MATHEMATICAL METHODS PART 1
MULTIPLE-CHOICE QUESTION BOOKLET

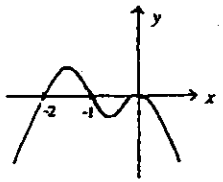
Specific Instructions for Section A

This part consists of 33 questions.
Answer all questions in this section on the answer sheet provided for multiple-choice questions.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers. You should attempt every question.
No credit will be given if two or more letters are marked for that question.

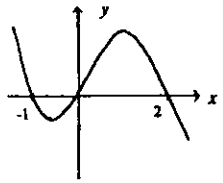
Question 1

Which one of the following graphs shows the graph with equation $y = x^2(2-x)(1+x)$.

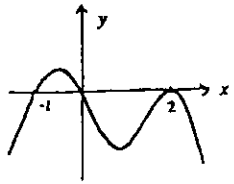
A.



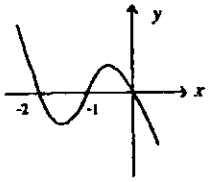
B.



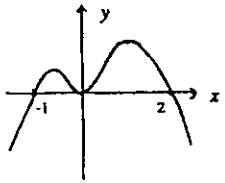
C.



D.



E.



Question 2

For the function $f : (-3,2] \to R,\ f(x) = (x+1)^2 - 4$ the range is

A. $(-3,5]$

B. $[-4,5]$

C. $(-4,5]$

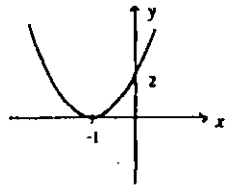
D. $(0,5]$

E. $[-1,2]$

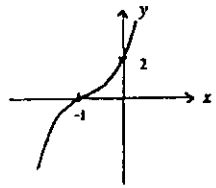
Question 3

From the following graphs select the graphs which represent a function and which also have an inverse function.

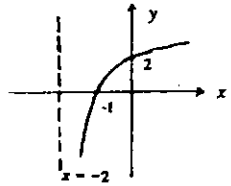
(i)



(ii)



(iii)



A. (i) only

B. (ii) only

C. (i) and (iii) only

D. (ii) and (iii) only

E. all of (i), (ii) and (iii)

Question 4

A possible equation for the graph shown is

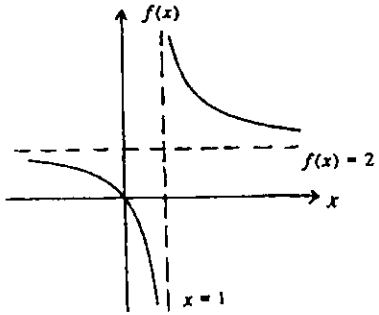
A. $f(x) = \dfrac{2x-1}{x-1}$

B. $f(x) = \dfrac{2x+3}{x+1}$

C. $f(x) = \dfrac{2x}{x-1}$

D. $f(x) = \dfrac{2x+4}{x+1}$

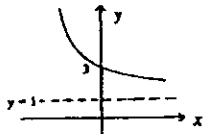
E. $f(x) = \dfrac{3-2x}{x-1}$



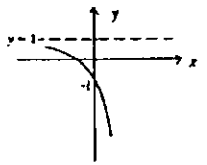
Question 5

Which one of the following graphs represents the relation $y = 1 + 2e^{-x}$.

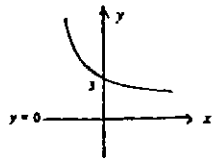
A.



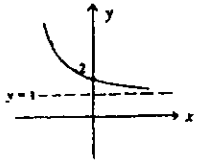
B.



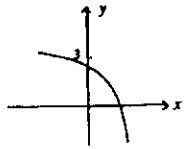
C.



D.



E.



Question 6

A possible equation for the graph shown is

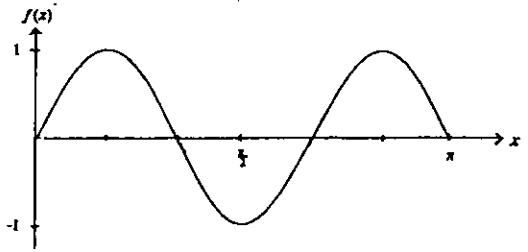
A. $f(x) = \sin 2x$

B. $f(x) = \sin(3x + \pi)$

C. $f(x) = \cos(3x - \frac{\pi}{2})$

D. $f(x) = \cos(3x + \frac{\pi}{2})$

E. $f(x) = \cos(2x - \frac{\pi}{2})$



Question 7

The solutions between 0 and π for which $\sqrt{2}\cos 3x = 1$ are

A. $\frac{5\pi}{12}, \frac{7\pi}{12}$

B. $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{11\pi}{12}$

C. $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

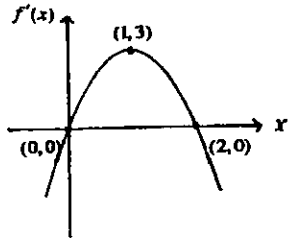
D. $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}$

E. $\frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}$

Question 8

The graph of the derived function $f'(x)$ is shown.

Which of the following statements relating to the function, $f(x)$, is false?



A. $f(x)$, is a polynomial of degree three.

B. $f(x)$ has exactly two stationary points.

C. $f(x)$ is decreasing over the domain $(2,\infty)$.

D. $f(x)$ has a maximum turning point at $x = 2$.

E. The gradient of $f(x)$ is positive over the domain $(-\infty,1)$.

Question 9

The derivative of $\frac{3x^2 + 2}{x^2}$ is equal to

A. $-\frac{4}{x^3}$

B. $-\frac{1}{x}$

C. $-\frac{1}{4x^3}$

D. $-\frac{1}{x^3}$

E. 3

Question 10

If $y = xe^{2x}$ then $\frac{dy}{dx}$ is

A. $2xe^{4x}$

B. $2xe^{3x}$

C. $2xe^{2x}$

D. $(2x+1)e^{2x}$

E. $2xe^{x} + e^{2x}$

Question 11

If $f(x) = \sqrt{x^2 - 4}$ then $f'(x)$ is equal to

A. $x\sqrt{x^2-4}$

B. $\frac{1}{2\sqrt{x^2-4}}$

C. $\frac{x}{\sqrt{x^2-4}}$

D. $\frac{x}{x-2}$

E. $\frac{1}{2(x-2)}$

Question 12

The derivative of $\frac{2t-1}{t+4}$ is equal to

A. $\frac{9}{(t+4)^2}$

B. $\frac{7}{(t+4)^2}$

C. $\frac{-9}{(t+4)^2}$

D. $\frac{-7}{(2t-1)^2}$

E. 2

Question 13

The minimum value of $4x^2 - 2x + 3$ is

A. 59

B. 4

C. $3\frac{1}{2}$

D. $2\frac{3}{4}$

E. $\frac{1}{4}$

Question 14

The gradient of the normal to the curve $f(x) = e^{-2x}$ at the point where $x = \frac{1}{2}$ is equal to

A. $-\frac{e}{2}$

B. $2e$

C. $-\frac{2}{e}$

D. $\frac{e}{2}$

E. $\frac{2}{e}$

Question 15

The volume of water in a container, V, after t minutes is given by $V(t) = \frac{2}{5}t^2(15 - \frac{1}{4}t)$, $0 \le t \le 60$. After how many minutes is the volume increasing at the greatest rate?

A. 10

B. 20

C. 30

D. 40

E. 50

Question 16

If x satisfies the equation $(e^x - 1)(e^{2x} - 4) = 0$ then x is equal to

A. 1 or $\log_e 2$

B. 1 or $\log_e 4$

C. 0 or $\log_e 2$

D. 0 or $\log_e 4$

E. 0 or $\log_e 16$

Question 17

The coefficient of x^3 in the expansion of $(3 - 2x)^5$ is equal to

A. -1080

B. -720

C. -360

D. -180

E. -90

Question 18

The function $f:[1,\infty) \to R,\ f(x) = (x-1)^2 - 4$ has an inverse function f^{-1} defined by

A. $f^{-1}:[1,\infty) \to R,\ f^{-1}(x) = 1+\sqrt{x+4}$

B. $f^{-1}:[-1,\infty) \to R,\ f^{-1}(x) = 4+\sqrt{x+1}$

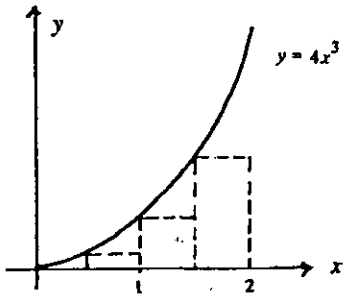
C. $f^{-1}:[-4,\infty) \to R,\ f^{-1}(x) = 1+\sqrt{x+4}$

D. $f^{-1}:[1,\infty) \to R,\ f^{-1}(x) = 4+\sqrt{x,+1}$

E. $f^{-1}:[-5,\infty) \to R,\ f^{-1}(x) = \sqrt{x+5}$

Question 19

The area under the curve $y = 4x^3$ between $x = 0$ and $x = 2$ is approximated by dividing the interval into four sections equal in width and calculating the area of the lower rectangles.
Using this technique, the approximate area is equal to



A. 25 square units

B. 24.75 square units

C. 16 square units

D. 12 square units

E. 9 square units

Question 20

Given that $\int_0^3 f(x)\,dx = 4$ and $g(x) = 2f(x) - 1$ then $\int_3^0 g(x)\,dx$ is equal to

A. -5

B. 5

C. 7

D. 11

E. -11

Question 21

Evaluate $\int_0^{\frac{\pi}{2}} 4\sin 2x\,dx$

A. -4

B. -2

C. 0

D. 2

E. 4

Question 22

If c is an arbitrary constant and $f'(x) = \dfrac{6}{\sqrt{3x-1}}$ then $f(x)$ is equal to

A. $12\sqrt{3x-1}+c$

B. $4\sqrt{3x-1}+c$

C. $\sqrt{3x-1}+c$

D. $\dfrac{4}{3\sqrt{3x-1}}+c$

E. $\dfrac{4}{\sqrt{3x-1}}+c$

Question 23

The area bounded by the curve $f(x) = \dfrac{3}{7-2x}$ and the x-axis from $x = \frac{1}{2}$ to $x = 2$ is equal to

A. $\frac{3}{2}\log_e 2$

B. $\frac{2}{3}\log_e 0.5$

C. $\frac{3}{2}\log_e 0.5$

D. $\frac{2}{3}\log_e 2$

E. $\frac{3}{2}\log_e 3$

Question 24
Calculate $\Pr(X > 2)$ where X has a probability distribution given by

x	1	2	3	4
$\Pr(X = x)$	$3c^2$	$8c^2$	c^2	$4c^2$

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{5}{16}$

D. $\frac{1}{16}$

E. $\frac{5}{11}$

Question 25
The random variable X represents the number of work place accidents in a factory per week.

x	0	1	2	4	5	6
$\Pr(X = x)$	0.4	0.3	0.1	0.1	0.05	0.05

The owner of this factory pays all employees a weekly bonus according to the following conditions:
if no accidents occur a bonus of \$10 is paid
if one accident occur a bonus of \$2 is paid
if two or more accidents occur no bonus is paid
The employee can expect to receive a weekly bonus of

A. \$1.45

B. \$3.60

C. \$4.00

D. \$4.60

E. \$4.90

Question 26
For a discrete random variable with mean 6.2 and variance 2.89, the interval in which 95% of the distribution would lie is

A. 3 to 10

B. 3 to 9

C. 2 to 10

D. 0 to 11

E. 0 to 12

The following information relates to questions 27 and 28

A lampshade requires four light globes. The probability that each light globe will last more than one year is 0.6. The lampshade has four new globes inserted on Anzac Day.

Question 27
The probability that no more than one of these globes will need to be replaced in the coming year is closest to

A. 0.026

B. 0.130

C. 0.154

D. 0.179

E. 0.475

Question 28
Over a ten year period, the number of globes the owner could expect to replace is

A. 4

B. 6

C. 16

D. 24

E. 60

Question 29
X is a binomial random variable with $p = 0.3$. If $\Pr(X \geq 1) = 0.7599$ the variance of X is equal to

A. 1.47

B. 1.26

C. 1.05

D. 0.84

E. 0.63

Question 30
Individual sweets are packaged in boxes with a recommended weight of 375 grams. The weight of these boxes of sweets is normally distributed with a mean of 375 g and variance of 4 g.
Boxes which weigh less than 372 g are rejected prior to distribution. Calculate the probability, correct to 4 decimal places, that a randomly selected box of sweets will be rejected.

A. 0.0668

B. 0.2266

C. 0.5000

D. 0.7734

E. 0.9932

Question 31
The diagram below shows two normal distributions, A and B, with means of μ_A and μ_B respectively and standard deviations of σ_A and σ_B respectively. Which of the following is true?

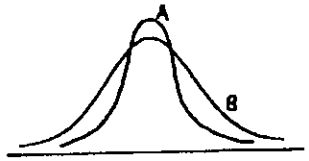
A. $\mu_A = \mu_B$ and $\sigma_A = \sigma_B$

B. $\mu_A > \mu_B$ and $\sigma_A = \sigma_B$

C. $\mu_A = \mu_B$ and $\sigma_A > \sigma_B$

D. $\mu_A > \mu_B$ and $\sigma_A < \sigma_B$

E. $\mu_A = \mu_B$ and $\sigma_A < \sigma_B$



Question 32
X is normally distributed with a mean of 20. Given that $\Pr(X > 24) = 0.4$, the variance of X is closest to

A. 250

B. 37.2

C. 30.4

D. 15.8

E. 6.1

Question 33
From a random sample of 25 primary school children, 10 are left-handed. An approximate 95% confidence interval for the proportion of primary school children who are left-handed is

A. 0.106 — 0.694

B. 0.204 — 0.596

C. 0.302 — 0.498

D. 0.371 — 0.429

E. 0.381 — 0.419

MATHEMATICAL METHODS
QUESTION AND ANSWER BOOKLET

Specific instructions to students

Answer **all** questions in this section in the spaces provided.

Question 1
Determine the largest possible domain for the function $f(x) = \sqrt{4x - x^2}$

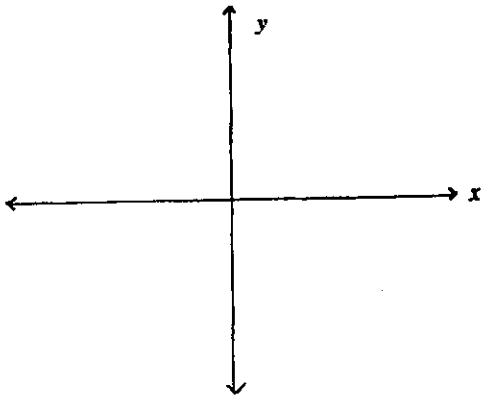
2 marks

Question 2
Find the rule for the inverse function for $y = 4e^{x-1} + 2$

3 marks

Question 3

On the set of axes below sketch the graph with equation $y = 4x^2 - x^4$. Label the coordinates of all intercepts and stationary points.



3 marks

Question 4

Find the area bounded by the x axis and the curve $f(x) = 3(1+x)(3-x)$ from $x = 1$ to $x = 4$.

3 marks

Question 5

a. Find the derivative of $4x^2 \log_e x$

b. Use your answer to part a to find $\int 4x \log_e x \, dx$.

3 marks

Question 6

X is normally distributed with a mean of 12 and variance of 9.

Find the value of a, correct to two decimal places, for which $\Pr(X < a) = 0.05$

3 marks

END 1995 MATHEMATICAL METHODS TRIAL CAT 2.
FACTS, SKILLS AND APPLICATIONS.