1995

MATHEMATICAL METHODS TRIAL CAT 2

Based on the Victorian Certificate of Education Mathematics Study Design.

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VCE MATHEMATICAL METHODS 1995

CAT 2: Facts, Skills and Applications

ANSWER SHEET FOR MULTIPLE CHOICE QUESTIONS

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SURNAME

GIVEN NAME (S)

STUDENT NUMBER

0	0	0	0	0	0	0	0	A
1	1	1	1	1	1	1	1	E
2	2	2	2	2	2	2	2	F
3	3	3	3	3	3	3	3	G
4	4	4	4	4	4	4	4	J
5	5	5	5	5	5	5	5	L
6	6	6	6	6	6	6	6	R
7	7	7	7	7	7	7	7	T
8	8	8	8	8	8	8	8	W
9	9	9	9	9	9	9	9	X

ABSENT

How to complete this form

Please use an HB PENCIL only. If you make a mistake, ERASE the incorrect answer – DO NOT just cross it out.

EXAMPLE ONLY

9 / 9 / 0 9 / 0 E

Enter your Student Number in the box on the right as shown in the example.

All answers must be completed like this. ONLY ONE answer per line.

(A) ■ (C) (D) (E)

One answer per line

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
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24	A	B	C	D	E
25	A	B	C	D	E
26	A	B	C	D	E
27	A	B	C	D	E
28	A	B	C	D	E
29	A	B	C	D	E
30	A	B	C	D	E
31	A	B	C	D	E
32	A	B	C	D	E
33	A	B	C	D	E

Instructions

Complete ALL the questions.

Marks will NOT be deducted for incorrect answers.

NO mark will be given if more than ONE answer is completed for any question.

Use only a HB PENCIL. If you make a mistake, ERASE the incorrect answer – DO NOT just cross it out.

Please DO NOT fold, bend or staple this form

Victorian Certificate of Education
Mathematics 1995

MATHEMATICAL METHODS
1995 TRIAL CAT 2
Facts, Skills and Applications

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

Part I

MULTIPLE-CHOICE QUESTION BOOKLET

This task has two parts: part I (multiple-choice questions) and part II (short answer questions). Part I consists of this question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part, continue immediately to the other part.
A detachable formula sheet for use in both parts is included with this booklet.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of the question and answer booklet (part II) and hand them in.
You may retain this question booklet.

Directions to students

Materials
Question booklet of 11 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the booklet.
An approved calculator may be used.

The task
Detach the formula sheet from this booklet during reading time.
Ensure that you write your **name and student number** on the answer sheet for multiple-choice questions.
Answer **all** questions.
There is a total of 33 marks available for part I.
All questions should be answered on the answer sheet for multiple-choice questions provided. Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of the question and answer booklet (part II) and hand them in.
You may retain this question booklet.

MATHEMATICAL METHODS PART 1
MULTIPLE-CHOICE QUESTION BOOKLET

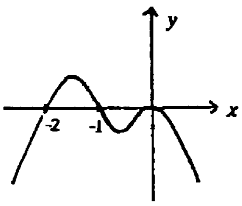
Specific Instructions for Section A

This part consists of 33 questions.
Answer all questions in this section on the answer sheet provided for multiple-choice questions.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers. You should attempt every question.
No credit will be given if two or more letters are marked for that question.

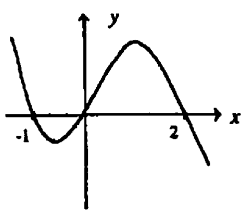
Question 1

Which one of the following graphs shows the graph with equation $y = x^2(2-x)(1+x)$.

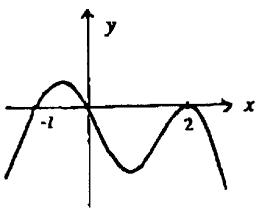
A.



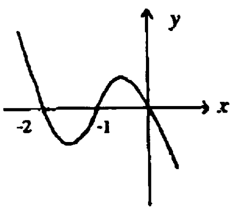
B.



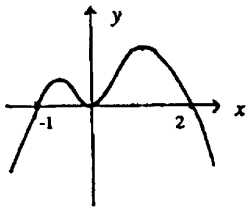
C.



D.



E.



Question 2

For the function $f : (-3,2] \to R,\ f(x) = (x+1)^2 - 4$ the range is

A. $(-3,5]$

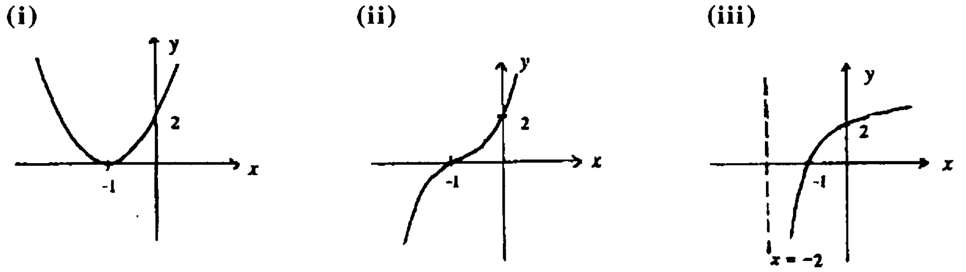
B. $[-4,5]$

C. $(-4,5]$

D. $(0,5]$

E. $[-1,2]$

Question 3
From the following graphs select the graphs which represent a function and which also have an inverse function.



A. (i) only

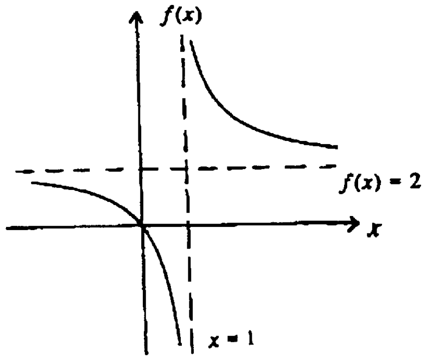
B. (ii) only

C. (i) and (iii) only

D. (ii) and (iii) only

E. all of (i), (ii) and (iii)

Question 4
A possible equation for the graph shown is



A. $f(x)=\dfrac{2x-1}{x-1}$

B. $f(x)=\dfrac{2x+3}{x+1}$

C. $f(x)=\dfrac{2x}{x-1}$

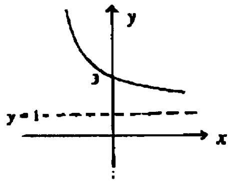
D. $f(x)=\dfrac{2x+4}{x+1}$

E. $f(x)=\dfrac{3-2x}{x-1}$

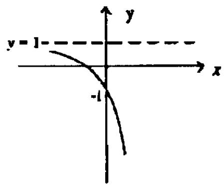
Question 5

Which one of the following graphs represents the relation $y = 1 + 2e^{-x}$.

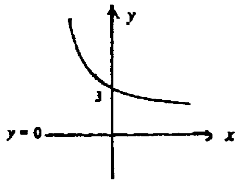
A.



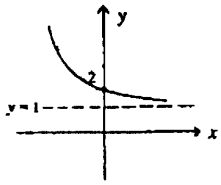
B.



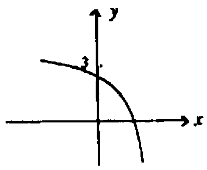
C.



D.



E.



Question 6

A possible equation for the graph shown is

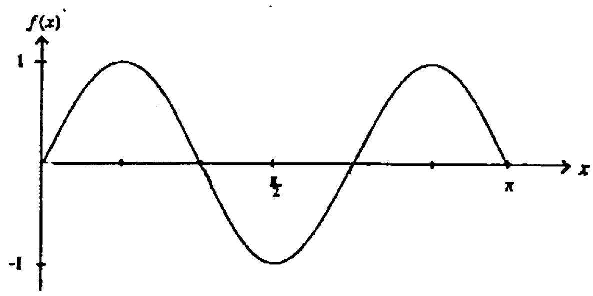
A. $f(x) = \sin 2x$

B. $f(x) = \sin(3x + \pi)$

C. $f(x) = \cos(3x - \frac{\pi}{2})$

D. $f(x) = \cos(3x + \frac{\pi}{2})$

E. $f(x) = \cos(2x - \frac{\pi}{2})$



Question 7

The solutions between 0 and π for which $\sqrt{2}\cos 3x = 1$ are

A. $\frac{5\pi}{12}, \frac{7\pi}{12}$

B. $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{11\pi}{12}$

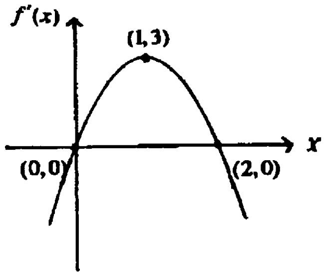
C. $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

D. $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}$

E. $\frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}$

Question 8

The graph of the derived function $f'(x)$ is shown.
Which of the following statements relating to the function, $f(x)$, is false?



A. $f(x)$, is a polynomial of degree three.

B. $f(x)$ has exactly two stationary points.

C. $f(x)$ is decreasing over the domain $(2,\infty)$.

D. $f(x)$ has a maximum turning point at $x = 2$.

E. The gradient of $f(x)$ is positive over the domain $(-\infty,1)$.

Question 9

The derivative of $\dfrac{3x^2+2}{x^2}$ is equal to

A. $-\dfrac{4}{x^3}$

B. $-\dfrac{1}{x}$

C. $-\dfrac{1}{4x^3}$

D. $-\dfrac{1}{x^3}$

E. 3

Question 10

If $y = xe^{2x}$ then $\frac{dy}{dx}$ is

A. $2xe^{4x}$

B. $2xe^{3x}$

C. $2xe^{2x}$

D. $(2x+1)e^{2x}$

E. $2xe^{x}+e^{2x}$

Question 11

If $f(x) = \sqrt{x^2 - 4}$ then $f'(x)$ is equal to

A. $x\sqrt{x^2 - 4}$

B. $\dfrac{1}{2\sqrt{x^2 - 4}}$

C. $\dfrac{x}{\sqrt{x^2 - 4}}$

D. $\dfrac{x}{x - 2}$

E. $\dfrac{1}{2(x - 2)}$

Question 12

The derivative of $\dfrac{2t - 1}{t + 4}$ is equal to

A. $\dfrac{9}{(t + 4)^2}$

B. $\dfrac{7}{(t + 4)^2}$

C. $\dfrac{-9}{(t + 4)^2}$

D. $\dfrac{-7}{(2t - 1)^2}$

E. 2

Question 13

The minimum value of $4x^2 - 2x + 3$ is

A. 59

B. 4

C. $3\frac{1}{2}$

D. $2\frac{3}{4}$

E. $\frac{1}{4}$

Question 14

The gradient of the normal to the curve $f(x) = e^{-2x}$ at the point where $x = \frac{1}{2}$ is equal to

A. $-\frac{e}{2}$

B. $2e$

C. $-\frac{2}{e}$

D. $\frac{e}{2}$

E. $\frac{2}{e}$

Question 15

The volume of water in a container, V, after t minutes is given by $V(t) = \frac{2}{5}t^2(15 - \frac{1}{4}t),\ 0 \le t \le 60$.
After how many minutes is the volume increasing at the greatest rate?

A. 10

B. 20

C. 30

D. 40

E. 50

Question 16

If x satisfies the equation $(e^x - 1)(e^{2x} - 4) = 0$ then x is equal to

A. 1 or $\log_e 2$

B. 1 or $\log_e 4$

C. 0 or $\log_e 2$

D. 0 or $\log_e 4$

E. 0 or $\log_e 16$

Question 17

The coefficient of x^3 in the expansion of $(3 - 2x)^5$ is equal to

A. -1080

B. -720

C. -360

D. -180

E. -90

Question 18

The function $f:[1,\infty)\to R,\ f(x)=(x-1)^2-4$ has an inverse function f^{-1} defined by

A. $f^{-1}:[1,\infty)\to R,\ f^{-1}(x)=1+\sqrt{x+4}$

B. $f^{-1}:[-1,\infty)\to R,\ f^{-1}(x)=4+\sqrt{x+1}$

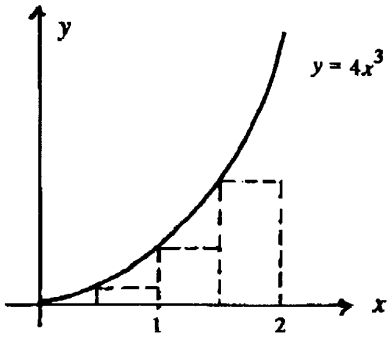
C. $f^{-1}:[-4,\infty)\to R,\ f^{-1}(x)=1+\sqrt{x+4}$

D. $f^{-1}:[1,\infty)\to R,\ f^{-1}(x)=4+\sqrt{x+1}$

E. $f^{-1}:[-5,\infty)\to R,\ f^{-1}(x)=\sqrt{x+5}$

Question 19

The area under the curve $y=4x^3$ between $x=0$ and $x=2$ is approximated by dividing the interval into four sections equal in width and calculating the area of the lower rectangles.
Using this technique, the approximate area is equal to



A. 25 square units

B. 24.75 square units

C. 16 square units

D. 12 square units

E. 9 square units

Question 20

Given that $\int_0^3 f(x)\,dx=4$ and $g(x)=2f(x)-1$ then $\int_3^0 g(x)\,dx$ is equal to

A. -5

B. 5

C. 7

D. 11

E. -11

Question 21

Evaluate $\int_0^{\frac{\pi}{2}} 4\sin 2x\, dx$

A. -4

B. -2

C. 0

D. 2

E. 4

Question 22

If c is an arbitrary constant and $f'(x) = \dfrac{6}{\sqrt{3x-1}}$ then $f(x)$ is equal to

A. $12\sqrt{3x-1}+c$

B. $4\sqrt{3x-1}+c$

C. $\sqrt{3x-1}+c$

D. $\dfrac{4}{3\sqrt{3x-1}}+c$

E. $\dfrac{4}{\sqrt{3x-1}}+c$

Question 23

The area bounded by the curve $f(x) = \dfrac{3}{7-2x}$ and the x-axis from $x = \frac{1}{2}$ to $x = 2$ is equal to

A. $\frac{3}{2}\log_e 2$

B. $\frac{2}{3}\log_e 0.5$

C. $\frac{3}{2}\log_e 0.5$

D. $\frac{2}{3}\log_e 2$

E. $\frac{3}{2}\log_e 3$

Question 24
Calculate $\Pr(X > 2)$ where X has a probability distribution given by

x	1	2	3	4
$\Pr(X = x)$	$3c^2$	$8c^2$	c^2	$4c^2$

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{5}{16}$

D. $\frac{1}{16}$

E. $\frac{5}{11}$

Question 25
The random variable X represents the number of work place accidents in a factory per week.

x	0	1	2	4	5	6
$\Pr(X = x)$	0.4	0.3	0.1	0.1	0.05	0.05

The owner of this factory pays all employees a weekly bonus according to the following conditions:
if no accidents occur a bonus of \$10 is paid
if one accident occur a bonus of \$2 is paid
if two or more accidents occur no bonus is paid
The employee can expect to receive a weekly bonus of

A. \$1.45

B. \$3.60

C. \$4.00

D. \$4.60

E. \$4.90

Question 26
For a discrete random variable with mean 6.2 and variance 2.89, the interval in which 95% of the distribution would lie is

A. 3 to 10

B. 3 to 9

C. 2 to 10

D. 0 to 11

E. 0 to 12

The following information relates to questions 27 and 28

A lampshade requires four light globes. The probability that each light globe will last more than one year is 0.6. The lampshade has four new globes inserted on Anzac Day.

Question 27
The probability that no more than one of these globes will need to be replaced in the coming year is closest to

A. 0.026

B. 0.130

C. 0.154

D. 0.179

E. 0.475

Question 28
Over a ten year period, the number of globes the owner could expect to replace is

A. 4

B. 6

C. 16

D. 24

E. 60

Question 29
X is a binomial random variable with $p = 0.3$. If $\Pr(X \geq 1) = 0.7599$ the variance of X is equal to

A. 1.47

B. 1.26

C. 1.05

D. 0.84

E. 0.63

Question 30
Individual sweets are packaged in boxes with a recommended weight of 375 grams. The weight of these boxes of sweets is normally distributed with a mean of 375 g and variance of 4 g.
Boxes which weigh less than 372 g are rejected prior to distribution. Calculate the probability, correct to 4 decimal places, that a randomly selected box of sweets will be rejected.

A. 0.0668

B. 0.2266

C. 0.5000

D. 0.7734

E. 0.9932

Question 31
The diagram below shows two normal distributions, A and B, with means of μ_A and μ_B respectively and standard deviations of σ_A and σ_B respectively. Which of the following is true?

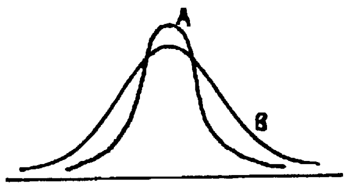
A. $\mu_A = \mu_B$ and $\sigma_A = \sigma_B$

B. $\mu_A > \mu_B$ and $\sigma_A = \sigma_B$

C. $\mu_A = \mu_B$ and $\sigma_A > \sigma_B$

D. $\mu_A > \mu_B$ and $\sigma_A < \sigma_B$

E. $\mu_A = \mu_B$ and $\sigma_A < \sigma_B$



Question 32
X is normally distributed with a mean of 20. Given that $\Pr(X > 24) = 0.4$, the variance of X is closest to

A. 250

B. 37.2

C. 30.4

D. 15.8

E. 6.1

Question 33
From a random sample of 25 primary school children, 10 are left-handed. An approximate 95% confidence interval for the proportion of primary school children who are left-handed is

A. 0.106 — 0.694

B. 0.204 — 0.596

C. 0.302 — 0.498

D. 0.371 — 0.429

E. 0.381 — 0.419

STUDENT NUMBER									LETTER
figures									
words									

Victorian Certificate of Education
Mathematics 1995

MATHEMATICAL METHODS
1995 TRIAL CAT 2
Facts, Skills and Applications

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

Part II

QUESTION AND ANSWER BOOKLET

This task has two parts: part I (multiple-choice questions) and part II (short answer questions).
Part I consists of a separate question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part, continue immediately to the other part.
A detachable formula sheet for use in both parts is included in the part I question booklet.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of this question and answer booklet (part II) and hand them in.

Directions to students

Materials
Question and answer booklet of 5 pages including one blank page for rough working.
Working space is provided throughout the booklet.
You may use an approved calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from the part I booklet during reading time.
Ensure that you write your **student number** in the space provide on the cover of this booklet.
The marks allotted to each question are indicated at the end of the question.
There is a total of 17 marks available for part II.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π,e, surds or fractions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses should be in English.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of this question and answer booklet (part II) and hand them in.

MATHEMATICAL METHODS
QUESTION AND ANSWER BOOKLET

Specific instructions to students

Answer **all** questions in this section in the spaces provided.

Question 1

Determine the largest possible domain for the function $f(x) = \sqrt{4x - x^2}$

2 marks

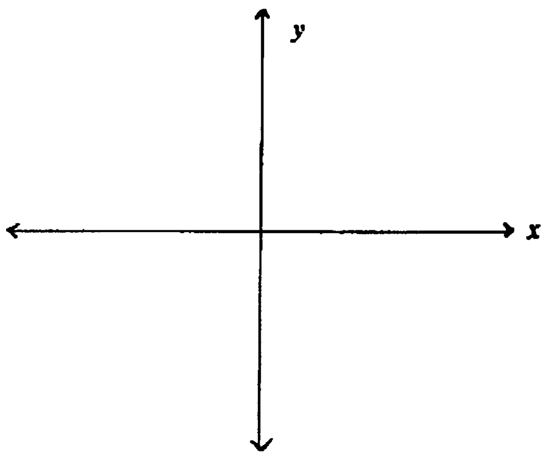
Question 2

Find the rule for the inverse function for $y = 4e^{x-1} + 2$

3 marks

Question 3

On the set of axes below sketch the graph with equation $y = 4x^2 - x^4$. Label the coordinates of all intercepts and stationary points.



3 marks

Question 4

Find the area bounded by the x axis and the curve $f(x) = 3(1 + x)(3 - x)$ from $x = 1$ to $x = 4$.

3 marks

Question 5

a. Find the derivative of $4x^2 \log_e x$

b. Use your answer to part a to find $\int 4x \log_e x \, dx$.

3 marks

Question 6

X is normally distributed with a mean of 12 and variance of 9.
Find the value of a, correct to two decimal places, for which $\Pr(X < a) = 0.05$

3 marks

ROUGH WORKING

End Of Questions 1995 Mathematical Methods Trial Cat 2

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Suggested solutions to 1995 Mathematical Methods CAT 2 - part I

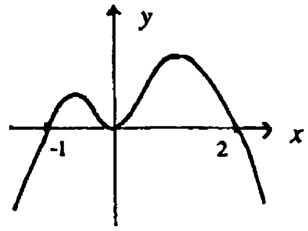
Question 1 **E**

x intercept : let $y = 0$

$\therefore 0 = x^2(2-x)(x+1)$

$\therefore x = 0,\ 2,\ -1$

$x = 0$ is a turning point



y intercept : let $x = 0$

$\therefore y = 0 \times 2 \times 1 = 0$

General shape is a negative quartic

Question 2 **B**

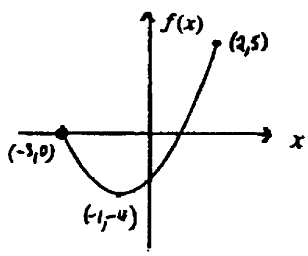
$f(x) = (x+1)^2 - 4$

From translations, turning point at $(-1,4)$

$f(-3) = (-2)^2 - 4 = 0$

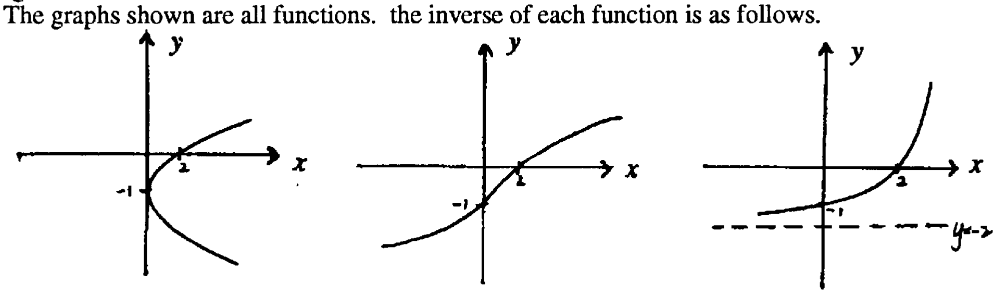
$f(2) = 3^2 - 4 = 5$

The minimum y value is -4 and the maximum y value is 5, therefore the range is $[-4,5]$



Question 3 **D**

The graphs shown are all functions. the inverse of each function is as follows.



The inverse of (i) is not a function, but the inverses of (ii) and (iii) are both functions.

Question 4 **C**

Using the asymptotes given, the equation is of the form: $f(x) = \frac{A}{x-1} + 2$

Substitute the point (0,0) : $\therefore 0 = \frac{A}{-1} + 2$

$\therefore A = 2$

$\therefore f(x) = \frac{2}{x-1} + 2$

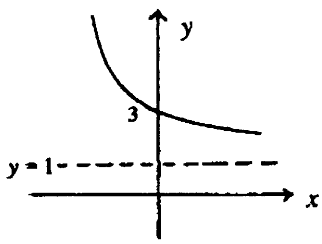
$= \frac{2+2(x-1)}{x-1}$

$= \frac{2x}{x-1}$

Question 5 **A**

$y = 1 + 2e^{-x}$

Horizontal asymptote: $y = 1$
Basic shape is reflected in the y axis.
y intercept : $y = 1 + 2e^0 = 3$



Question 6 **C**

Let the model be of the form $y = A\cos n(x+b)$

amplitude = 1, $\therefore A = 1$ period $= \frac{2\pi}{3}$, $\therefore n = 3$ $\therefore y = \cos 3(x+b)$

The cosine curve is translated $\frac{\pi}{6}$ units to the right, $\therefore b = -\frac{\pi}{6}$

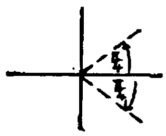
The equation of the curve is $y = \cos 3(x - \frac{\pi}{6}) = \cos(3x - \frac{\pi}{2})$

Question 7. **D**

$\sqrt{2}\cos 3x = 1$

$\therefore \cos 3x = \frac{1}{\sqrt{2}}$

Cosine is positive, angles in 1st & 4th quadrants
Basic angle is $\frac{\pi}{4}$ as $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$



$\therefore 3x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$

$\therefore \ x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}$

Question 8. **E**

The gradient of $f(x)$ is negative over the domain $(-\infty, 0)$, therefore statement E is incorrect.

Question 9. **A**

Let $f(x) = \frac{3x^2+2}{x^2} = 3 + 2x^{-2}$

$\therefore \ f'(x) = -4x^{-3} = -\frac{4}{x^3}$

Question 10. D

Using the Product rule:

$\frac{dy}{dx} = x(2e^{2x}) + e^{2x}(1)$

$= 2xe^{2x} + e^{2x}$

$= (2x+1)e^{2x}$

Question 11. C

Using the Chain rule:

Let $u(x) = x^2 - 4$ $\quad \therefore f(u) = u^{\frac{1}{2}}$

$\therefore u'(x) = 2x$ $\quad f'(u) = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}} = \frac{1}{2\sqrt{x^2-4}}$

$$f'(x) = \frac{2x}{2\sqrt{x^2-4}} = \frac{x}{\sqrt{x^2-4}}$$

Question 12. A

Using the Quotient rule:

Let $f(t) = \dfrac{2t-1}{t+4}$

then $f'(t) = \dfrac{(t+4)(2)-(2t+1)(1)}{(t+4)^2} = \dfrac{2t+8-2t+1}{(t+4)^2} = \dfrac{9}{(t+4)^2}$

Question 13. D

Let $f(x) = 4x^2 - 2x + 3$

For local maximum or minimum solve $f'(x) = 0$

$\therefore 8x - 2 = 0$

$\therefore \quad x = \frac{1}{4}$

Test for local minimum:

$f'(0) = -2 < 0$

$f'(1) = 6 > 0$

x	$< \frac{1}{4}$	$\frac{1}{4}$	$> \frac{1}{4}$
$f'(x)$	< 0	0	> 0
	\	_	/

$x = \frac{1}{4}$ gives minimum value

minimum value $= 4(\frac{1}{4})^2 - 2(\frac{1}{4}) + 3 = 2\frac{3}{4}$

Question 14. D

Gradient of tangent $f'(x) = -2e^{-2x}$

At $x = \frac{1}{2}$ gradient of tangent $= -2e^{-1} = -\frac{2}{e}$

At $x = \frac{1}{2}$ gradient of normal $= -1 \div -\frac{2}{e} = \frac{e}{2}$

Question 15. B

$V(t) = \frac{2}{5}(15t^2 - \frac{1}{4}t^3)$

Rate of change $= \frac{2}{5}(30t - \frac{3}{4}t^2)$

For maximum rate of change let $V''(t) = 0$

$\therefore \frac{2}{5}(30 - \frac{3}{2}t) = 0$

$\therefore \quad 30 - \frac{3}{2}t = 0$

$\therefore \quad t = 20$

Volume is changing at the greatest rate after 20 minutes.

Test for maximum:

$V'(19) = \frac{2}{5}\left(30 - \frac{3}{2}(19)\right) > 0$

$V'(21) = \frac{2}{5}\left(30 - \frac{3}{2}(21)\right) < 0$

t	< 20	20	> 21
$V'(t)$	> 0	0	< 0
	/	–	\

Question 16. C

$(e^x - 1)(e^{2x} - 4) = 0$

either $e^x = 1$ or $e^{2x} = 4$

$\therefore \quad x = 0$ or $2x = \log_e 4$

$x = \frac{1}{2}\log_e 4 = \log_e 4^{\frac{1}{2}} = \log_e 2$

Question 17. B

$(3 - 2x)^5 = (3)^5 - 5(3)^4(2x) + 10(3)^3(2x)^2 - 10(3)^2(2x)^3 + 5(3)(2x)^4 - (2x)^5$

coefficient of $x^3 = -10 \times 3^2 \times 2^3 = -720$

Question 18. C

Let $y = (x-1)^2 - 4$

Interchanging x and y gives

$x = (y-1)^2 - 4$

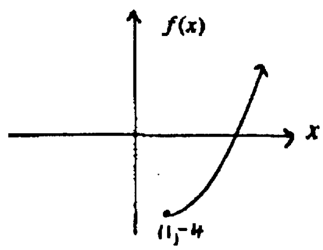
$\therefore \quad x + 4 = (y-1)^2$

$\therefore \sqrt{x+4} = y - 1$

$\therefore \quad y = 1 + \sqrt{x+4}$

$\therefore f^{-1}(x) = 1 + \sqrt{x+4}$

Inverse of $f = [-4, \infty) \to R,\ f^{-1}(x) = 1 + \sqrt{x+4}$



Domain of f^{-1} = range of $f = [-4, \infty)$

Question 19. E

If $x = 0.5,\ y = 4(0.5)^3 = 0.5$

If $x = 1,\ y = 4(1)^3 = 4$

If $x = 1.5,\ y = 4(1.5)^3 = 13.5$

Approximate area $= 0.5 \times 0.5 + 0.5 \times 4 + 0.5 \times 13.5 = 9$ square units

Question 20. **A**

$$\int_3^0 g(x)\,dx = \int_3^0 2f(x)-1\,dx$$
$$= 2\int_3^0 f(x)\,dx - \int_3^0 1\,dx$$
$$= -2\int_0^3 f(x)\,dx - [x]_3^0$$
$$= -2(4)-(0-3)$$
$$= -5$$

Question 21. **E**

$$\int_0^{\frac{\pi}{2}} 4\sin 2x\,dx = [-2\cos 2x]_0^{\frac{\pi}{2}}$$
$$= -2\cos\pi + 2\cos 0$$
$$= 2+2$$
$$= 4$$

Question 22. **B**

$$f'(x) = \frac{6}{\sqrt{3x-1}} = 6(3x-1)^{-\frac{1}{2}}$$
$$f(x) = \frac{6}{\frac{1}{2}\times 3}(3x-1)^{-\frac{1}{2}} + c$$
$$= 4\sqrt{3x-1}+c$$

Question 23. **A**

$$\text{Area} = \int_{\frac{1}{2}}^2 \tfrac{3}{7-2x}\,dx = -\tfrac{3}{2}\int_{\frac{1}{2}}^2 \tfrac{-2}{7-2x}\,dx$$
$$= -\tfrac{3}{2}\left[\log_e(7-2x)\right]_{\frac{1}{2}}^2$$
$$= -\tfrac{3}{2}\left(\log_e 3 - \log_e 6\right)$$
$$= \tfrac{3}{2}\left(\log_e 6 - \log_e 3\right)$$
$$= \tfrac{3}{2}\log_e 2$$

Question 24. **C**

$$\sum \Pr(X=x) = 1$$
$$\therefore 3c^2 + 8c^2 + c^2 + 4c^2 = 1$$
$$\therefore \quad 16c^2 = 1$$
$$\therefore \quad c^2 = \tfrac{1}{16}$$

$$\Pr(X>2) = \Pr(X=3) + \Pr(X=4)$$
$$= \tfrac{1}{16} + \tfrac{4}{16}$$
$$= \tfrac{5}{16}$$

Question 25. D
Let B denote the bonus paid

b	$\Pr(B=b)$	$b\Pr(B=b)$
10	0.4	4
2	0.3	0.6
0	0.3	0
		4.6

The expected weekly bonus is $4.60

Question 26. B
$\mu = 6.2, \quad \sigma = \sqrt{2.89} = 1.7$

$\mu + 2\sigma = 6.2 + 2(1.7) = 9.6 \qquad \mu - 2\sigma = 6.2 - 2(1.7) = 2.8$

Since X is a discrete random variable, the 95% confidence interval is 3 to 9.

Question 27. E
Let X denote the number of globes which need to be replaced in the year.
$n = 4, \quad p = 0.4$

$$\Pr(X \le 1) = \Pr(X=0) + \Pr(X=1)$$
$$= \binom{4}{0}(0.4)^0(0.6)^4 + \binom{4}{1}(0.4)^1(0.6)^3$$
$$= 0.475$$

Question 28. C
$E(X) = np = 4 \times 0.4 = 1.6$

On average 1.6 globes per year would need to be replaced. Therefore it would be expected that 16 globes would need to be replaced over a ten year period.

Question 29. D

$\Pr(X \geq 1) = 0.7599$

$\therefore \Pr(X = 0) = 1 - 0.7599 = 0.2401$

$\therefore \quad \binom{n}{0}(0.3)^0(0.7)^n = 0.2401$

$\therefore \quad (0.7)^n = 0.2401$

$\therefore \quad \log_{10}(0.7)^n = \log_{10} 0.2401$

$\therefore \quad n = \dfrac{\log_{10} 0.2401}{\log_{10} 0.7}$

$= 4$

$n = 4, \ p = 0.3, \ \sigma^2 = np(1-p) = 4 \times 0.3 \times 0.7 = 0.84$

Question 30. A

$\mu = 375, \quad \sigma = \sqrt{4} = 2$

$\Pr(X < 372) = \Pr(Z < \frac{372-375}{2})$

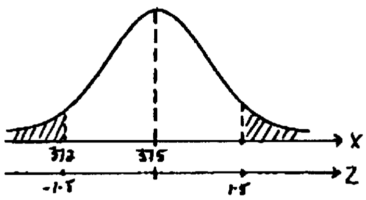
$= \Pr(Z < -1.5)$

$= \Pr(Z > 1.5)$

$= 1 - \Pr(Z < 1.5)$

$= 1 - 0.9332$

$= 0.0668$



Question 31. E

Both normal distributions are centred about the same value, $\therefore \mu_A = \mu_B$

Distribution A has a smaller spread than B, $\therefore \sigma_A < \sigma_B$

Question 32. A

$\mu = 20$

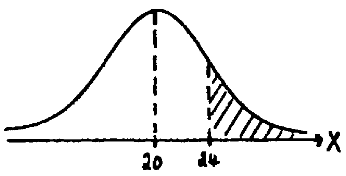
$\Pr(X > 24) = 0.4$

$\therefore \Pr(X < 24) = 0.6$

$\therefore \frac{24-20}{\sigma} = 0.253$

$\therefore \quad \sigma = 15.8$

$\therefore \quad \sigma^2 = 250$



Question 33. **B**

$\hat{p} = \frac{10}{25} = 0.4$

$se(\hat{p}) = \sqrt{\frac{0.4 \times 0.6}{25}} = 0.098$

lower limit = 0.4 - 2(0.098) = 0.204

Upper limit = 0.4 + 2(0.098) = 0.596

95% confidence interval is 0.204 to 0.596

Suggested solutions to 1995 Mathematical Methods CAT 2 - part II

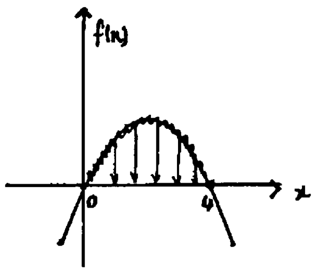
Question 1

$f(x)$ is defined when $4x - x^2 \geq 0$

$\therefore\ x(4-x) \geq 0$

$\therefore\ 0 \leq x \leq 4$

The largest possible domain is $[0, 4]$



Question 2

Interchanging x and y gives:

$\therefore\quad x = 4e^{y-1} + 2$

$\therefore\ \frac{x-2}{4} = e^{y-1}$

$\therefore y - 1 = \log_e\left(\frac{x-2}{4}\right)$

$\therefore\quad y = 1 + \log_e\left(\frac{x-2}{4}\right)$

The inverse of the function is $y = 1 + \log_e\left(\frac{x-2}{4}\right)$

Question 3

x intercepts: let $y = 0$

$\therefore 0 = 4x^2 - x^4$

$\therefore 0 = x^2(4 - x^2)$

$\therefore 0 = x^2(2 - x)(2 + x)$

$\therefore x = 0, \quad 2, \ -2$

stationary points: let $\frac{dy}{dx} = 0$

$\therefore 0 = 8x - 4x^3$

$\therefore 0 = 4x(2 - x^2)$

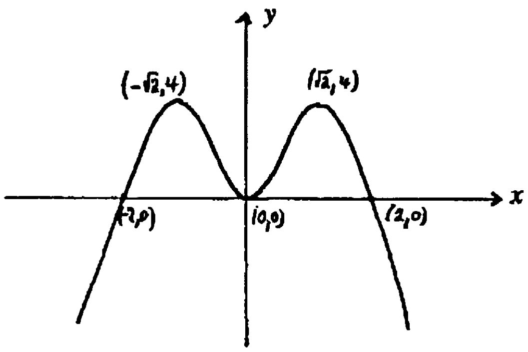
$\therefore 0 = 4x(\sqrt{2} - x)(\sqrt{2} + x)$

$\therefore x = 0, \quad \sqrt{2}, \ -\sqrt{2}$

When $x = 0, \ y = 0$

When $x = \sqrt{2}, \ y = 4(\sqrt{2})^2 - (\sqrt{2})^4 = 4$

When $x = -\sqrt{2}, \ y = 4(-\sqrt{2})^2 - (-\sqrt{2})^4 = 4$



Question 4

$f(x) = 3(3 + 2x - x^2) = 9 + 6x - 3x^2$

$$\text{Area} = \int_1^3 (9 + 6x - 3x^2)\,dx + \left|\int_3^4 (9 + 6x - 3x^2)\,dx\right|$$

$$= \left[9x + 3x^2 - x^3\right]_1^3 + \left|\left[9x + 3x^2 - x^3\right]_3^4\right|$$

$$= (27 + 27 - 27) - (9 + 3 - 1) + |(36 + 48 - 64) - (27 + 27 - 27)|$$

$$= 27 - 11 + |20 - 27|$$

$$= 27 - 11 + 7$$

$$= 23 \text{ square units}$$

Question 5

a. Using the Product Rule:

$$f'(x) = 4x^2\left(\tfrac{1}{x}\right) + 8x\log_e x = 4x + 8x\log_e x = 4x(1 + 2\log_e x)$$

b. From part a

$$\int (4x + 8x\log_e x)\, dx = 4x^2\log_e x + C$$

$$\therefore \int 8x\log_e x\, dx = 4x^2\log_e x - \int 4x\, dx + C$$

$$\therefore 2\int 4x\log_e x\, dx = 4x^2\log_e x - 2x^2 + C$$

$$\therefore \int 4x\log_e x\, dx = 2x^2\log_e x - x^2 + C$$

Question 6

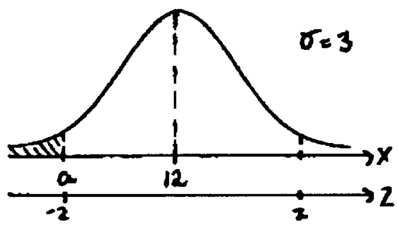
$\Pr(X < a) = 0.05$

$\therefore \Pr(Z < -z) = 0.05$

$\therefore \Pr(Z < z) = 0.95$

$\therefore \dfrac{a-12}{3} = -1.645$

$\therefore a = 7.065$



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