

Question 1

Should a state of emergency occur in a nuclear power station, it has been modelled that the total number of atomic particles, N , of a particular type released into the environment t minutes after the alert siren is sounded, will be approximated by the equation

$$N(t) = (10^{15} - \frac{1}{3}t^3 + 8t^2) \times 10^{10}$$

Normal operations at the plant are continued until any sign of emergency is activated (i.e., until the alert siren is sounded).

You may leave all your answers in exponent form.

- a. How many particles are there at the instant when the alert siren is sounded ?

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2 marks

It is believed that the danger period (for a spill) lies in the first sixteen minutes.

- b. What is the average rate at which the particles are spilled over this danger period ?

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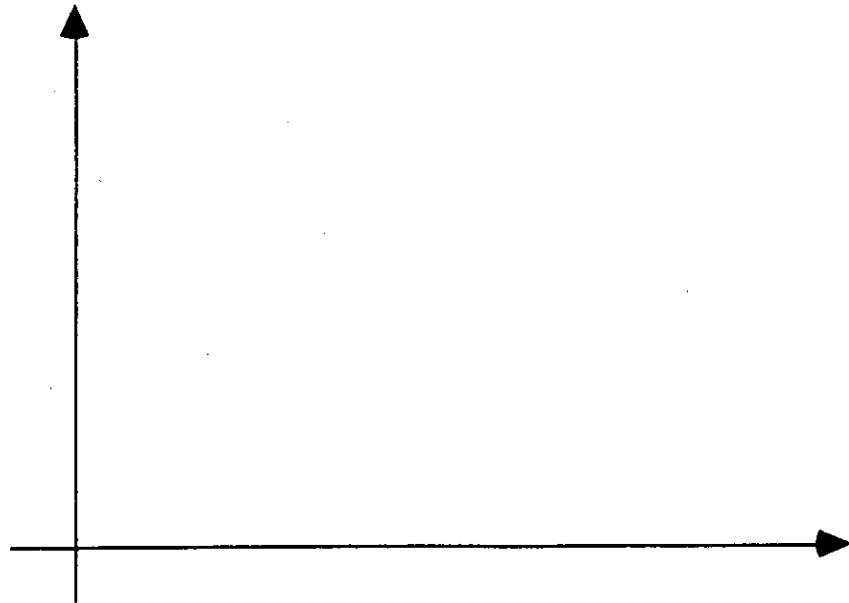
2 marks

- c. How quickly are particles spilling into the environment at any time t minutes after the siren is sounded ?

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2 marks

- d. Sketch the graph which shows the rate of change of the number of particles which spill into the environment during the danger period.



2 marks

- e. What is the maximum number of particles that could spill into the atmosphere ?

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4 marks

- f. When will the rate at which the spill is occurring be a maximum ?

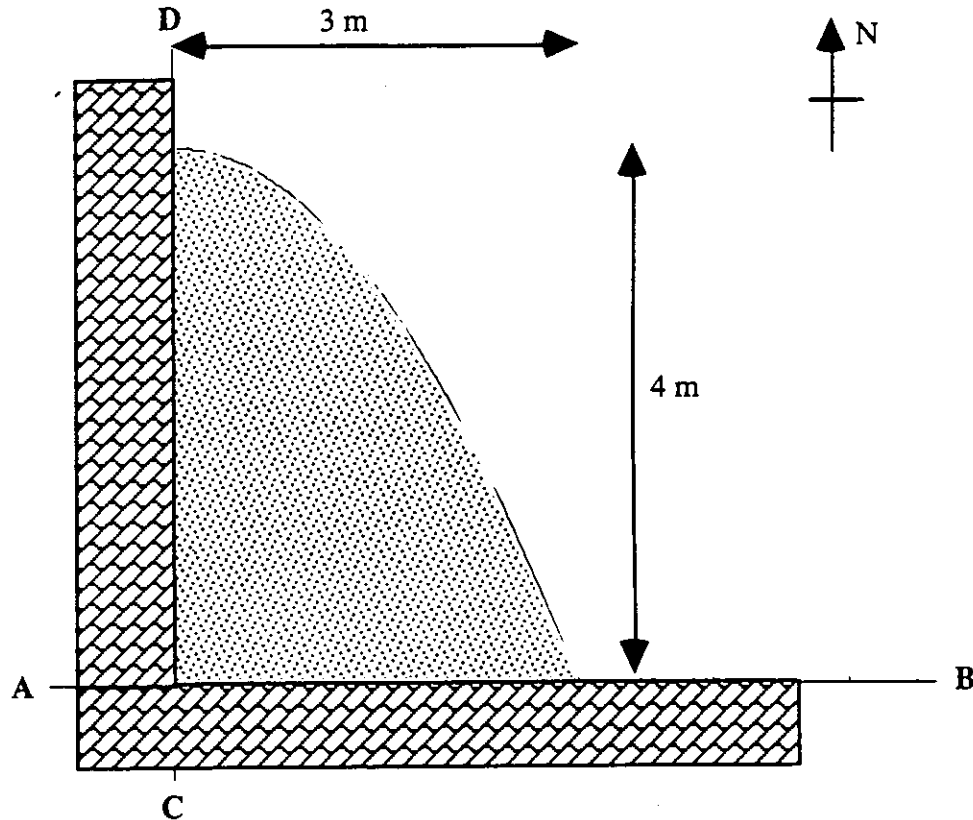
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2 marks

Total 14 marks

Question 2

A sandpit is being constructed in a backyard. The sandpit is to be placed at one of the corners of the backyard in such a way that two of the sides of the sandpit are to be against the brick fences that are at right angles to each other. The boundary of the sandpit will then be a curve in the shape of a parabola. The sandpit meets with the north-south wall at right angles, 4 metres from the corner and meets the east-west wall 3 metres from the corner (a rough construction plan is shown).



The builder decides to use the fences to represent a set of co ordinate axes; AB will represent the x-axis and CD will represent the y-axis.

- a. Give the coordinates of the points at which the edge of the sandpit meets the fences.

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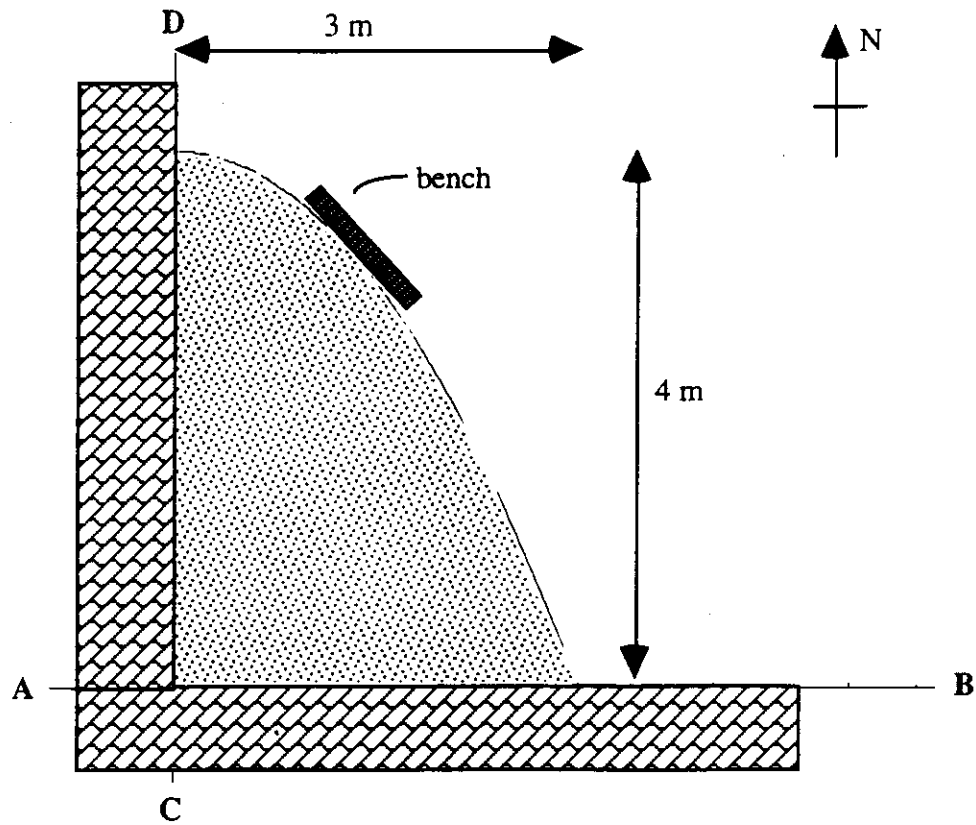
2 marks

- b. Prove that the equation of the section which defines the edge of the sandpit not against a wall is $f(x) = 4 - \frac{4}{9}x^2, 0 \leq x \leq 3$.

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3 marks

It has been decided that a straight bench, 1 metre long, will be placed somewhere against the edge of the sandpit. The owners wish for this bench to be aligned in a north-west direction.



c. Find the distance that this bench is from each of the two fences.

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4 marks

The cost of construction is made up of;

\$ 40.00 for a plank of wood which will be used for the curved border of the sandpit,

\$ 56.00 per cubic metre of sand, and

\$ 190.00 flat fee for labour.

d. i. Find the surface area covered by the sandpit.

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3 marks

The sandpit is to be filled (with sand) to a uniform depth of 25 cm.

ii. Find the total cost for constructing this sandpit.

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2 marks

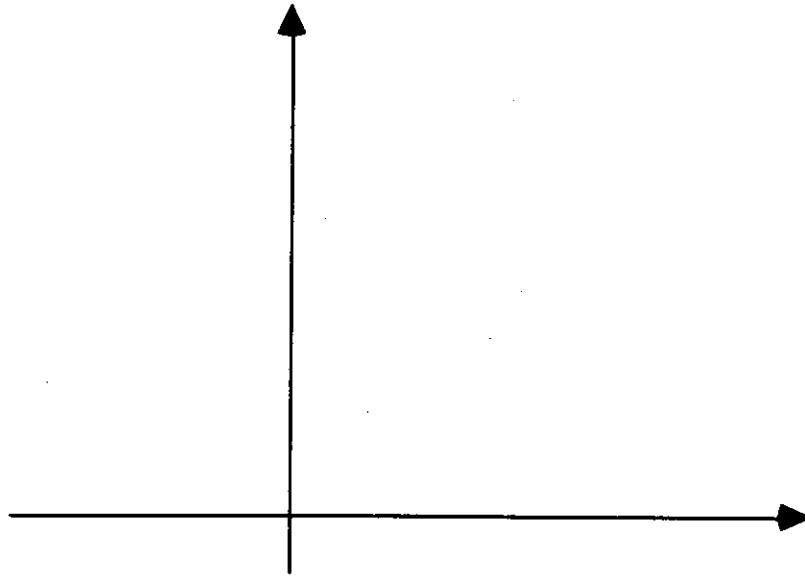
Total 14 marks

Question 3

a. On the same set of axes, clearly sketch the graphs with equations

i. $y = e^{-0.2x}$

ii. $y = 1 + e^{-0.2x}$



3 marks

In a group project on learning theory, a model for the proportion P of correct responses after n trials was constructed.

The model was found to approximately obey the rule with equation

$$P = \frac{0.80}{1 + e^{-0.2n}}, n \geq 0$$

b. What proportion of responses were correct after

i. the first trial

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1 mark

ii. the tenth trial

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1 mark

c. After how many trials will 60% of the responses be correct ?

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3 marks

d. What proportion of responses will eventually be correct ?

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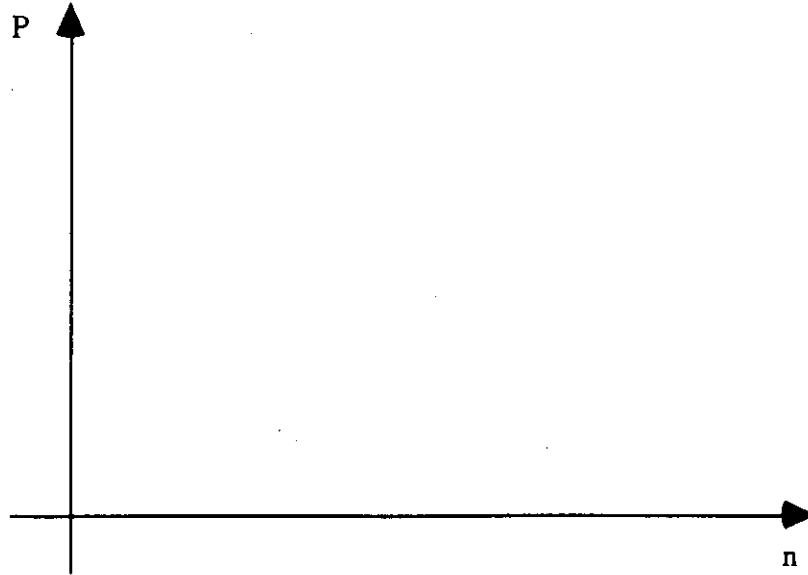
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2 marks

e. Making use of parts a. to d., sketch the graph of P.



4 marks

f. i. Find $\frac{dP}{dn}$

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2 marks

It can be shown that $e^{-0.2n} = \left(\frac{0.80}{P} - 1\right)$

ii. Show that $\frac{dP}{dn} = 0.2P(1 - 1.25P)$

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3 marks

Total 19 marks

Question 4

From a large batch of electric parts, a random sample of 20 such parts is selected. It is found that the proportion of defective electric parts in this batch is 0.07. Let X denote the number of defective parts found in the sample of 20.

a. Find

i. $P(X = 0)$

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1 mark

ii. $P(X = 1)$

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1 mark

A quality control procedure is set in place for accepting or rejecting a large batch of electrical parts as described above. The quality control engineer sets up the following process:

From the batch a random sample of 20 is selected. If the sample contains not more than 1 defective then the batch is accepted. If the batch contains at least two defectives it is rejected.

b. Using this process, find the probability of accepting a batch of electrical parts.

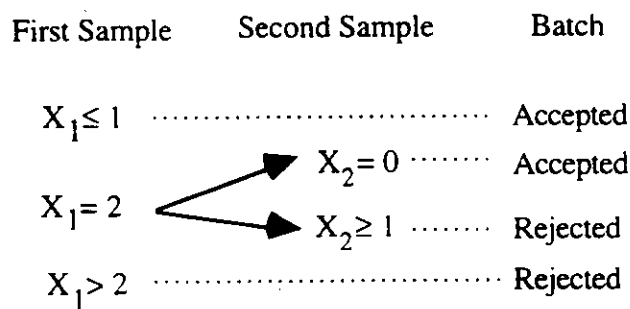
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2 marks

It is found that this process is not sufficiently discriminative, and so, an additional check is implemented into the quality control procedure.

The engineer now draws a chart describing the procedure:

Let X_1 denote the number of defectives in the first sample of 20 and X_2 denote the number of defectives in the second sample of 20 (if a second sample is necessary).



Because of the size of the batch it can be assumed that the two samples are independent and that the proportion of defectives in each sample is not altered.

c. Find the probability that a second sample is necessary.

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2 marks

d. Show that the probability that the batch is accepted is given by 0.6459

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3 marks

One hundred such batches are inspected, each having the same properties as already described.

e. Find an approximate 95% confidence interval for the proportion of batches accepted.

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5 marks

Total 14 marks