YEAR 12
IARTV TEST — OCTOBER 1994
MATHEMATICAL METHODS CAT 3
ANSWERS & SOLUTIONS

1

(a) $h'(x) = 6 - 2x$
$h'(x) = 0$ implies $x = 3$
Therefore height = 900 m (3)

(b) $6x - x^2 = 0$ implies $x = 0$ or $x = 6$.
Therefore B is at the point (6,0) (3)

(c) $\text{Area} = \int_2^6 (6x - x^2) - (-2x + 12) \, dx = \int_2^6 (-x^2 + 8x - 12) \, dx$ (2)

$= \left[\frac{-x^3}{3} + 4x^2 - 12x \right]_2^6$ (2)

$= 10\frac{2}{3}$ sq. units (2)

(d) Volume = $500 \times 10\frac{2}{3} = 5333\frac{1}{3}$ cubic units (3)

2

(a) Grade A : $P(z \geq 0.8) = 0.21184$ (1)
Grade B : $P(-1.2 < z < 0.8) = 0.67308$ (1)
Grade C : $P(z \leq -1.2) = 0.11507$ (1)

(b) Proportion of Grade A $= \frac{P(A)}{P(A) + P(B)} = \frac{0.21184}{0.88492} = 0.2394$ (3)

(c) $40(0.21184) + 30(0.67308) = (0.21184 + 0.67308) x$
Therefore x = 32.39 cents (3)

(d) $E(x) = \$50000[0.21184 \times 40 + 0.67308 \times 30 + (0.11507) \times 10] = \$14908.35$ (2)

(e) $P(z > m) = 0.15$ implies $P(z < m) = 0.85$
Therefore $m = 1.04$ (2)

Therefore $1.04 = \frac{x - 84}{5}$ and so x = 89.2 mm (2)

3

(a)

t	0	0.5	1
C(t)	0	0.507	0.275

(3)

(b) $C'(t) = 30t\,e^{-4t} + 15t^2(-4)\,e^{-4t} = 30t\,e^{-4t}(1 - 2t)$ (1)

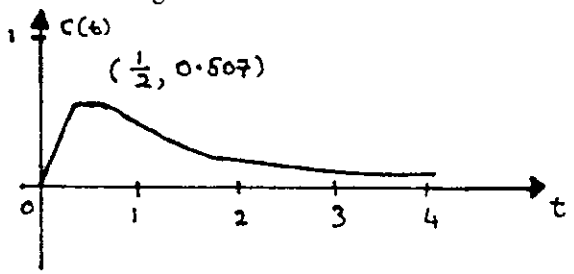
$C'(t) = 0$ implies $t = 0$ or $t = \frac{1}{2}$ (4)

Therefore maximum concentration is 0.507 units and this occurs $\frac{1}{2}$ an hour after the injection. (1)

(c) The concentration decreases to zero

because $\lim\left(\frac{15t^2}{e^{4t}}\right) = 0$ as t tends to infinity. (2)

(d)



(4)

4

(a) $P'(x) = 4 - \frac{10000}{x^2}$ and $P'(x) = 0$ implies $x = 50$ (3)

Therefore 50 leg Newtons minimises the pressure.

($P''(x) > 0$ when $x = 50$) (2)

(b)(1) $\log_e P = -6t + \log_e A$

$\log_e\left(\frac{P}{A}\right) = -6t$

$\frac{P}{A} = e^{-6t}$ and so $P = Ae^{-6t}$ (4)

Therefore $\frac{1}{2} = Ae^0$ implies $A = \frac{1}{2}$

Therefore $P = \frac{1}{2}e^{-6t}$ (3)

(2) If $t = 10$ min $= \frac{1}{6}$ hour, then $P = \frac{1}{2}e^{-1} = 0.1839$

Therefore $P = 0.184$ (3 d.p.) (3)
