SECTION A: Multiple-choice questions

Question 1

The domain of the function $f(x) = \frac{1}{\sqrt{x - 2}}$ is

A. $[2,\infty)$

B. $[-2,\infty)$

C. $(-\infty,2)$

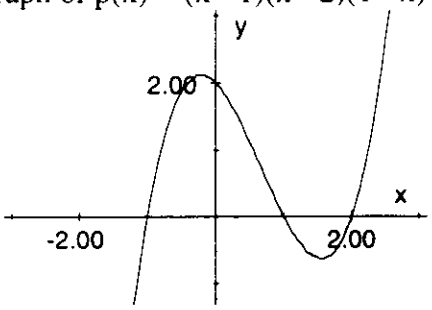
D. $(2,\infty)$

E. $R\backslash\{2\}$

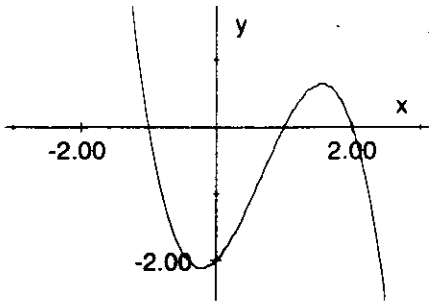
Question 2

The graph of $p(x) = (x - 1)(x - 2)(1 - x)$ is given by

A.



B.



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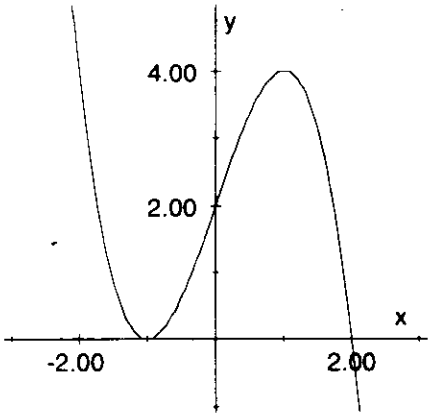
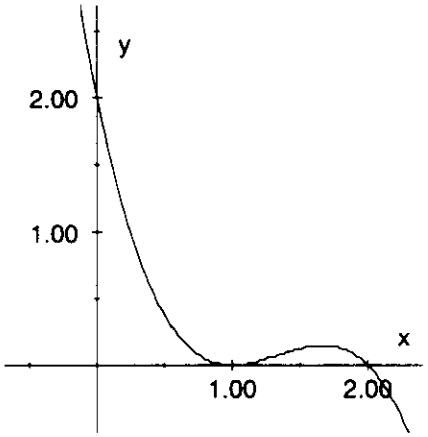
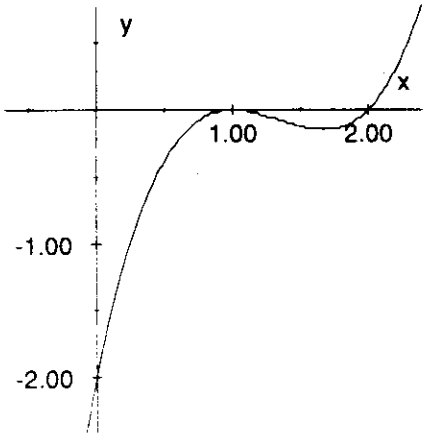


D.



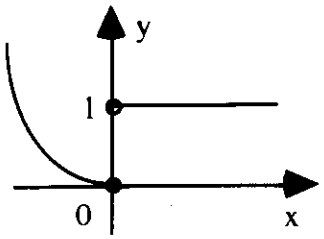
E.



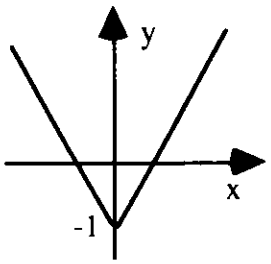
Question 3

Which one of the following relations is not a function ?

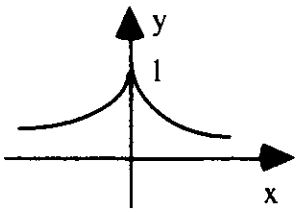
A.



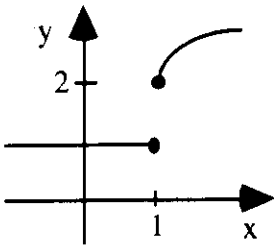
B.



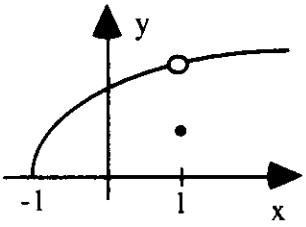
C.



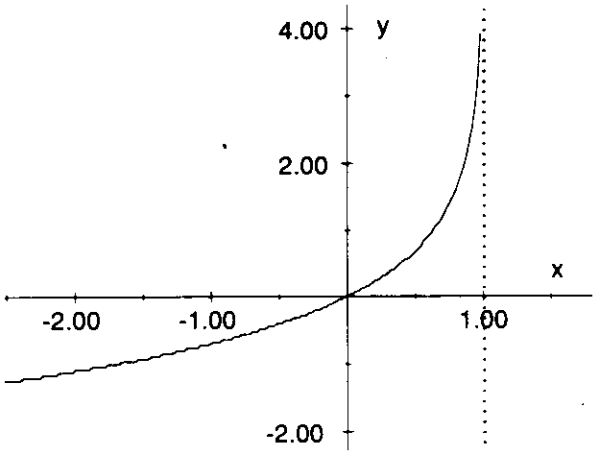
D.



E.



Question 4



The equation which corresponds to the graph above is

A. $y = -\log_e(x + 1)$

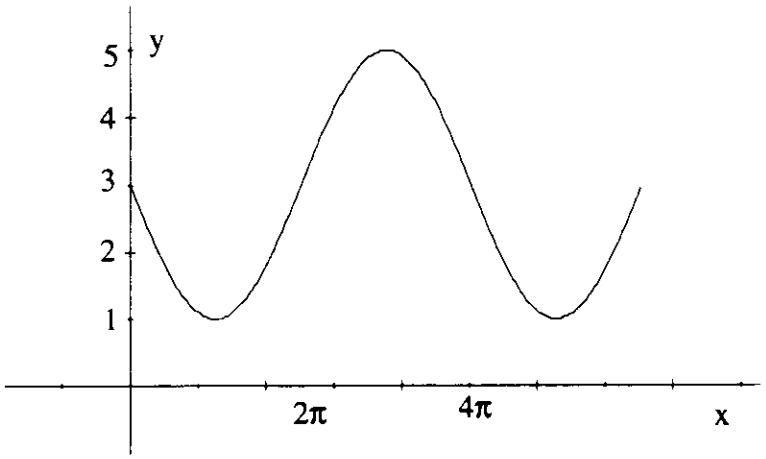
B. $y = \log_e(1 - x)$

C. $y = -\log_e(x - 1)$

D. $y = \log_e(x - 1) + 1$

E. $y = -\log_e(1 - x)$

Question 5



For the graph shown above, we have that $y = 3 + b\sin kx$. This means that

A. $b = 2, k = 2$.

B. $b = 2, k = \frac{1}{2}$.

C. $b = -2, k = 2$.

D. $b = -2, k = \frac{1}{2}$.

E. $b = -5, k = \frac{1}{2}$.

Question 6

$\{x: 2\sin x = \sqrt{3}, 0 \leq x \leq 2\pi\}$ is equal to

A. $\{\frac{2\pi}{3}\}$

B. $\{\frac{\pi}{3}, \frac{4\pi}{3}\}$

C. $\{\frac{\pi}{3}, \frac{2\pi}{3}\}$

D. $\{\frac{\pi}{6}, \frac{\pi}{3}\}$

E. $\{\frac{\pi}{6}\}$

Question 7

The equation $e^{2x} - 2e^{x} - 3 = 0$ has solution set

A. $\{-1, 3\}$

B. $\{ln\ 3\}$

C. $\{-3, 1\}$

D. $\{ln\ 1\}$

E. $\{ln\ 2\}$

Question 8

The graph of the function whose rule is given by $f(x) = \frac{1}{x}$, undergoes the following transformations;

A reflection about the x-axis, followed by a dilation of factor 3 parallel to the x-axis and then a translation of +2 units parallel to the y-axis.

The equation of this transformed function is

A. $y = -\frac{1}{3x} - 2$

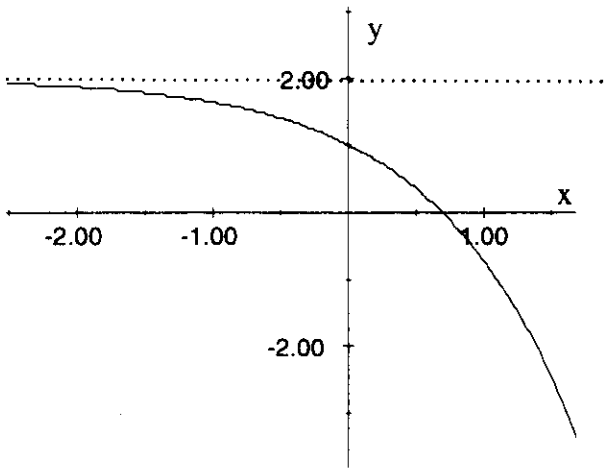
B. $y = -\frac{3}{x} + 2$

C. $y = \frac{3}{x} + 2$

D. $y = -\frac{1}{3x} + 2$

E. $y = -\frac{3}{x+2}$

Question 9



The equation corresponding to the graph above is

A. $y = -e^x + 2$

B. $y = -e^{-x} + 2$

C. $y = 2e^{-x} + 1$

D. $y = -(e^{-x} + 2)$

E. $y = -(e^x + 2)$

Question 10

The value of x, to two decimal places, which satisfies the equation $10^{0.2x} + 4 = 9$ is equal to

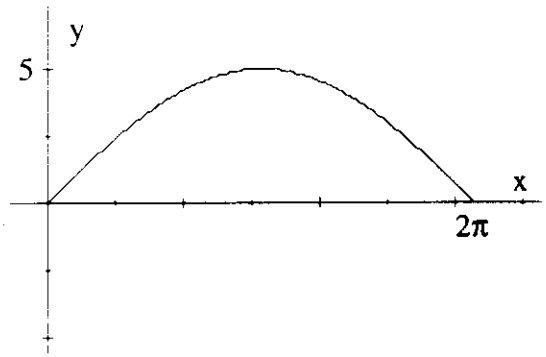
A. 80.47
B. 1.609
C. 65
D. 55.70
E. 3.495

Question 11

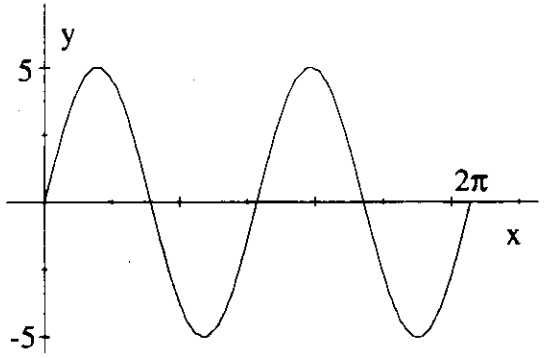
Which one of the following graphs represents the function defined as

$$f:[0,2\pi] \rightarrow R, \text{ where } f(x) = 5\sin\left(\frac{x}{2} + \pi\right) ?$$

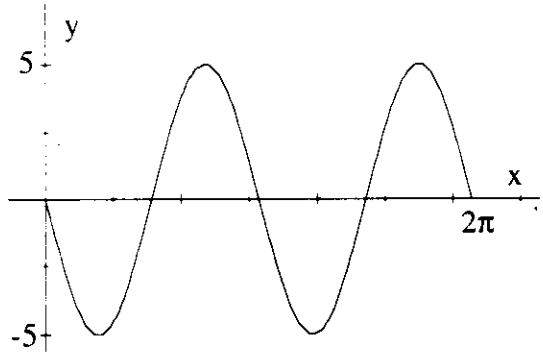
A.



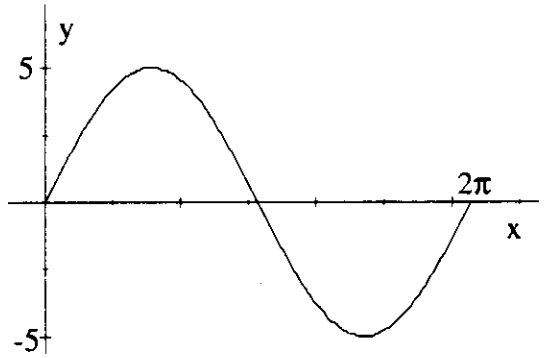
B.



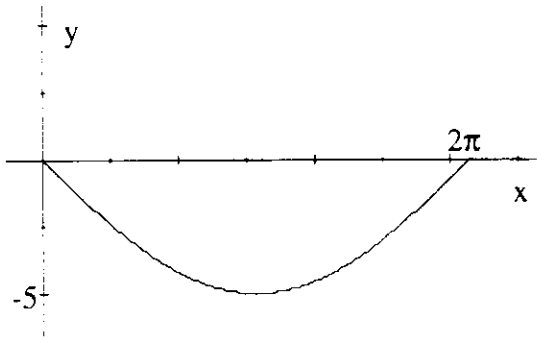
C.



D.



E.

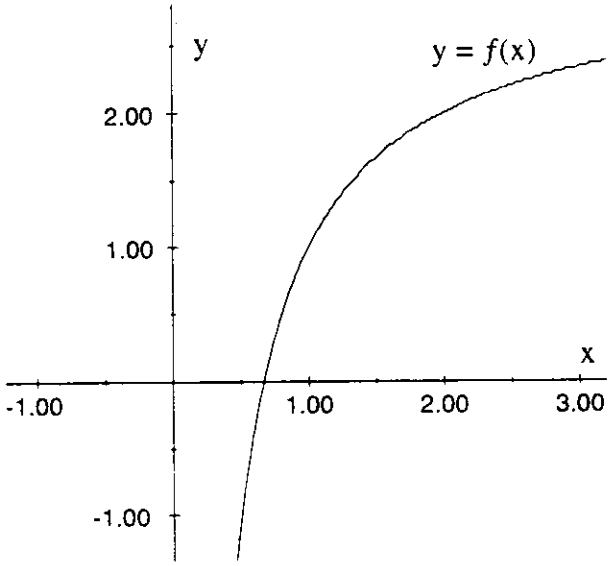


Question 12

The range of the function $f:[0,\infty) \rightarrow R$, where $f(x) = \sqrt{x+4} - 1$ is given by

A. $[1,\infty)$
B. $(1,\infty)$
C. $[-1,\infty)$
D. $(-1,\infty)$
E. $[-4,\infty)$

Question 13



From the graph of $y = f(x)$, shown above, the number of solutions to the equation $f(x) = f^{-1}(x)$ is

A. 0
B. 1
C. 2
D. ± 1
E. 1 and 2

Question 14

The velocity, v m/s, of a particle is given by the equation $v(t) = 3 + 2\cos(\pi t)$. Its minimum velocity during the interval $0 \leq t \leq 0.5$ is

A. 1
B. 2
C. 3
D. 3.99
E. 5

Question 15

$\frac{d}{dx}(x^2\cos x)$ is equal to

A. $-2x\sin x$

B. $2x\cos x + x^2\sin x$

C. $2x - \sin x$

D. $2x\cos x - x^2\sin x$

E. $\frac{1}{3}x^2\cos x + x^2\sin x$

Question 16

Find $f'(x)$ if $f(x) = \sqrt{x^4+9}$

A. $2x^3\sqrt{x^4+9}$

B. $\frac{4x^3}{\sqrt{x^4+9}}$

C. $\frac{2x}{\sqrt{x^4+9}}$

D. $\frac{2x^3}{\sqrt{x^4+9}}$

E. $\frac{2x^3}{x^2+3}$

Question 17

Find the gradient of the curve $y = \log_e(x^2+1)$ when $x = 5$.

A. $\frac{1}{26}$

B. $\frac{10}{26}$

C. $\frac{1}{25}$

D. $\frac{2}{5}$

E. $\frac{26}{5} + 10\log_e 5$

Question 18

If $g(x) = e^{3x^2+2}$, then $g'(3)$ is equal to

A. $18e^{29}$
B. $18e^{28}$
C. $27e^{28}$
D. $3e^{29}$
E. $\frac{1}{3}e^{29}$

Question 19

If $f'(x) = e^{\frac{1}{3}x} - \frac{1}{x^2}$, then, for any real number c, $f(x)$ is equal to

A. $\frac{1}{3}e^{\frac{1}{3}x} - \frac{1}{x} + c$
B. $\frac{1}{3}e^{\frac{1}{3}x} + \frac{1}{x} + c$
C. $3e^{\frac{1}{3}x} - \frac{1}{x} + c$
D. $3e^{\frac{1}{3}x} + \frac{1}{x} + c$
E. $\frac{1}{3}e^{\frac{1}{3}x} + \frac{2}{x^3} + c$

Question 20

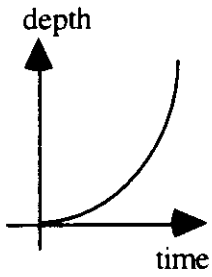
$\int_{-1}^{1}(2x - e^{-x})dx$ is equal to

A. 0
B. $e - e^{-1}$
C. $e^{-1} - e$
D. $2(1+e^{-1})$
E. $2(1-e^{-1})$

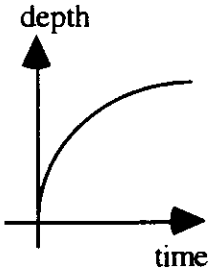
Question 21

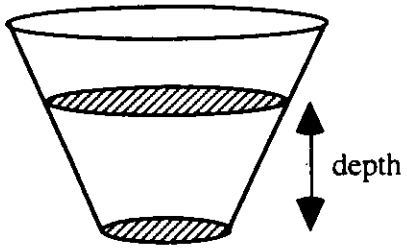
Water is being poured into a pot at a constant rate. The graph which shows how the depth of water changes with time for the pot shown below is

A.

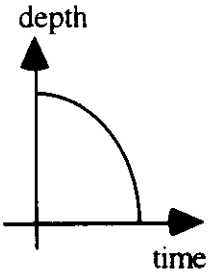


B.

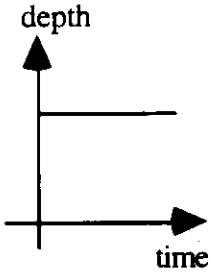




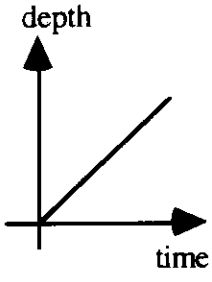
C.



D.



E.



Question 22

$\int_0^{\frac{\pi}{2}} (\sin x + \cos x)dx$ is equal to

A. 0
B. 1
C. - 1
D. 2
E. - 2

Question 23

$\int_2^4 (5f(x)+10)dx$ is equal to

A. $5\int_2^4 f(x)dx + 2$

B. $5\int_2^4 f(x)dx + 10$

C. $5\int_2^4 f(x)dx + 10x$

D. $\int_2^4 5f(x)dx + 50$

E. $5\int_2^4 f(x)dx + 20$

Question 24

The area, in square units, enclosed by the curve $y = 4 - \frac{1}{x}$, the x-axis and the lines $x = 1$ and $x = 4$ is

A. $16 - \log_e 4$

B. $12 + \log_e 4$

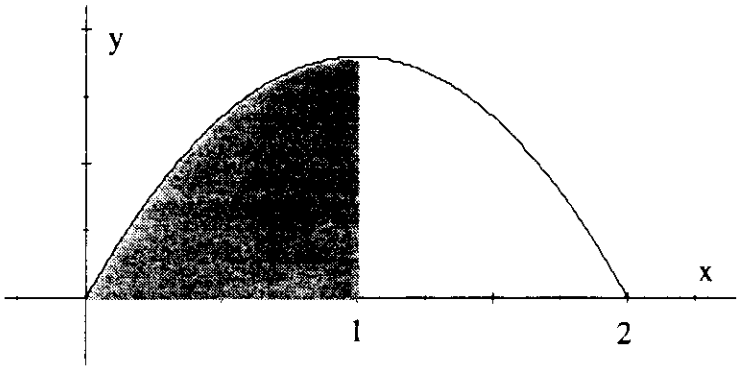
C. 12

D. $12 - \log_e 4$

E. $11\frac{1}{16}$

Question 25

The graph with equation $y = ax(x-2)$ is shown below. Given that the shaded region has an area of 6 square units, the value of a must be



A. 9

B. -9

C. 4

D. -4

E. $-\frac{9}{2}$

Question 26

An antiderivative of $\frac{1}{(2x-1)^2}$ is equal to

A. $\frac{1}{2}\log_e(2x-1)^2$

B. $-\frac{1}{2x-1}$

C. $-\frac{1}{2(2x-1)}$

D. $\frac{4}{(2x-1)^3}$

E. $-\frac{4}{(2x-1)^3}$

Question 27

Which one of the following represents a probability distribution function ?

A.

x	1	2	3	4	5
P(X=x)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{5}$

B.

x	2	3	4	5	6
P(X=x)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$

C.

x	1	2	3	4	5
P(X=x)	-0.7	0.4	0.3	0.6	0.4

D.

x	-2	-1	0	1	2
P(X=x)	$\frac{1}{8}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{8}$

E.

x	-2	-1	0	1	2
P(X=x)	0	0.4	0.1	0	0.5

Questions 28 and 29 refer to the following information.

The random variable X denotes the number of cars that cross a particular intersection in a ten minute interval. This random variable is found to have the following probability distribution:

x	0	1	2	3	4
P(X=x)	0.05	0.2	0.3	0.4	0.05

Question 28

The average number of cars expected to cross this intersection in a **20 minute** interval is

A. 1
B. 1.1
C. 2
D. 2.2
E. 4.4

Question 29

During peak hour traffic it is thought that a better model would be described by the random variable Y, where $Y = 2X + 1$. The variance of Y is given by

A. $E(X^2) - (E(X)^2)$
B. $2Var(X) + 1$
C. $4Var(X) + 1$
D. $2Var(X)$
E. $4Var(X)$

Question 30

A bag contains 4 blue balls and 5 red balls. Geoff selects one ball, notes its colour and then replaces it into the bag. He continues this process another 4 times. The probability that Geoff selects at least 2 red balls is given by

A. $\binom{9}{2}(0.55)^2(0.45)^7 + \binom{9}{3}(0.55)^3(0.45)^6 + \binom{9}{4}(0.55)^4(0.45)^5 + \binom{9}{5}(0.55)^5(0.45)^4$
B. $1 - \left(\binom{9}{0}(0.55)^0(0.45)^9 + \binom{9}{1}(0.55)^1(0.45)^8\right)$
C. $\binom{5}{2}(0.55)^3(0.45)^2 + \binom{5}{3}(0.55)^2(0.45)^3 + \binom{5}{4}(0.55)^1(0.45)^4 + (0.45)^5$
D. $\binom{5}{2}(0.55)^2(0.45)^3 + \binom{5}{3}(0.55)^3(0.45)^2 + \binom{5}{4}(0.55)^4(0.45)^1 + (0.55)^5$
E. $1 - \left(\binom{5}{2}(0.55)^2(0.45)^3 + \binom{5}{1}(0.55)^1(0.45)^4 + \binom{5}{0}(0.55)^0(0.45)^5\right)$

Question 31

Researchers took a random sample of 200 fish from a very large fishing farm. It was found that 55 of these fish were at least 68 cm long.
The best statement that the researchers can make about the proportion of fish on the farm that are less than 68 cm long is that:

A. 72.5% of the fish on the farm are less than 68 cm
B. they are approximately 95% confident that the proportion shorter than 68 cm lies between 0.21 & 0.34
C. they are approximately 95% confident that the proportion shorter than 68 cm lies between 0.66 & 0.79
D. they are approximately 95% confident that the proportion shorter than 68 cm lies between 0.24 & 0.31
E. they are approximately 95% confident that the proportion shorter than 68 cm lies between 0.69 & 0.76

Question 32

During a physics experiment, the rebound height of a plastic ball is found to be normally distributed with mean 0.90 m and variance 0.09 m.
The probability that on any one trial the ball rises to a height greater than 0.95 is equal to
(*Give answer correct to four decimal places*)

A. 0.5660
B. 0.1667
C. 0.5556
D. 0.2895
E. 0.4340

Question 33

Given that $X \overset{d}{=} N(1,1)$. If $P(X \geq a) = 0.05$, then a must be equal to

A. 0.6449
B. 1.6449
C. 1.9600
D. 2.6449
E. 2.9600

SECTION B: Short answer questions

Question 1

A biased coin is tossed 8 times. The probability of obtaining a tail on any one throw is 0.4. Let X denote the number of tails observed.

a. Calculate E(X)

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b. Calculate $Pr(X \geq 1)$

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1 + 2 marks

Question 2

a. Find $\int \frac{1}{(3x+1)^3}\,dx$

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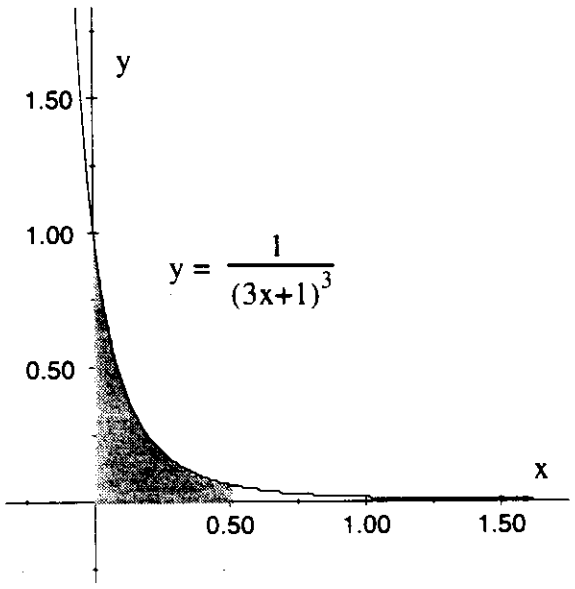
b. **Hence** find the shaded area for the graph shown.



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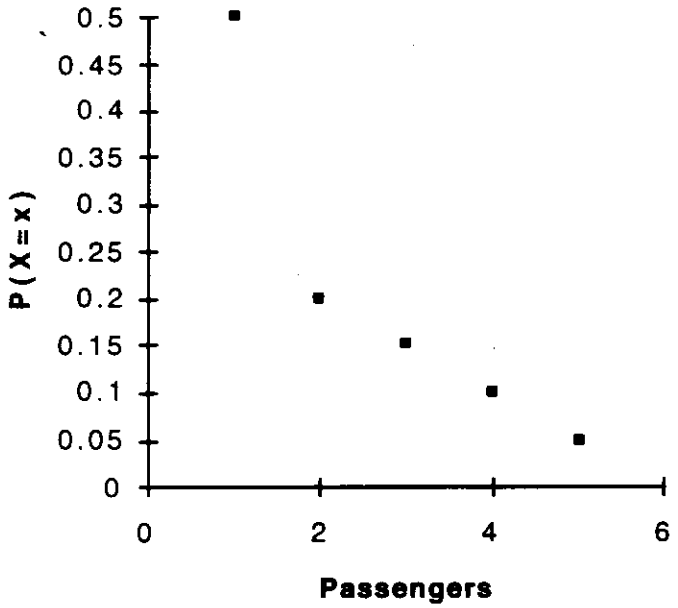
2 + 1 marks

Question 3

The number, X, of passengers in a car crossing a toll bridge has the following probability distribution:



a. Find the mean of X

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b. If the toll is 10 cents per passenger and $1.00 per car, find the expected toll charge paid.

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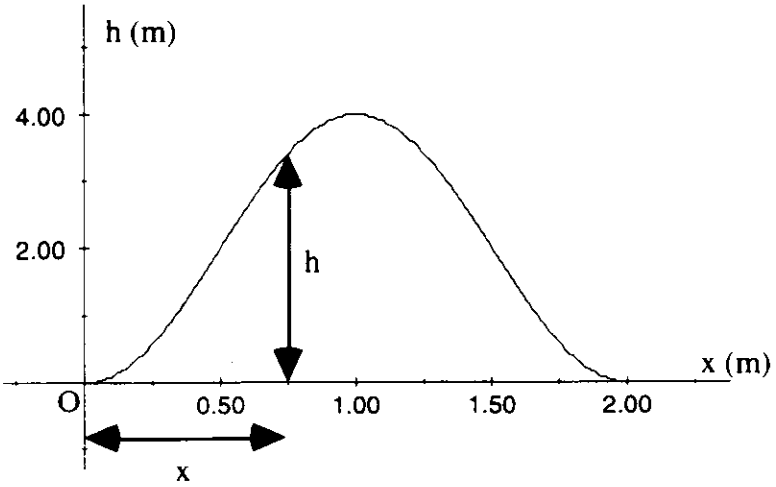
1 + 2 marks

Question 4

A hill has its cross-section modelled by the function, $h:[0,2] \rightarrow R$, where $h(x) = 2 - 2\cos(\pi x)$, and $h(x)$ measures the height of the hill relative to the horizontal distance, x m, from O.(See figure below).



a. How far horizontally from O, would a person climbing this hill be, when first ascending to a height of 1 metre ?

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b. Find the gradient of hill at this particular instant.

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2 + 2 marks

Question 5

The cost C, in dollars, of storing x containers in a warehouse is modelled by the function

$$C(x) = 10x + \frac{4000}{x} + 5000,\ x > 0.$$

a. Find the rate of change, C'(x).

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b. Find the number of boxes which should be stored in order to minimize the cost.

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1 + 3 marks