M1 indicates a method mark awarded for use of a correct method

A1 indicates an answer mark for correct answer in the correct format.

Question 1

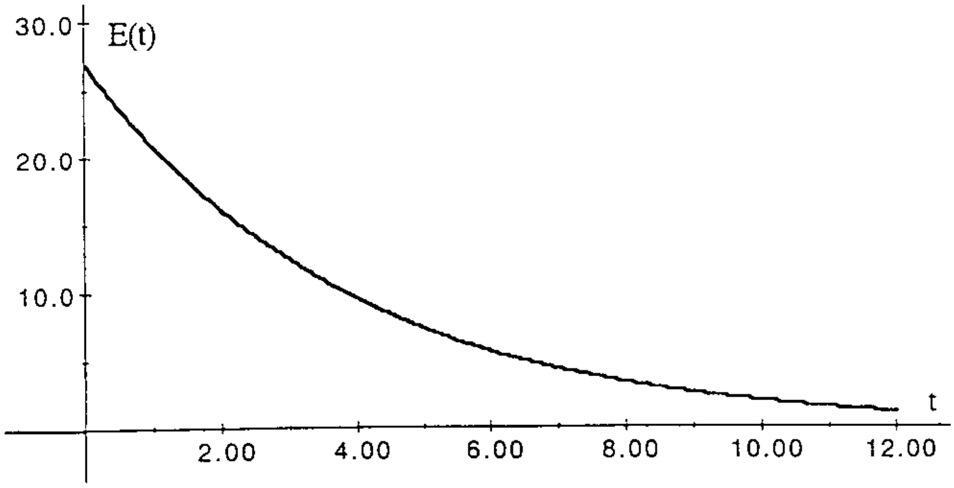
i. Use of $t = 0$ in $E(t) = 27.e^{-0.26t}$ M1

Initial concentration = 27 gms/litre A1

ii. Realising that the process lasts 12 hours ($0 \le t \le 12$) and that $t = 6$ is the correct time. M1

$E(6) \approx 5.67$ gms/litre (no accuracy specified) A1

iii.



Negative exponential shape A1

Correct t=0 intercept taken from their answer to (i) A1

correct domain $0 \le t \le 12$ A1

iv. It is necessary to solve $27.e^{-0.26t} = \frac{27}{2}$ or $e^{-0.26t} = 0.5$ M1

Use of log as the inverse function of exp. M1

to get $-0.26t = \log_e 0.5$ which has the solution $t \approx 2.666$ or 2 hours 40 minutes A1

v. $D'(t) = \frac{13}{45}E(t) \qquad 0 \le t \le 12$ M1

$= \frac{13}{45} \times 27.e^{-0.26t} \qquad 0 \le t \le 12$

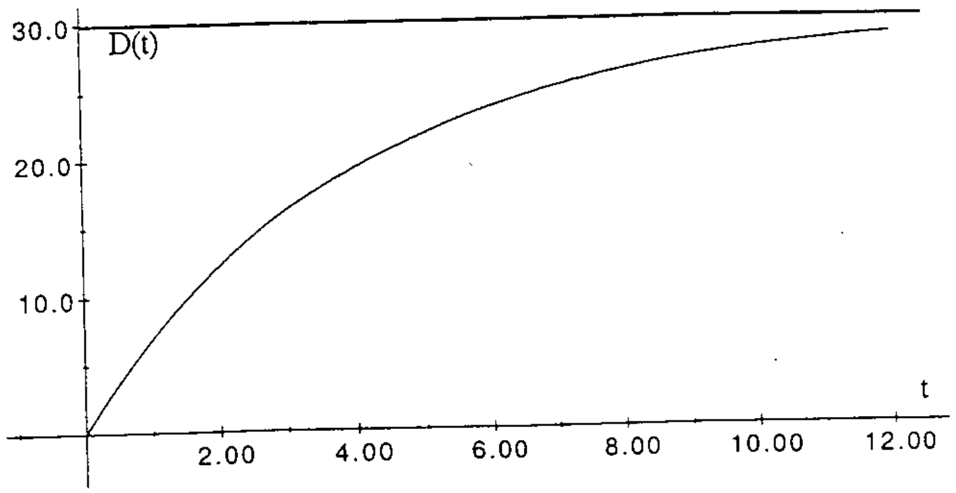
$= 7.8 \times e^{-0.26t} \quad 0 \le t \le 12$ A1

vi. $D'(t) = 7.8 \times e^{-0.26t}$ Use of antidifferentiation M1

to get

$$D(t) = \frac{7.8}{-0.26} \times e^{-0.26t} + c = -30 \times e^{-0.26t} + c$$ A1

use of the condition t=0 D=0 to get c = 30 and hence $D(t) = -30 \times e^{-0.26t} + 30$ A1



Shape (ie. bending in the correct manner) A1

Correct value at t = 0 and approximately correct value at t = 12 A1

vii. $$-30 \times e^{-0.26t} + 30 = 27.e^{-0.26t}$$ M1

$$30 = 57.e^{-0.26t}$$

$$e^{-0.26t} = \frac{30}{57}$$

$$t = \frac{1}{-0.26} \log_e\left(\frac{30}{57}\right)$$

Sensible attempt to solve, including the use of logarithms M1

$$t \approx 2.46866 \text{ hours}$$

2 hours 28 mins is the required time in the required format. A1

Question 2

i. The width is given by the domain statement of the function $-4 \le x \le 4$ and is 8 metres.

A1

The maximum depth needs care as it is not sufficient to say that the maximum depth is 2 metres just by looking at the rule for the function. It is necessary to observe that the period is 8 metres (or other appropriate method).

M1

before: arriving at the conclusion that the maximum depth is 2 metres.

A1

ii. $h(x) = 1 - \cos\left(\frac{\pi x}{4}\right) \quad -4 \le x \le 4$

If the width of the water surface is 2 metres, the appropriate value of x to use is 1.

M1

$$h(1) = 1 - \cos\left(\frac{\pi}{4}\right) \approx 0.29 \text{ m} = 29\text{cm}.$$

$\therefore$ Depth at this point is 29cm (0.29m).

A1

iii. $h(x) = 1 - \cos\left(\frac{\pi x}{4}\right) = 1.2$

M1

$$1 - \cos\left(\frac{\pi x}{4}\right) = 1.2$$

$$\cos\left(\frac{\pi x}{4}\right) = -0.2$$

$$\frac{\pi x}{4} \approx 1.7722$$

Taking reasonable steps to solve the trigonometric equations (must be using radians).

M1

x = 4.51 metres (correct to 2 d.p. as required)

A1

iv. The area **under** the curve is: $\int_{-1.5}^{1.5}\left(1 - \cos\left(\frac{\pi x}{4}\right)\right)dx$

Use of definite integration to find an area using the correct integrand.

M1

Correct terminals.

A1

$$\int_{-1.5}^{1.5}\left(1 - \cos\left(\frac{\pi x}{4}\right)\right)dx = \left[x - \frac{4}{\pi}\sin\left(\frac{\pi x}{4}\right)\right]_{-1.5}^{1.5}$$

Correct anti-derivative

A1

Substitution of terminals

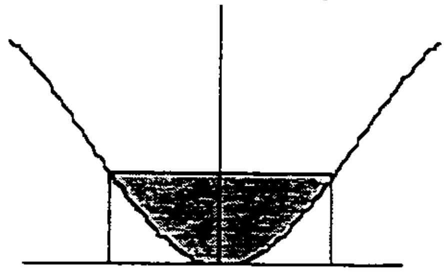
M1

$$= \left(1.5 - \frac{4}{\pi}\sin\left(\frac{1.5\pi}{4}\right)\right) - \left(-1.5 - \frac{4}{\pi}\sin\left(\frac{-1.5\pi}{4}\right)\right)$$

$$\approx 0.64736$$

A1

The required area is area of enclosing rectangle - area under curve



The width of this rectangle is 3 metres (given) and the height is $h(1.5) \approx 0.6173$ A1

Required area of cross-section is $3 \times 0.6173 - 0.64736 = 1.20\ m^2$ to 2 dec. pl. A1

Question 3

a. i. $\mu = 7.1$ and $\sigma = 0.12$. Let x be the weight of the bags and z be the corresponding weight appropriate to using the standard normal curve.

Using $z = \dfrac{x - \mu}{\sigma}$ M1

Use of $x = 6.9$ M1

To get the z value $-1\frac{2}{3}$ which will need to be expressed as the rounded decimal 1.667 A1

before using the tables. It is also necessary to argue that:

$$\Pr(x<6.9) = \Pr(z<-1.667) = \Pr(z>1.667) = 1 - \Pr(z<1.667)$$ M1

$$= 1 - 0.9522$$
$$= 0.0478$$

which means that 5% (to the nearest whole number) of the bags are underfilled. A1

ii. If Y = the number of defective bags in the crate then

$$\Pr(\text{crate rejected}) = \Pr(Y=0) + \Pr(Y=1)$$ M1

$$= \binom{10}{0} 0.9521^{10}.0.0479^{0} + \binom{10}{1} 0.9521^{9}.0.0479^{1}$$

correct binomial expressions using their values from part (i) M1

$$\approx 0.6121 + 0.3079$$

for getting 1 part right A1

$$\approx 0.9201$$

Pr(more than 1 reject) $\approx 1 - 0.9201$

or 0.0799 M1

A1

iii. Expected profit = Profit×Pr(crate accepted) - loss×Pr(crate rejected) M1

$\approx 20\times0.9201 - 55\times0.0799$
$\approx \$14.01$

A1

b. i. The proportion of bags underweight in the sample $\hat{p} = 0.1$

A1

Use of the approx. 95% interval formula $\hat{p} \pm 2\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

M1

The interval is $0.1 \pm 2\sqrt{\dfrac{0.1\times0.9}{10}}$ or ≈ -0.090 to 0.290

A1

ii. The only reasonable conclusion from the evidence is that no conclusion is possible as the interval calculated is too wide. This is because the sample is too small. The company should do a much more extensive test on the new machine.

A1

Question 4

a. i. Attempt to solve $F(t) = \dfrac{7(0.9t+3)^4}{15} = 300$

M1

$$(0.9t+3)^4 = \frac{300\times15}{7}$$

$$0.9t + 3 = \sqrt[4]{\frac{300\times15}{7}}$$

M1

$$0.9t = \sqrt[4]{\frac{300\times15}{7}} - 3$$

$$t \approx 2.2615 \text{ hrs}$$

or 2 hours and 16 minutes

A1

ii. Finding F(0) and F(0.5)

M1

$$F(0) = \frac{7(0+3)^4}{15} = 37.8^\circ C$$

$$F(0.5) = \frac{7(0.5\times0.9+3)^4}{15} \approx 66.11^\circ C$$

A1

$$\text{The average rate of change} = \frac{F(0.5) - F(0)}{0.5}$$

$$\approx \frac{66.11 - 37.8}{0.5}$$

= 57 °C per hour (to the nearest whole number)

A1

iii. Use of the chain rule to find the derivative

M1

$$F(t) = \frac{7(0.9t+3)^4}{15}$$

$$F'(t) = \frac{7}{15}\times0.9\times4(0.9t+3)^3$$

A1

$$F'(1.5) = \frac{7}{15}\times0.9\times4(0.9\times1.5+3)^3$$ (use of t = 1.5 in their derivative)

M1

≈ 138.286 or 138 °C per hour to the nearest whole number.

A1

b. i. If the graph is to be continuous, the F(1) = F*(1)

M1

$$15\times1^2 - 45\times1 + k = \frac{7(0.9\times1+3)^4}{15}$$

M1

$$15 - 45 + k \approx 107.96$$

k = 138 to the nearest whole number

A1

ii. $F^*(t) = 15t^2 - 45t + 138 \quad t > 1$

$F^{*\prime}(t) = 30t - 45 \quad t > 1$

A1

iii. Using $F^{*\prime}(t) = 0$ and attempting to solve

M1

$30t - 45 = 0$

$t = 1.5$ hours (1 hour 30 mins)

A1

The coordinates of the point are (1.5,104.25) or (1.5,104)

A1

iv. Students should observe that the stationary point just calculated is below the temperature at the moment of the power cut and is a minimum.

A1

The temperature falls for the first 30 minutes after the power cut and then starts to rise again.

A1