Specific instructions to students

Answer **all** questions in the spaces provided.

Question 1

In a biochemical process used to manufacture a drug, an enzyme (E) is used to make the drug (D).

During the process, the concentration of the enzyme decreases with time. The concentration of the enzyme is modelled by the function:

$$E(t) = 27.e^{-0.26t} \quad 0 \leq t \leq 12$$

t is measured in hours and E in grams per litre.

i. Find the concentration of the enzyme at the start of the process.

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[2 marks]

ii. Find the concentration of the enzyme half way through the process.

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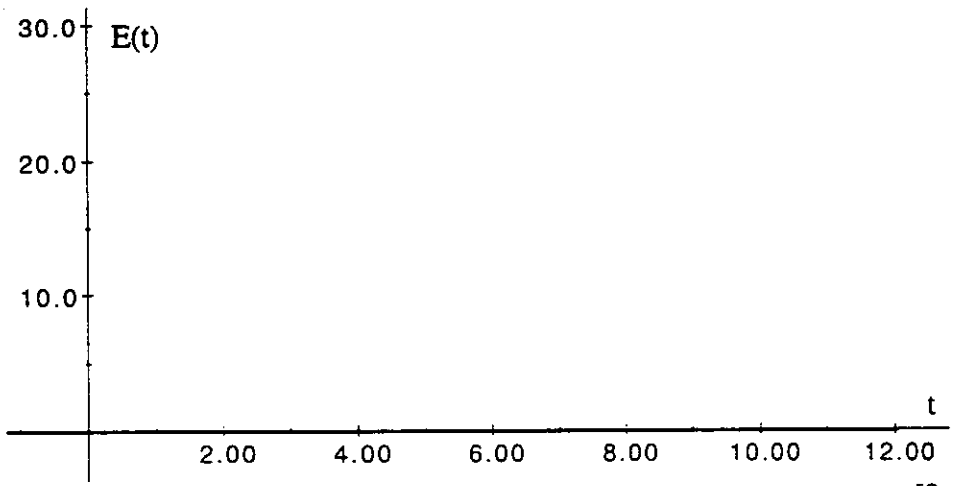
[2 marks]

iii. Sketch the graph of E(t) against t



[3 marks]

iv. Find, to the nearest minute, the time taken for the concentration of the enzyme to fall to half it original value.

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[3 marks]

The **rate** at which the drug is produced by the enzyme is proportional to the **concentration** of the **enzyme**. D(t) is the concentration (gm/litre) at time t (hours) of the drug and the initial concentration of the drug is 0 gm/litre.

The constant of proportionality is $\frac{13}{45}$

v. Show that the rate of change of concentration of the drug with time, D'(t) is:

$$D'(t) = 7.8e^{-0.26t} \quad 0 \leq t \leq 12$$

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[2 marks]

vi. Find the rule, D(t), that gives the concentration of the drug in terms of time and sketch its graph.

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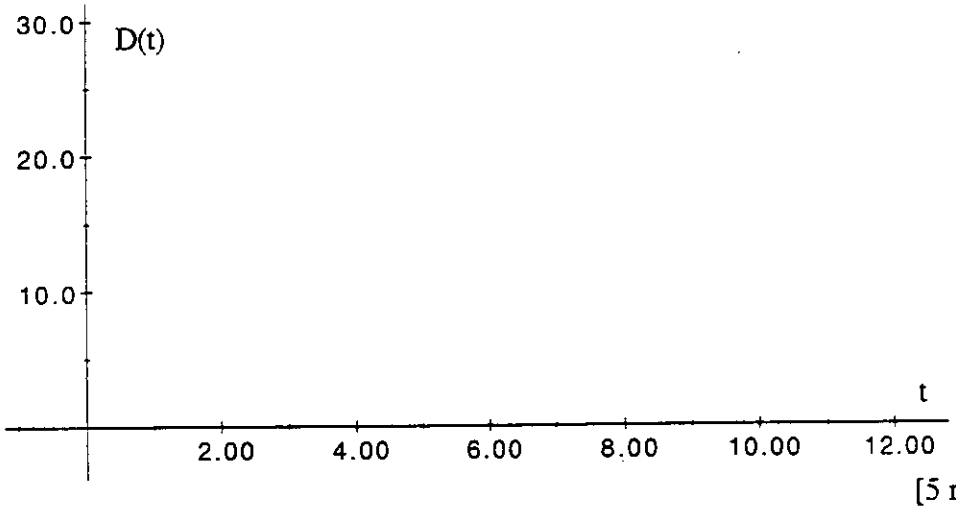

[5 marks]

vii. Find, correct to the nearest minute, the time at which the concentrations of the enzyme and the drug are equal.

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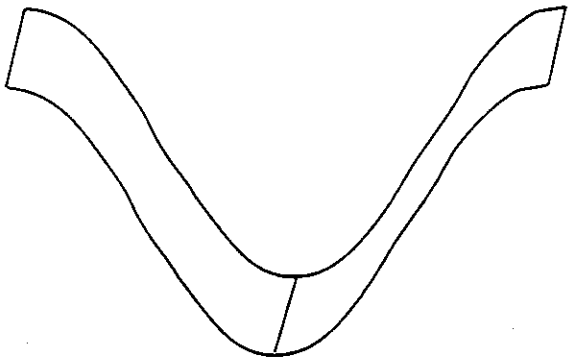
[3 marks]

Question 2

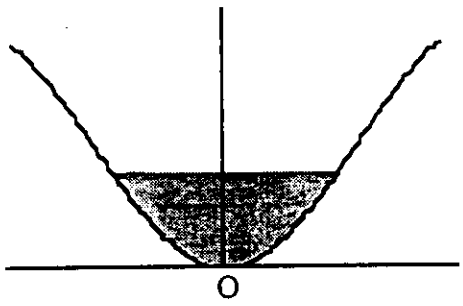
The cross-section of a storm drain is in the shape of a trigonometric curve



If an origin is placed at the base of the cross-section at point O, then the equation of the drain is:

$$h(x) = 1 - \cos\left(\frac{\pi x}{4}\right), \; -4 \leq x \leq 4$$

With the drain in operation, it fills with water to a depth appropriate to the amount of water flowing into the drain.



i. Find the maximum width and depth of the drain.

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[3 marks]

ii. After a fall of rain, the width of the surface of the water in the drain is 2 metres, find the maximum depth of the water, correct to 2 decimal places.

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[2 marks]

iii. If the depth of the water is 1.2 metres, find the width of the water surface in metres correct to two decimal places.

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[3 marks]

iv. After a rainfall, the width of the surface of the water in the drain is 3 metres. Find the area of cross-section of the water in square metres, correct to two decimal places.

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[7 marks]

Question 3
(requires a normal distribution table)

Multiple Manufacturing Pty. Ltd. produce a bagged organic fertiliser. The bags are filled by a machine that, overall, fills the bags with a mean contents (μ) of 7.1 kilos of fertiliser. The standard deviation (σ) is 0.12 kilos.

Bags containing 6.9 kg. or less are rejected as being underweight.

a. i. Assuming that the weights in the bags are normally distributed what percentage of the bags are rejected. Answer in percent, to the nearest whole number.

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[5 marks]

The bags are shipped in crates, each of which contains ten bags. The crates are filled with bags straight from the production line with no preliminary testing. Multiple Manufacturing can produce each crate for $55 and sell it for $75. If a crate is found to contain more than one underweight bag, Multiple Manufacturing will refund the purchase price to the customer, with the crate remaining the property of the customer.

ii. Find the probability that a crate will contain more than one underweight bag.

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[5 marks]

iii. Find the expected profit per crate.

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[2 marks]

Multiple Manufacturing are considering buying a new bagging machine. In a trial, they found that the weights (kilos) filled into ten test bags were:

7.0 7.2 7.0 7.0 7.1 7.1 7.0 6.8 7.0 7.1

b. i. Find approximate 95% confidence limits for the proportion of bags filled by the new machine that the company can expect to be underweight if they buy the new machine.

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[3 marks]

ii. On the basis of the above calculations, would you advise the company to buy and operate the new machine ? Explain your answer.

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[1 mark]

Question 4

The temperature, F°C, in an industrial furnace t hours after the power has been switched on is modelled by the function:

$$F(t) = \frac{7(0.9t + 3)^4}{15}, t \geq 0$$

The operating temperature of the furnace is 300°C.

a. i. Find the time that the furnace takes to reach its operating temperature. Express your answer in hours and minutes, correct to the nearest minute.

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[3 marks]

ii. Find the average rate of change of temperature expressed in °C per hour over the first half hour after the power is turned on. Express your answer to the nearest whole number.

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[3 marks]

iii. Find the rate at which the temperature is changing after one and a half hours in °C per hour. Express your answer to the nearest whole number.

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[4 marks]

Instead of reaching its operating temperature, there is a power cut one hour after the furnace was switched on. An emergency power system takes over after the power cut so that the temperature of the furnace is modelled by the function:

$$F^*(t) = 15t^2 - 45t + k \quad t > 1$$

b. i. Find the value of k such that the functions $F(t)$ $0 \leq t \leq 1$ & $F^*(t)$ $t > 1$ form a continuous graph. Give your answer to the nearest whole number.

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[3 marks]

ii. Find the derivative of the function F^*, $F^{*\prime}(t)$.

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[1 mark]

iii. Find the coordinates of the stationary point(s) on the graph of F*(t).

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[3 marks]

iv. Use your results to describe **briefly** what happens to the temperature after the power cut.

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[2 marks]