vertical asymptote through $x = -4$

See diagram above for the sketch of the inverse of $f$.

### Question 3

The graph of $y = \cos x$ is transformed into the graph of $y = 2\cos x$ by a **dilation in the $y$-direction by a scale factor of 2**. In each case below state the type of transformation together with any relevant scale factors, distances and directions required to transform the graph of the first equation into the graph of the second equation.

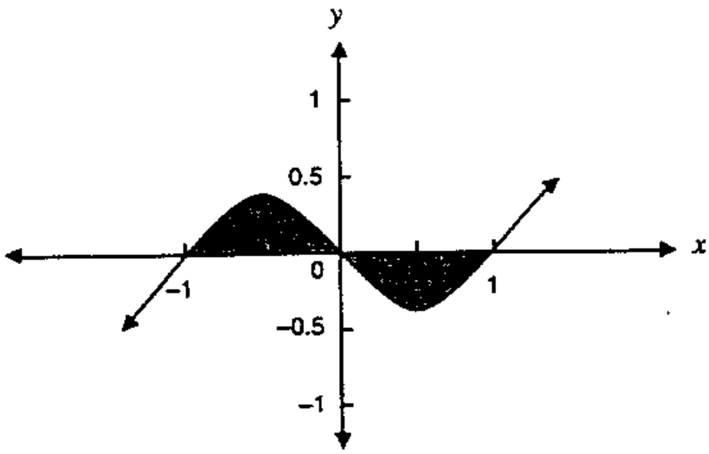
**i.** $y = 2\cos x$ to $y = 2\cos(0.5x)$
dilation in the $x$-direction by a scale factor of 0.5

**ii.** $y = 2\cos(0.5x)$ to $y = 2\cos\left(0.5(x - \frac{\pi}{4})\right)$
translation of $\frac{\pi}{4}$ units in the positive $x$-direction

**iii.** $y = 2\cos\left(0.5(x - \frac{\pi}{4})\right)$ to $y = 2\cos\left(0.5(x - \frac{\pi}{4})\right) + 2$
translation of 2 units in the positive $y$-direction

### Question 4

The graph with equation $y = x(x+1)(x-1)$ is shown below.




Find the exact value of the shaded region.

$y = x(x+1)(x-1) = x(x^2-1) = x^3 - x$

$$\text{Area shaded} = \int_{-1}^{0} (x^3 - x)\,dx + \left|\int_0^1 (x^3 - x)\,dx\right| = 2\int_{-1}^{0} (x^3 - x)\,dx$$

$$= 2\left[\tfrac{1}{4}x^4 - \tfrac{1}{2}x^2\right]_{-1}^{0}$$

$$= 2\left(0 - (\tfrac{1}{4}(-1)^4 - \tfrac{1}{2}(-1)^2\right)$$

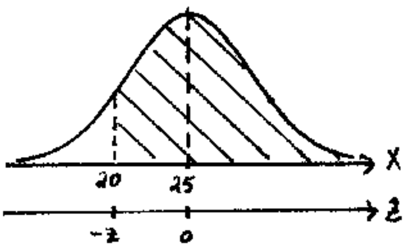
$$= 2 \times \tfrac{1}{4}$$

$$= \tfrac{1}{2} \text{ square unit}$$

## Question 5

Stanley rides a bicycle to work each day. it may be assumed that the journey time is a normally distributed random variable with a mean of 25 minutes. Calculate the standard deviation of Stanley's journey time , if the journey exceeds 20 minutes on 90 per cent of occasions.

Let X Stanley's journey time




$$\Pr(X > 20) = 0.9$$
$$\Pr(Z > -z) = 0.9$$
$$\Pr(Z < z) = 0.9$$
$$z = 1.282$$
$$\frac{20-25}{\sigma} = -1.282$$
$$\sigma = \frac{-5}{-1.282} = 3.9 \quad \text{(1 decimal place)}$$

The standard deviation of Stanley's journey time is 3.9 minutes.

## Question 6

In this question $p$ represents the population proportion and $\hat{p}$ represents a sample proportion.
A random sample of 100 people is selected from the population of a country. Of this sample 30 people believed that their president was doing an excellent job.

**a.** Find approximate 95 per cent confidence interval estimate for the proportion of the population, $p$, who believed that their president was doing an excellent job.

$$\hat{p} = \frac{30}{100} = 0.3$$

$$\text{se}(\hat{p}) = \sqrt{\frac{0.3 \times 0.7}{100}} = \sqrt{0.0021}$$

95% confidence limit: lower limit $= \hat{p} - 2\text{se}(\hat{p}) = 0.3 - 2 \times \sqrt{0.0021} = 0.21$

upper limit $= \hat{p} + 2\text{se}(\hat{p}) = 0.3 + 2 \times \sqrt{0.0021} = 0.39$

95% confidence interval correct to 2 decimal places is $(0.21, 0.39)$

**b.** What is the minimum sample size so that the standard error of the sample proportion, $\hat{p}$, is less than 0.01?

$$\sqrt{\frac{0.3 \times 0.7}{n}} < 0.01$$
$$\frac{0.21}{n} < 0.0001$$
$$\frac{0.21}{0.0001} < n$$
$$n > 2100$$

Since $n$ must be a positive whole number, minimum sample size is 2101.

**END OF SUGGESTED SOLUTIONS**


**1995 VCE MATHEMATICAL METHODS CAT 2**

**CHEMISTRY ASSOCIATES**
**P.O. BOX 2227**
**KEW, VIC., 3101**
**AUSTRALIA**
**TEL:(03) 9817 5374 FAX: (03) 9817 4334**