Mathematical Association of Victoria

Mathematical Methods

Trial CAT 2 1995

Student Name:.....

Section A
Question Booklet

Reading time: **15 minutes**
Writing time: **90 minutes**

SECTION A 33 Marks
SECTION B 17 Marks

Directions to students

Answer all questions in **Section A** on the answer sheet provided, by placing a straight line through the answer you have chosen.

Answer all questions in **Section B** in the spaces provided.

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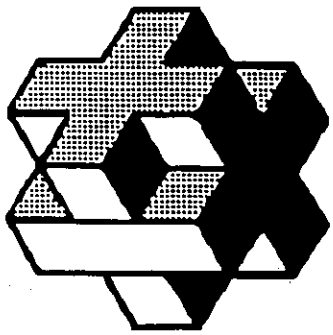


SECTION A

Question 1

The linear factors of $x^3 - 4x$ are

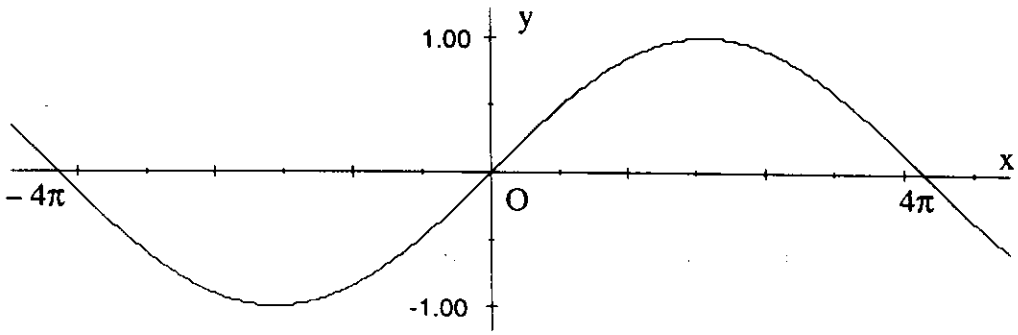
A. $x,\ x^2 - 4$
B. $x,\ x-4,\ x-4$
C. $x,\ x-2$
D. $x,\ x+2$
E. $x,\ x-2,\ x-2$

Question 2

The amplitude of the function with the rule $f(x) = -2\sin(3x)$ is

A. $\frac{2\pi}{3}$
B. 3
C. 2
D. -2
E. π

Question 3



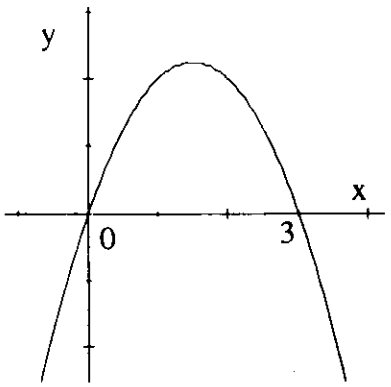
A possible equation for the graph shown above is

A. $y = sin\left(\frac{x}{4}\right)$
B. $y = sin(4x)$
C. $y = \sin(2x + 2\pi)$
D. $y = -\sin(2x)$
E. $y = \sin\left(\frac{x}{2} + 2\pi\right)$

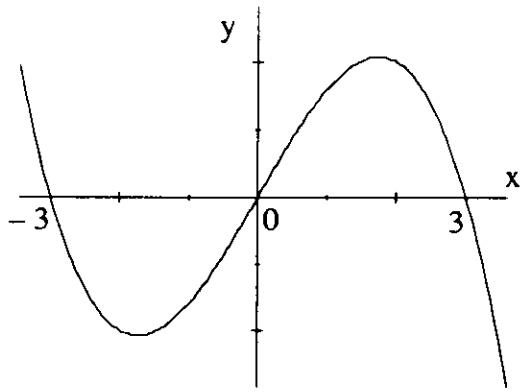
Question 4

Which one of the following could have the equation $y = x^2(x^2 - 9)$

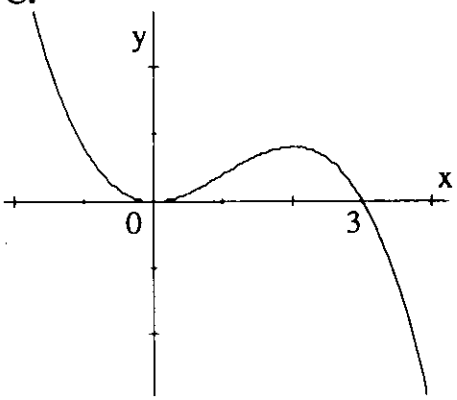
A.



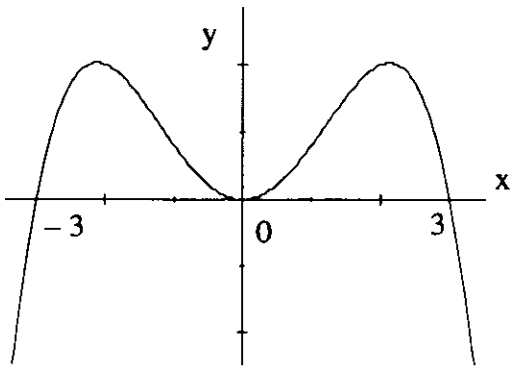
B.



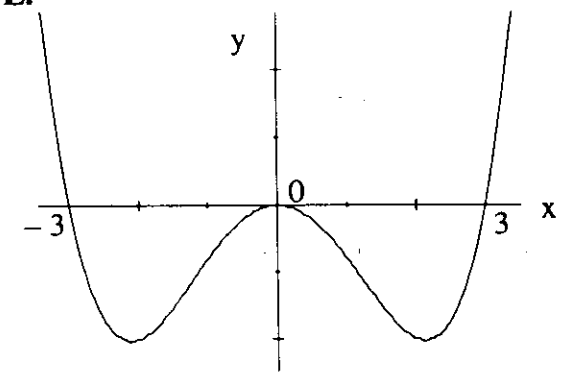
C.



D.



E.



Question 5

Which one of the following is not a one-to-one function ?

A. $f:[0,\infty) \to R$, where $f(x) = \sqrt{x}$

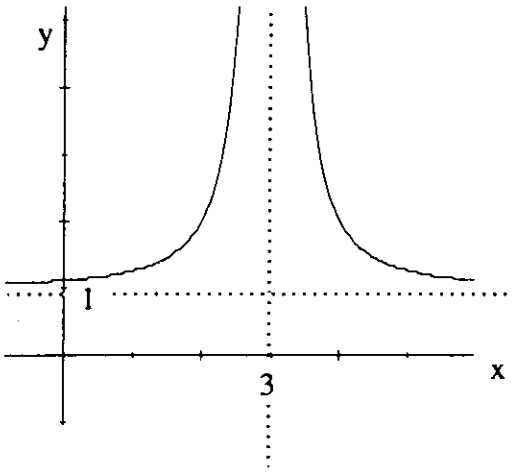
B. $f:[0,\infty) \to R$, where $f(x) = x^2 - 1$

C. $f:[0,\infty) \to R$, where $f(x) = 9$

D. $f:[0,\infty) \to R$, where $f(x) = 3 - x^2$

E. $f:[0,\infty) \to R$, where $f(x) = x\left(x^2 - 1\right)$

Question 6



The equation of the curve shown above is given by

A. $y - 1 = \dfrac{1}{x - 3}$

B. $y - 1 = \dfrac{1}{x + 3}$

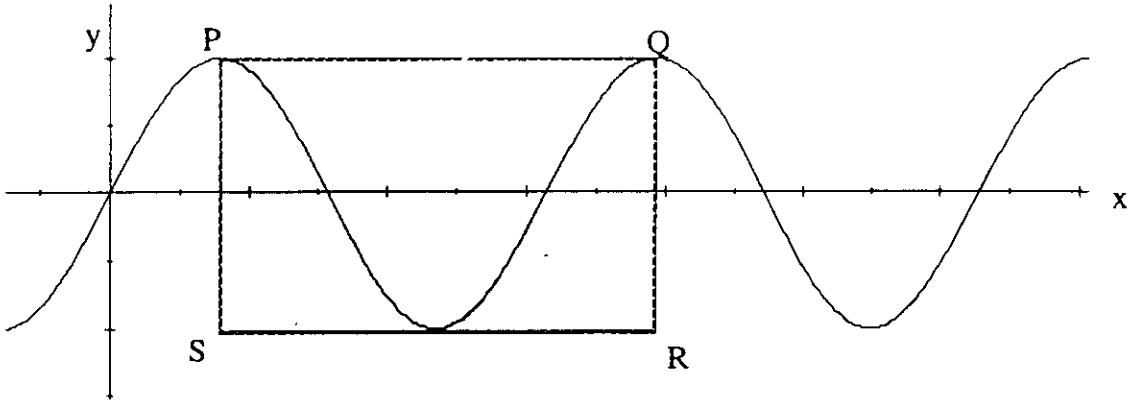
C. $y - 1 = \dfrac{1}{(x - 3)^2}$

D. $y + 1 = \dfrac{1}{(x + 3)^2}$

E. $y - 3 = \dfrac{1}{(x + 1)^2}$

Question 7

The graph of the function with equation $y = 3\sin(2x)$ is shown below:



The area of the shaded rectangle PQRS is equal to

A. 6π square units
B. 12 square units
C. 3π square units
D. 24 square units
E. 12π square units

Question 8

The solution to the equation $\cos(2x) = \frac{1}{2}x$ where $-2\pi \leq x \leq 2\pi$

A. is given by $-\frac{\pi}{3}$ and $\frac{\pi}{3}$
B. is given by $-\frac{\pi}{6}$ and $\frac{\pi}{6}$
C. lies in the range $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$
D. lies in the range $-\frac{3\pi}{4} \leq x \leq \frac{\pi}{4}$
E. lies in the range $-\frac{\pi}{2} \leq x \leq \frac{\pi}{4}$

Question 9

Find $\{x: 2^{2x} - 6 \times 2^{x+1} + 32 = 0\}$

A. $\{2,-2\}$
B. $\{4,8\}$
C. $\{2,3\}$
D. $\left\{\frac{1}{2},\frac{1}{3}\right\}$
E. $\{6,-1\}$

Question 10

The inverse of $f: R \to R$, *where* $f(x) = 3e^{2-x} + 5$ is

A. $f^{-1}: R \to R$, where $f^{-1}(x) = 2 - \log_e\left(\frac{x-5}{3}\right)$
B. $f^{-1}: (5,\infty) \to R$, where $f^{-1}(x) = 2 - \log_e\left(\frac{x-5}{3}\right)$
C. $f^{-1}: (5,\infty) \to R$, where $f^{-1}(x) = -3e^{3-x} - 5$
D. $f^{-1}: R \to R$, where $f^{-1}(x) = -3e^{3-x} - 5$
E. $f^{-1}: (5,\infty) \to R$, where $f^{-1}(x) = \frac{1}{3e^{3-x} + 5}$

Question 11

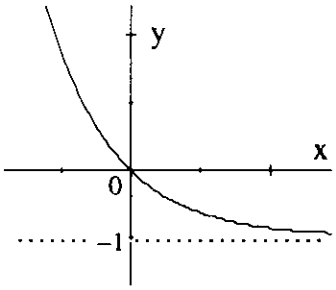
The domain of the function $f(x) = \frac{2}{\sqrt{3-x}}$ is

A. $[0,\infty)$
B. $(3,\infty)$
C. $[3,\infty)$
D. $(-\infty,3]$
E. $(-\infty,3)$

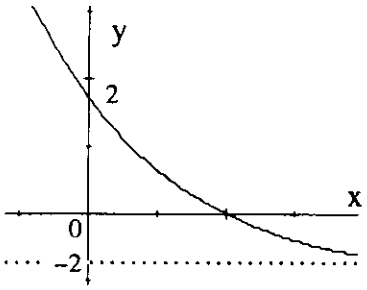
Question 12

Which one of the following graphs best represents the graph of the function defined by $f: R \to R$, where $f(x) = 2\left(e^{-x} - 1\right)$?

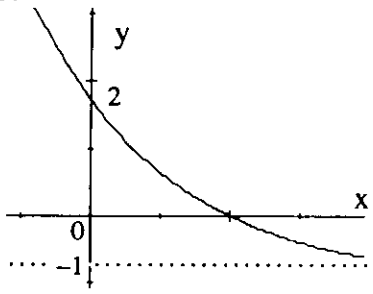
A.



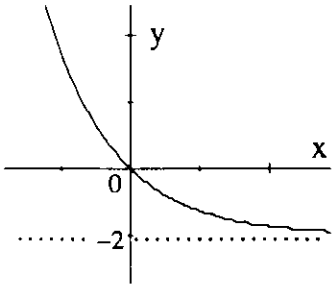
B.



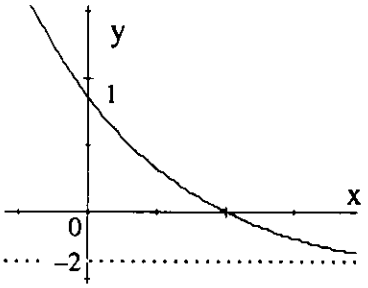
C.



D.



E.



Questions 13 and 14 refer to the following information:
The graph of a function f is shown below, in **Figure 1 (a)**.

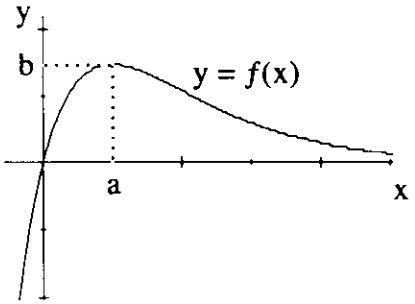


Figure 1 (a)

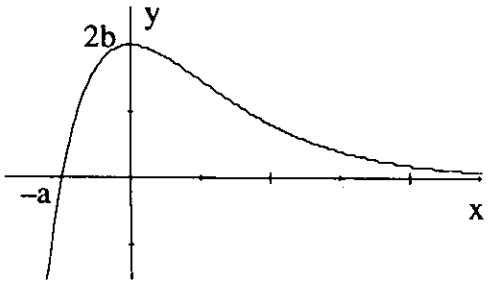


Figure 1 (b)

Question 13

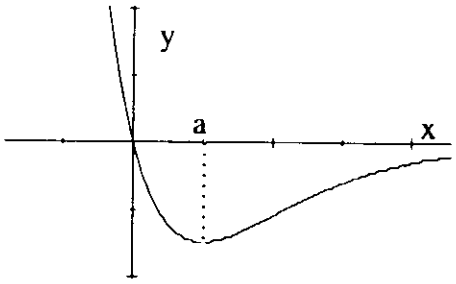
The graph shown in **Figure 1 (b)** has the equation

A. $y = f(x - a) + b$
B. $y = 2f(x) - a$
C. $y = f(x + a) + b$
D. $y = 2f(x + a)$
E. $y = 2f(x - a)$

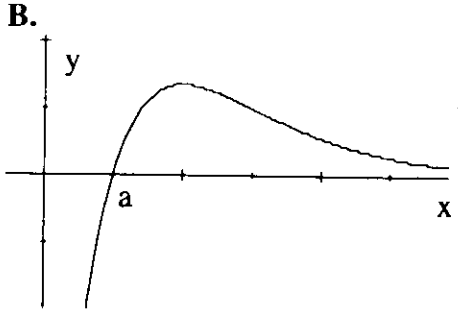
Question 14

The graph of $y = f'(x)$ is best represented by

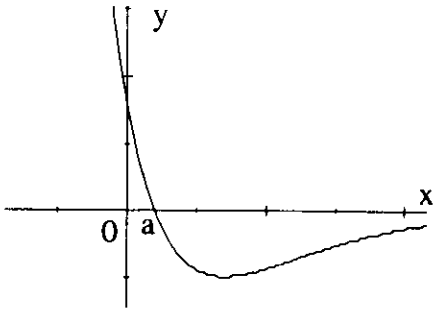
A.



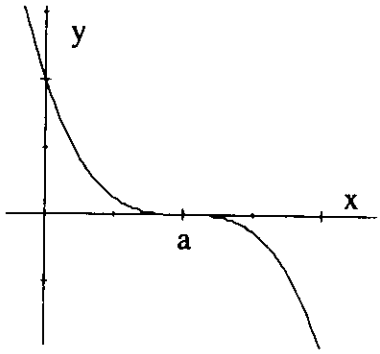
B.



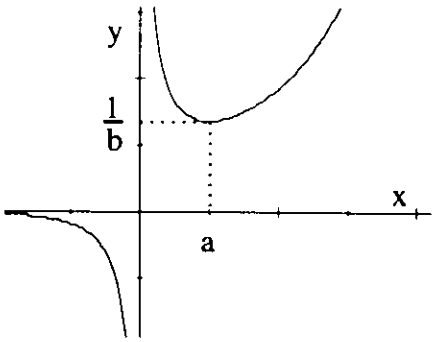
C.



D.



E.



Question 15

The derivative of $x^2 \cos(x)$ is equal to

A. $2x\cos(x)$

B. $-2x\sin(x)$

C. $2x\cos(x) + x^2\cos(x)$

D. $2x\cos(x) - x^2\sin(x)$

E. $2x\cos(x) + x^2\sin(x)$

Question 16

If $y = (x^3 - 1)^{\frac{1}{3}}$ then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{3}(x^3 - 1)^{-\frac{1}{3}}$

B. $\frac{1}{3}(x^3 - 1)^{\frac{4}{3}}$

C. $3x^2(x^3 - 1)^{-\frac{2}{3}}$

D. $x^2(x^3 - 1)^{\frac{4}{3}}$

E. $x^2(x^3 - 1)^{-\frac{2}{3}}$

Question 17

If $f(x) = \sin\left(\frac{x}{3}\right)$, then $f'(x)$ is equal to

A. $3\cos(x)$

B. $3\cos\left(\frac{x}{3}\right)$

C. $\frac{1}{3}\cos\left(\frac{x}{3}\right)$

D. $\cos\left(\frac{x}{3}\right)$

E. $\frac{1}{3}\cos(x)$

Question 18

$\int \frac{x + e^x}{xe^x} dx$ where $x > 0$ is equal to

A. $-\frac{1}{e^x} + \log_e x + c$

B. $\frac{1}{e^x} + \log_e x + c$

C. $-\frac{1}{e^x} - \log_e x + c$

D. $-\frac{1}{e^x} + \frac{1}{x^2} + c$

E. $-\frac{1}{e^x} - \frac{1}{x^2} + c$

Question 19

The function with equation $f(x) = x + \frac{1}{x}$ has a relative minimum of

A. -2

B. 2

C. $-\frac{5}{2}$

D. $\frac{5}{2}$

E. 1

Question 20

$$\int_0^1 \frac{1}{(2x+1)^3} dx =$$

A. $\frac{2}{9}$

B. $\frac{8}{9}$

C. $-\frac{5}{18}$

D. $-\frac{10}{9}$

E. $-\frac{3}{2}$

Question 21

The equation of the tangent to the graph of $y = e^{2x} + 1$, at its y–intercept, is

A. $y = 3x + 2$

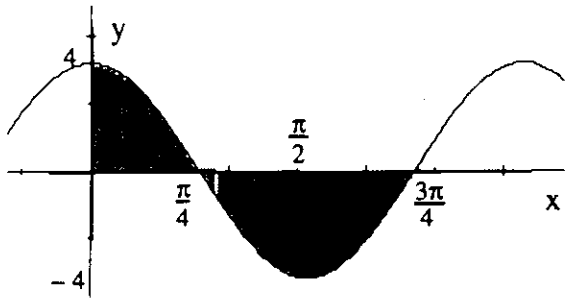
B. $y = 2x + 2$

C. $y = \frac{3}{2}x + 1$

D. $y = \frac{1}{2}x + 2$

E. $y = 2x + 1$

Question 22



The graph shown above has equation $y = 4\cos(2x)$. The area shaded is equal to

A. 2 sq units

B. 6 sq units

C. 4 sq units

D. 8 sq units

E. $\int_0^{\frac{3\pi}{4}} 4\cos(2x)dx$ sq units

Question 23

The volume, V cm^3, of unmelted ice cream in a container, t seconds after it has been removed from a freezer is modelled by the equation

$$V(t) = 0.02t^2 - 4t + 200$$

The rate (in cm^3/sec) at which the ice cream is melting, 40 sec after it is removed from the freezer is

A. 12

B. 2.4

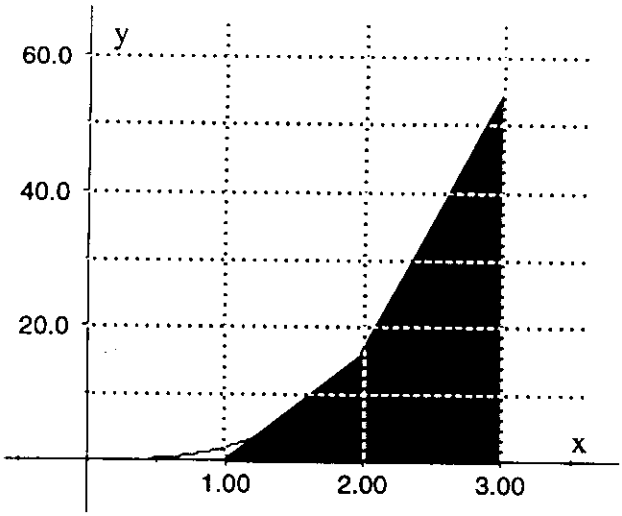
C. 0

D. –2.4

E. –2.6

Question 24

For the graph with equation $y = 2x^3, 0 \le x \le 3$ (shown below), the approximate area enclosed by the curve, the x–axis and the lines x = 1 and x = 3, using the trapezia shown is



A. 88 sq units

B. 70 sq units

C. 43 sq units

D. 40 sq units

E. 35 sq units

Question 25

Given that k > 1 and x > 1, $\int_1^x \frac{1}{t^{1-k}}\,dt$ is equal to

A. $\frac{1}{k}\left(x^k - 1\right)$

B. $\frac{1}{2-k}\left(x^{2-k} - 1\right)$

C. $\frac{1}{2+k}\left(x^{2+k} - 1\right)$

D. $\frac{1}{2-k}\left(\frac{1}{x^{2-k}} - 1\right)$

E. $(k-1)\left(x^{k-2} - 1\right)$

Questions 26 and 27 refer to the following information.

The random variable X denotes the number of worms in an apple that comes from a particular region in Victoria.

X	0	1	2	> 2
Pr(X = x)	$\frac{5}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	0

Question 26

The expected number of worms that can be found in each apple that comes from this region is equal to

A. $\frac{15}{10}$
B. $\frac{12}{10}$
C. 1
D. $\frac{7}{10}$
E. $\frac{5}{10}$

Question 27

The standard deviation of X is approximately equal to

A. 0.78
B. 0.11
C. 0.61
D. 0.37
E. 0.85

Question 28

If the random variable X has a binomial distribution such that $E(X) = 5$ and $Var(X) = \frac{10}{3}$, then $\Pr(X \geq 1)$ is equal to

A. $\left(\frac{1}{3}\right)^{15}$

B. $\left(\frac{2}{3}\right)^{15}$

C. $1-\left(\frac{1}{3}\right)^{15}$

D. $1-\left(\frac{2}{3}\right)^{15}$

E. ${}^{15}C_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{14}$

Question 29

The value of n and the missing term a in the expansion $\left(x^2-\frac{2}{x}\right)^n = x^8 - 2^4C_1x^5 + 4^4C_2x^2 + a + \frac{8}{x^4}$ are respectively

A. $8,\ -2^4C_3$

B. $8,\ -2^4C_3\left(\frac{1}{x^2}\right)$

C. $4,\ 8^4C_3\left(\frac{1}{x}\right)$

D. $4,\ -8^4C_3\left(\frac{1}{x^2}\right)$

E. $4,\ -8^4C_3\left(\frac{1}{x}\right)$

Question 30

A sample of 500 mice was taken from a very large population of mice. It was found that 50 of these mice had contracted a disease. The approximate 95% confidence interval for the proportion of mice from this population that have this disease is

A. (0.372, 0.217)
B. (0.073, 0.127)
C. (0.732, 0.163)
D. (0.087, 0.113)
E. (0.273, 0.227)

Question 31

The random variable X has a normal distribution with a mean of 1 and standard deviation of 1. If $Pr(X \leq a) = 0.95$, then

A. $a = 2.6449$
B. $a = 1.6449$
C. $a = 1.96$
D. $a = -1.6449$
E. $a = 0.6449$

Question 32

The random variable X is normally distributed with mean 20.5 and varaiance 9. The probability that $X > 23$ is closest to

A. 0.3909
B. 0.8333
C. 0.6091
D. 0.7975
E. 0.2025

Question 33

At the BUTTER–UP processing plant, packages of butter are found to have a mean weight of 505 gm and standard deviation of σ. It is found that 98% of all packages weigh in excess of 504 gm. The variance of the weight of butter packages at this plant is

A. 0.2371
B. 1.7084
C. 0.6275
D. 1.5937
E. 1.1955

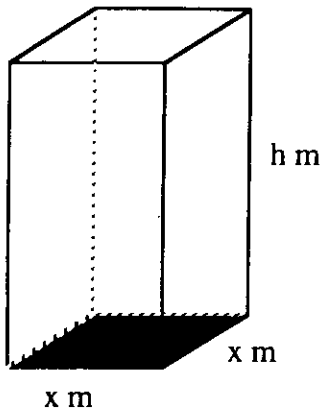
SECTION B: Short Answer Questions

Question 1

A large solid rectangular box with a square base of side length x m and height h m has a surface area, S m^2, given by

$$S = 4x^2 + 4xh$$

The sum of the dimensions is 3 metres.



a. Find h in terms of x.

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b. For what value of x will the surface area be a maximum ?

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3 marks

Question 2

Find the gradient of the function $y = \log_e(x^2 + 1)$ at the point $x = 1$.

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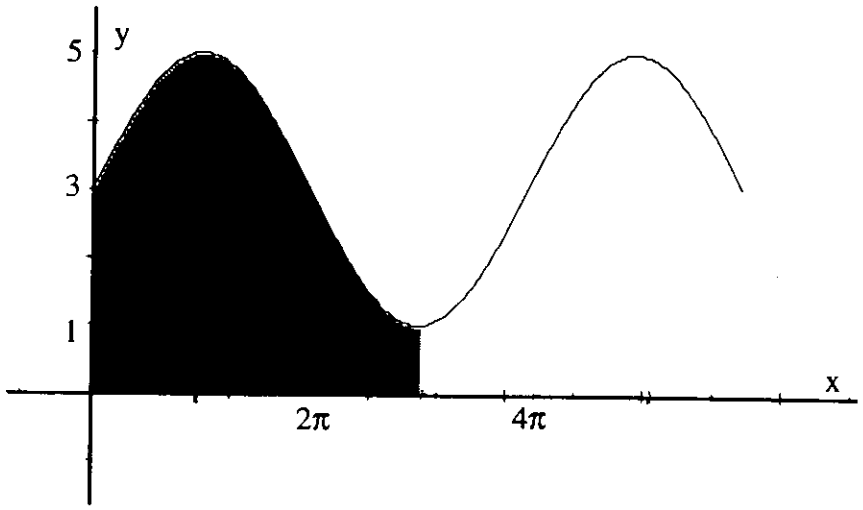
3 marks

Question 3



The graph of the function $y = a + b\sin(2x)$ is shown above.

a. Find the values of a and b

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b. Find the area of the shaded region.

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3 marks

Question 4

The random variable X is such that one any one trial, there are only one of two possible outcomes. The probability of "success" on any one trial is 0.3. If an experiment consists of 10 such trials

a. determine $\Pr(X \geq 1)$,

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.....

b. state any assumption(s) that you have made.

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.....

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3 marks

Question 5

A discrete random variable X has a probability distribution defined by

$$\Pr(X = x) = kx + 0.1, \quad x = 0, 1, 2, 3$$

where k is some real constant.

a. Show that k = 0.1.

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b. Hence, find the expected value of X.

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3 marks

Question 6

In a calculus test, the scores of 100 students were found to be normally distributed with mean 65 and standard deviation 9. A student obtained a score of 74. What is the student's ranking on this test ?

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2 marks

Total 17 marks