Question 1

a. Choose t = 0.

When $t = 0$, $N = 10^{15} \times 10^{10} = 10^{25}$ M1, A1

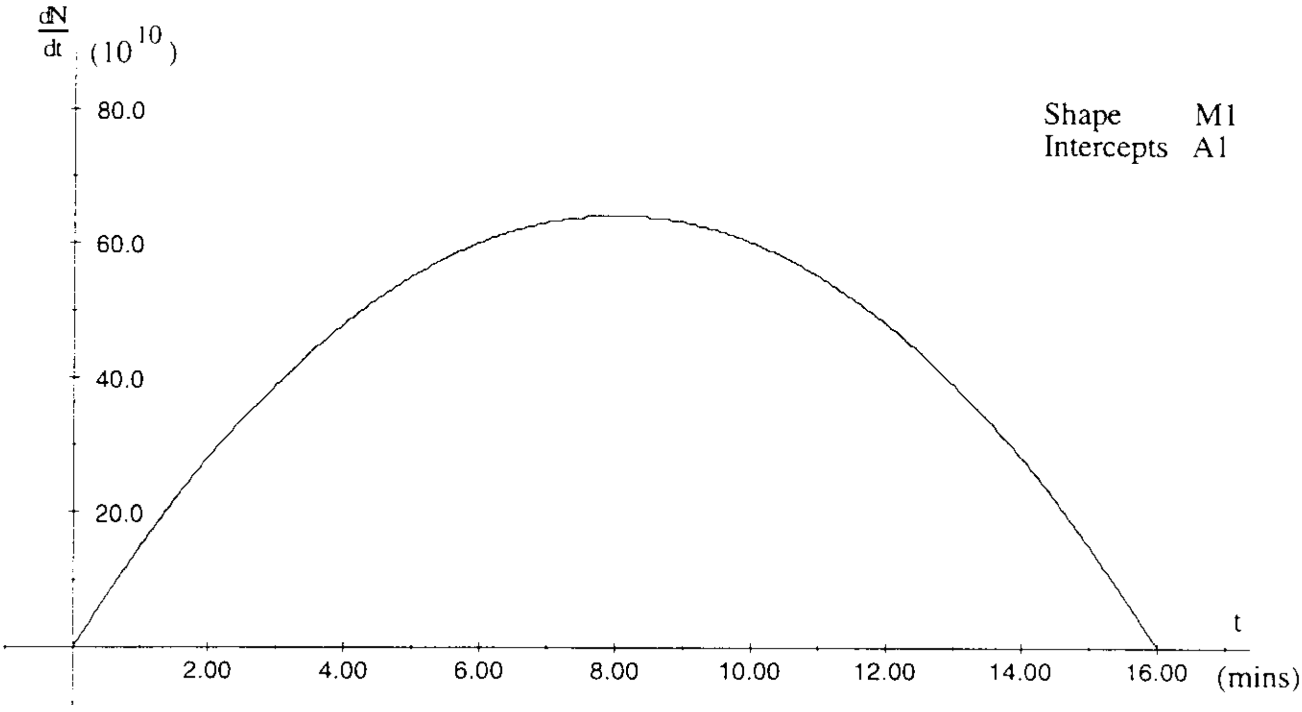
b. Average rate of change $= \dfrac{N(16) - N(0)}{16 - 0}$ M1

$= \dfrac{(10^{15} - \frac{1}{3}(16)^3 + 8(16)^2)10^{10} - 10^{25}}{16}$

$= \frac{128}{3} \times 10^{10}$ A1

c. $\dfrac{dN}{dt} = (0 - t^2 + 16t) \times 10^{10} = (16t - t^2) \times 10^{10}$ M1,A1

d. $\dfrac{dN}{dt} = t(16 - t)10^{10}$.



Shape M1
Intercepts A1

e. Maximum number of particles occur when $\dfrac{dN}{dt} = 0$.

$t(16 - t)10^{10} = 0$ when $t = 0$ or $t = 16$ M1,A1

When $t = 0$, $N = 10^{25}$

$t = 16$, $N = \left(10^{15} + \dfrac{2048}{3}\right)10^{10}$.(Maximum occurs when t = 16). A1,A1

Use of sign of first derivative to show maximum is obtained when t = 16 must be shown to gain both A1 *marks.*

f. From the graph in part d., $\dfrac{dN}{dt}$ is a maximum when t = 8. That is, at 8 minutes. M1,A1

Question 2

a. (0,4) A1

(3,0) A1

b. $f(x) = a + bx^2$ M1

(0,4): $4 = a + 0$, therefore $a = 4$ A1

(3,0): $0 = 4 + b(3)^2$, therefore $b = -\frac{4}{9}$ A1

c. $f'(x) = -\frac{8}{9}x$ M1

$-\frac{8}{9}x = -1$ M1

Therefore $x = \frac{9}{8}$ (distance from CD) A1

$f(\frac{9}{8}) = \frac{55}{16}$ (distance from AB) A1

d. i. Surface area $= \int_0^3 (4 - \frac{4}{9}x^2)\,dx$ M1

$= \left[4x - \frac{4}{27}x^3\right]_0^3$ A1

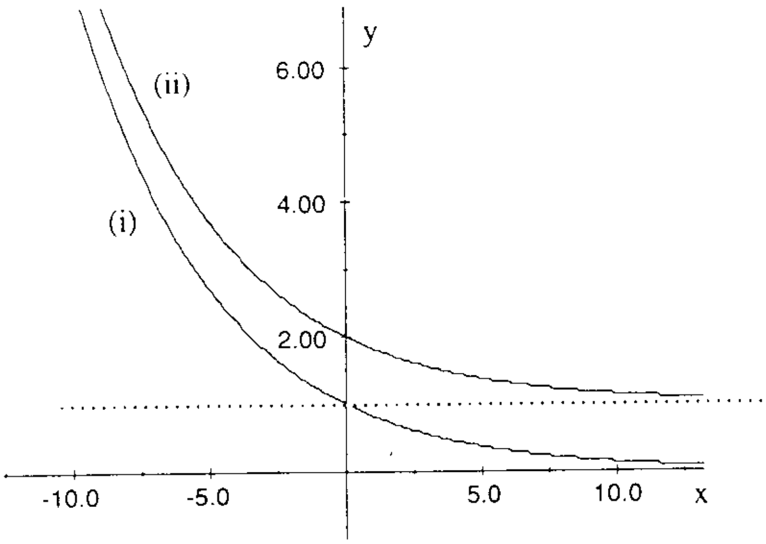
$= 8$ A1

ii. Cost = 190 + 40 + 56(8)(0.25) M1

That is, cost is \$ 342.00 A1

Question 3

a.



Graph i. A1

Graph ii. A1,M1

b. **i.** n = 1, $P = \frac{0.8}{1+e^{-0.2}} = 0.4399$ A1

ii. n = 10, $P = \frac{0.8}{1+e^{-2}} = 0.7046$ A1

c. $P = 0.60,\quad 0.60 = \dfrac{0.8}{1+e^{-0.2n}}$

$1 + e^{-0.2n} = \dfrac{0.8}{0.6}$ M1

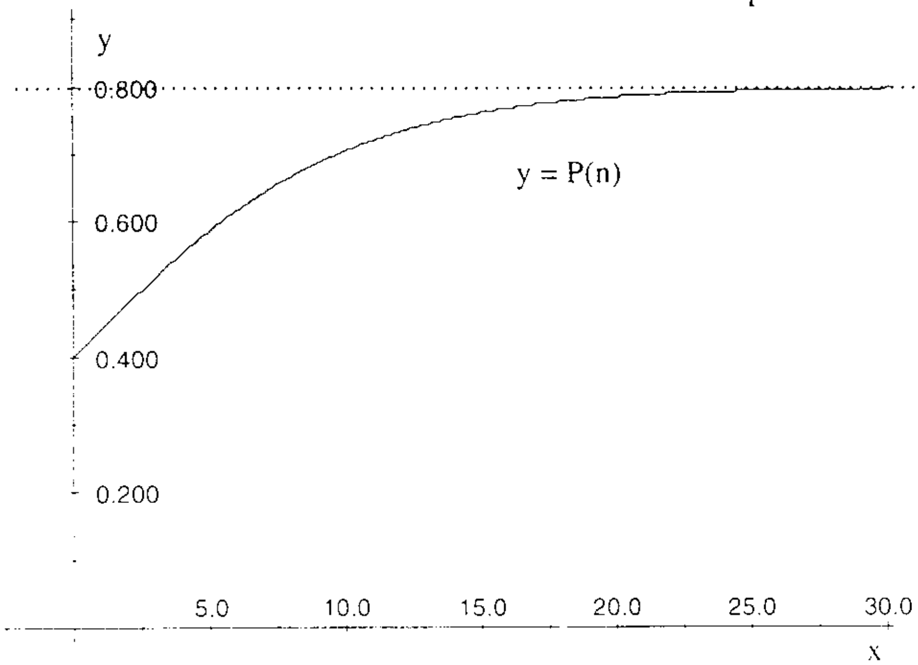
$-0.2n = \log_e\left(\dfrac{1}{3}\right)$ M1

$n = 5.49$

Therefore need at least 6 trials A1

d. $n \to \infty,\ P \to \dfrac{0.8}{1+0} = 0.8$ M1 A1

e. Shape A1 Asymptote A1



f. **i.** $\dfrac{dP}{dn} = (0.8)(-1)(-0.2e^{-0.2n})\left(1+e^{-0.2n}\right)^{-2}$ M1

$= \dfrac{0.16e^{-0.2n}}{\left(1+e^{-0.2n}\right)^2}$ A1

ii. $\dfrac{dP}{dn} = (0.8)(-1)(-0.2e^{-0.2n})\left(1+e^{-0.2n}\right)^{-2}$

$= \dfrac{(0.2)e^{-0.2n}}{0.8}\left(\dfrac{0.8}{\left(1+e^{-0.2n}\right)}\right)^2$ M1

$= (0.2)\left(\dfrac{1}{P} - 1.25\right)P^2$ M1

$= 0.2P(1 - 1.25P)$ A1

Question 4

a. **i.** $P(X=0) = {}^{20}C_0(0.07)^0(0.93)^{20} = 0.2342$ A1

ii. $P(X=1) = {}^{20}C_1(0.07)^1(0.93)^{19} = 0.3526$ A1

b. $P(\text{Accepting}) = P(X \leq 1) = P(X=0) + P(X=1)$

$= {}^{20}C_0(0.07)^0(0.93)^{20} + {}^{20}C_1(0.07)^1(0.93)^{19}$ M1

$= 0.2342 + 0.3526$

$= 0.5868$ A1

c. $P(X_1=2) = {}^{20}C_2(0.07)^2(0.93)^{18}$ M1

$= 0.2521$ A1

d. $P(\text{Accepted}) = P(X_1 \leq 1) + P(X_1=2)P(X_2=0)$ M1

$= 0.5868 + \left({}^{20}C_2(0.07)^2(0.93)^{18} \times {}^{20}C_0(0.07)^0(0.93)^{20} \right)$ A1

$= 0.5868 + (0.2521)(0.2342)$

$= 0.6459$ A1

e. Let Y = the number of batches accepted from the 100.

Then $Y \overset{d}{=} Bi(100, 0.6459)$

Then, $P = \frac{Y}{100}$ is the proportion of batches accepted.

$E(P) = \hat{p} = 0.6459$ A1

$Var(P) = \frac{\hat{p}(1-\hat{p})}{n} = \frac{0.6459(0.3541)}{100} = 0.0023$ A1

Approx 95% C.I is given by $0.6459 \pm 2\sqrt{0.0023}$ M1,A1

0.5502 to 0.7415 A1