Neap

VCE General Mathematics Units 3&4

Question and Answer Booklet

2024 Trial Examination 2

Reading time: 15 minutes

Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Approved materials

- One bound reference
- One approved technology (calculator or software)
- One scientific calculator

Materials supplied

- Question and Answer Booklet of 23 pages
- Formula Sheet

Instructions

- Write your responses in English.
- Write **your name** and your **teacher's name** in the space above on this page.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2024 VCE General Mathematics Units 3&4 Examination.

Instructions

- Answer **all** questions in the spaces provided.
- In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Data analysis

Question 1 (6 marks)

The following back-to-back stem plot shows the distribution of the maximum temperatures, in degrees Celsius (°C), of the cities Barcelona and Florence over 21 days in March.

Barcelona		Florence
9 8 7 5	1	8 9
4 3 2 1 1 1 0 0	2	
9 9 8 7 6 5	2	8 9
3 2	3	3 3 4
8	3	5 5 6 7 7 7 8 8
	4	0 0 1 2 5 6

Key
1 | 8 = 18°C

a. Which variable, *city* or *temperature*, is a categorical variable? 1 mark

b. Describe the distribution of the maximum temperatures for Florence. 1 mark

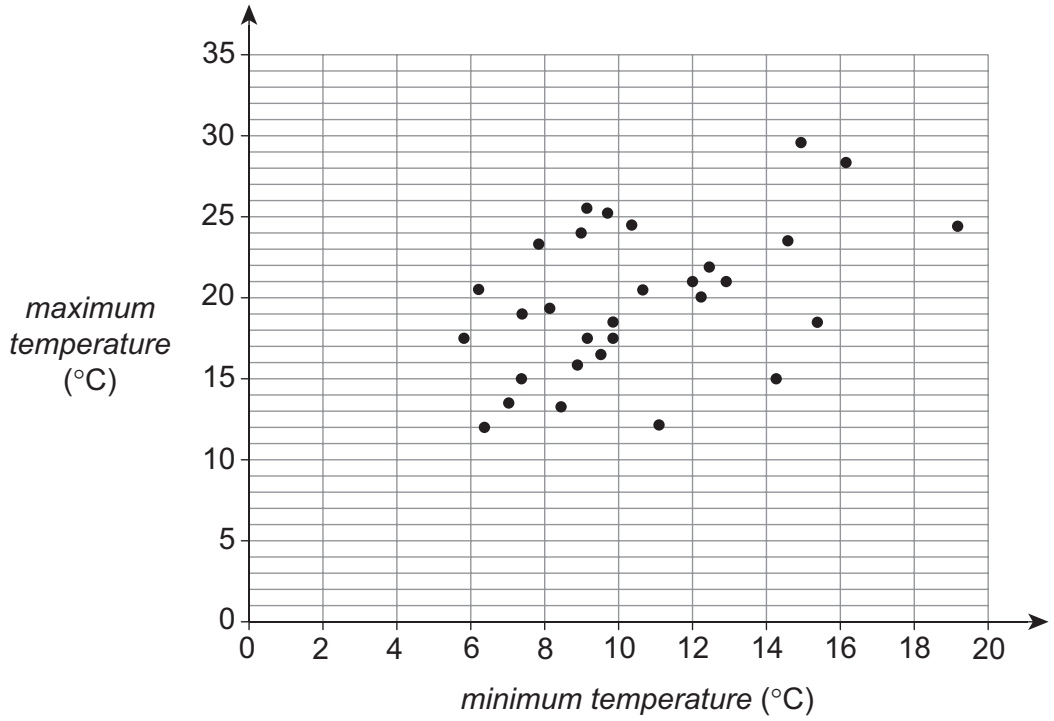
c. Use the information in the back-to-back stem plot on page 2 to complete the table below. 2 marks

	Minimum	**Lower quartile**	**Median**	**Upper quartile**	**Maximum**
Barcelona	15		23	28.5	38
Florence	18	33	37		46

d. Show that there are outliers in the maximum temperatures for Florence. 2 marks

Question 2 (6 marks)

The scatterplot below shows the *maximum temperature* and *minimum temperature* in Barcelona for each day in June.



Based on the scatterplot above, the correlation coefficient $r = 0.63$.

a. Describe the association between the variables *maximum temperature* and *minimum temperature* in terms of strength and direction. 1 mark

The equation of the least squares line for the scatterplot is shown below.

$$\textit{maximum temperature} = 15 + 0.73 \times \textit{minimum temperature}$$

b. Interpret the slope of the least squares line in terms of the variables *maximum temperature* and *minimum temperature*. 1 mark

c. **i.** Determine the coefficient of determination. 1 mark

ii. Interpret the coefficient of determination in terms of the variables *maximum temperature* and *minimum temperature*. 1 mark

d. On 13 June, the *minimum temperature* was 14°C and the *maximum temperature* was 20°C.

Determine the residual value for 13 June if the least squares line is used to predict the *maximum temperature*.

Give your answer correct to two decimal places. 2 marks

Question 3 (8 marks)

A meteorologist in Barcelona collected data on the number of extremely hot days over a 10-year period. The results are shown in the table below.

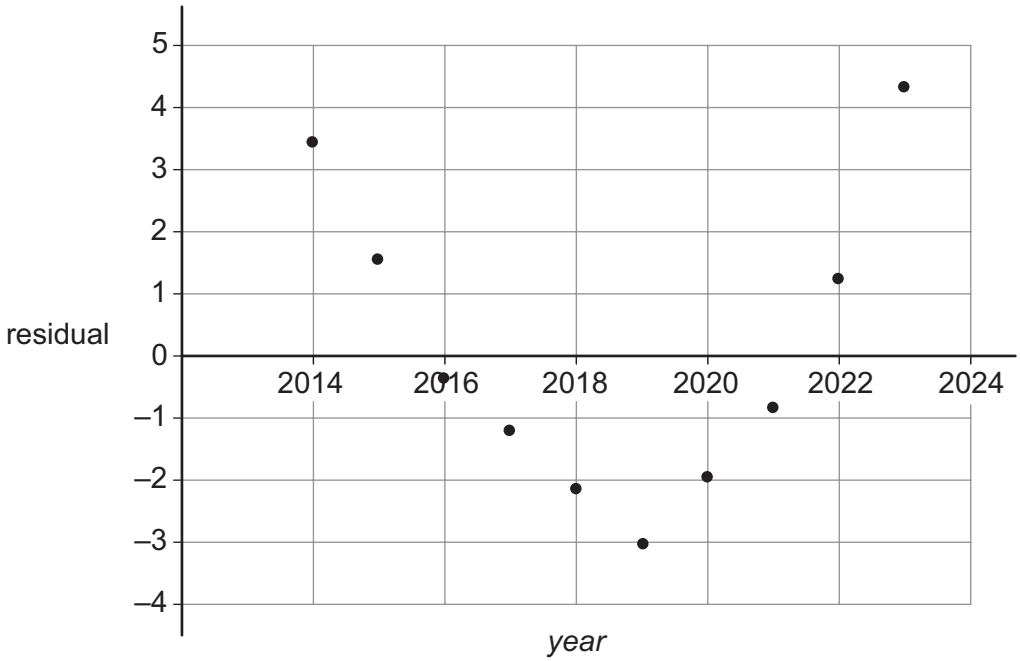
Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Number of extremely hot days	11	12	13	15	17	19	23	27	32	38

a. A least squares line can be used to model the association between *number of extremely hot days* and *year*, where the explanatory variable is *year*.

Using data in the table above, determine the equation of the least squares line. Write the values of the intercept and slope in the boxes provided, correct to four significant figures. 1 mark

number of extremely hot days = ☐ + ☐ × *year*

The least squares line is fitted to the data and the following residual plot is produced.



b. Does the residual plot support the assumptions of linearity? Justify your response. 1 mark

A square transformation can be applied to the variable *year* to linearise the data.

c. i. Using this square transformation, determine the equation of the least squares line that can be used to predict the *number of extremely hot days* from $year^2$.
Write the values of the intercept and slope in the boxes provided, correct to five significant figures. 1 mark

number of extremely hot days = ☐ + ☐ × $year^2$

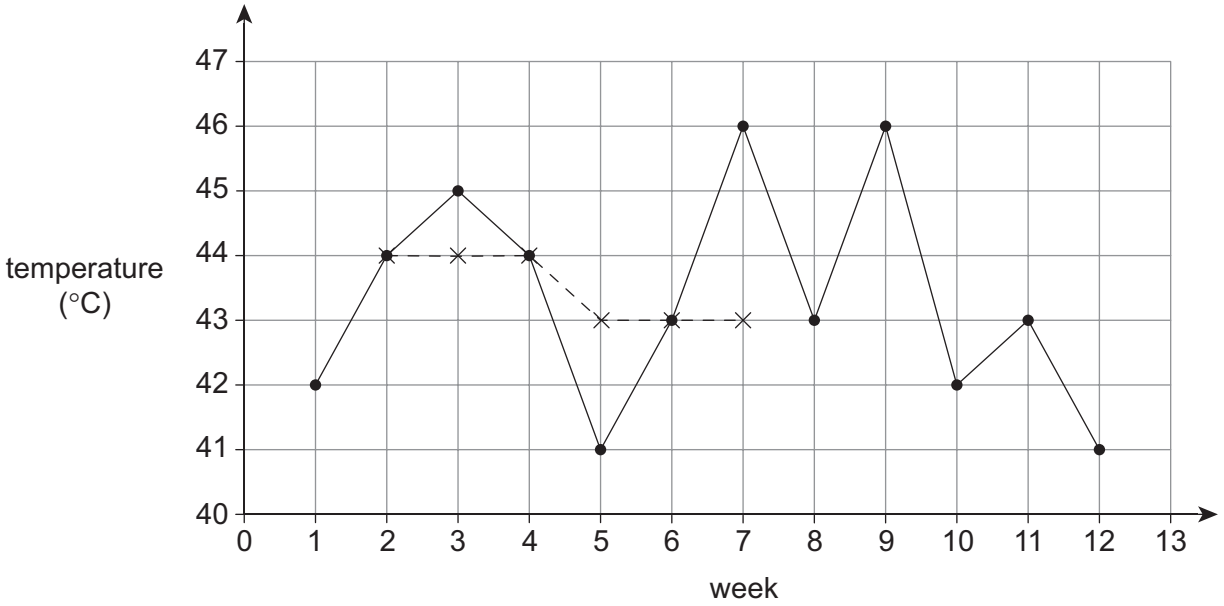
ii. Determine the correlation coefficient of the transformed data.
Give your answer correct to three decimal places. 1 mark

d. i. Using the equation of the transformed least squares line from **part c.i.**, predict the number of extremely hot days for 2027.
Give your answer correct to the nearest day. 1 mark

ii. Comment on the reliability of this prediction. Justify your response. 1 mark

In 2013, Barcelona experienced a summer heatwave. The following time series plot shows the average temperature during each week of the heatwave.

The data was collected over a period of 12 consecutive weeks.



Three-median smoothing has been applied to the data. The first six smoothed points are marked by crosses.

e. Sketch the remaining six smoothed values on the time series plot above. 2 marks

Question 4 (4 marks)

The total rainfall in Barcelona, in millimetres, for each season in 2021 and 2022 is shown in table 1 below.

Table 1

Year	Total rainfall (mm)			
	Summer	**Autumn**	**Winter**	**Spring**
2021	45	30	22	51
2022	53	29	26	60

a. Use the data in table 1 to complete the seasonal indices for table 2, correct to two decimal places. 2 marks

Table 2

	Summer	**Autumn**	**Winter**	**Spring**
Seasonal index				1.40

b. What does the seasonal index of 1.40 indicate in terms of Barcelona's total rainfall in spring? 1 mark

c. The total rainfall in Barcelona during each season in 2023 is shown in table 3 below.

Table 3

Year	Total rainfall (mm)			
	Summer	**Autumn**	**Winter**	**Spring**
2023	54	41	29	72

Use the appropriate seasonal index from table 2 to de-seasonalise the total rainfall for spring 2023.

Give your answer correct to the nearest whole number. 1 mark

Recursion and financial modelling

Question 5 (4 marks)

Carmen purchases a machine that costs $7600 to use in her business. Carmen uses the unit cost method to depreciate the machine. On average, the machine will depreciate by 20 cents per hour of use and will be used for 3800 hours each year.

a. Calculate the value of the machine after 3 years. 1 mark

b. Carmen is considering using flat rate depreciation at a rate of 10% per annum.

Show that, in any given year, the flat rate and unit cost methods will depreciate the cost of the machine by the same amount. 1 mark

c. If Carmen uses the reducing balance method, it would depreciate the value of the machine at a rate of 12% per annum.

Let V_n be the value of the equipment, in dollars, after n years.

Write a recurrence relation in terms of V_0, V_{n+1} and V_n to model the value of the machine after n years using reducing balance depreciation. 1 mark

d. At the end of which year will flat rate depreciation first result in a lower value for the machine than reducing balance depreciation? 1 mark

Question 6 (4 marks)

Carmen wants to purchase a block of land. She takes out a loan of $500 000 with an interest rate of 5.2% per annum, compounding monthly.

After three years of equal monthly repayments, the balance of Carmen's loan will be $462 740.82.

a. What is Carmen's monthly repayment during these three years?

Give your answer correct to the nearest cent. 1 mark

b. During the next four years, Carmen moves her loan to another bank with an interest rate that compounds fortnightly. She makes fortnightly repayments of $1907.15.

The balance of the loan, in dollars, after n fortnights, C_n, is modelled by the following recurrence relation.

$$C_0 = 462\,740.82,\ C_{n+1} = 1.004C_n - 1907.15$$

i. What is the annual percentage interest rate for this loan? 1 mark

ii. If the interest rate is changed to 5.5% per annum, and Carmen makes the same fortnightly repayments of $1907.15, what will be the remaining balance of the loan after four more years?

Give your answer correct to the nearest cent. 1 mark

c. After a certain number of years, the balance of Carmen's loan will be $182 567.87.

If Carmen changes to weekly repayments of $2010.45 and the interest rate remains at 5.5%, how many weekly repayments are needed to finish repaying her loan? 1 mark

Question 7 (4 marks)

Carmen received an inheritance from her aunt. She decides to invest $110 000 in a perpetuity that returns $1512.50 per quarter.

Interest is calculated quarterly.

a. Calculate the annual interest rate of Carmen's investment. 1 mark

b. Carmen invests part of her inheritance at an interest rate of 6.3% per annum, compounding monthly, from which she withdraws $1000 per month.

After eight years, the balance of Carmen's investment is reduced to $10 000.

Determine the initial value of Carmen's investment.

Give your answer correct to the nearest dollar. 1 mark

c. Carmen deposits $90 000 into an annuity at an interest rate of 4.45% per annum, compounding monthly. She withdraws $2500 per month.

i. How long will Carmen's investment last?

Give your answer correct to the nearest month. 1 mark

ii. Including interest, what will be the value of Carmen's final withdrawal? 1 mark

Matrices

Question 8 (5 marks)

Fran has a number of goats on his farm. The age, initial population, average yearly birth rate and average yearly survival rate of the goats are shown in the table below.

Age	Initial population	Average yearly birth rate	Average yearly survival rate
2	6	0.5	0.8
3	8	0.7	0.9
4	10	0.9	0

a. Find the Leslie matrix, L, for the data. 1 mark

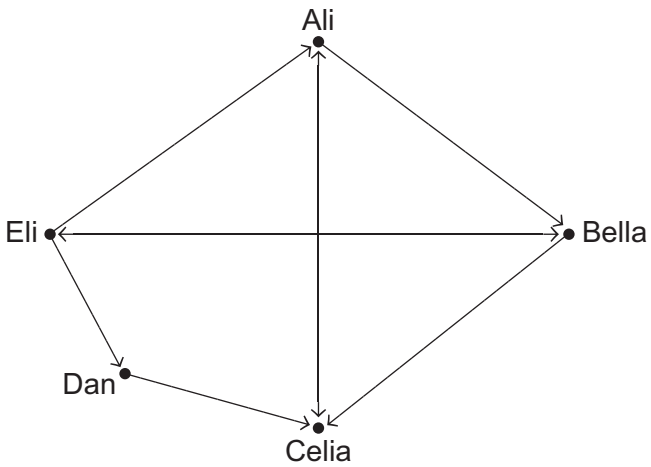
b. Write the matrix for the initial population, S_0. 1 mark

c. Write the matrix recurrence relation that predicts the number of goats on the farm. 1 mark

d. Determine the total number of goats Fran will have after four years. 2 marks

Question 9 (2 marks)

Fran employs five workers on the farm: Ali, Bella, Celia, Dan and Eli. The network diagram below shows the number of ways that the workers can communicate with each other.



a. Identify **two** pairs of workers who can communicate with each other. 1 mark

b. Write the communication matrix, N, that represents the one-step communications between the workers. 1 mark

Question 10 (5 marks)

To make a suitable fertiliser for his crops, Fran mixes the following chemicals: nitrogen (*N*), phosphorus (*P*), potassium (*K*) and zinc (*Z*).

In 2021, Fran purchased 4 kg of each chemical.

The state matrix, S_0, represents the weight of the chemicals, in grams, purchased in 2021.

$$S_0 = \begin{bmatrix} 4000 \\ 4000 \\ 4000 \\ 4000 \end{bmatrix} \begin{matrix} N \\ P \\ K \\ Z \end{matrix}$$

To make the fertiliser, Fran uses a different amount of each chemical. The amount of each chemical that he uses varies depending on the requirements of each crop. The transition matrix, *T*, predicts the weight of the chemicals, in grams, in the fertiliser mix.

$$T = \begin{array}{c} \text{this year} \\ \begin{matrix} N & P & K & Z \end{matrix} \\ \begin{bmatrix} 0.73 & 0.07 & 0.05 & 0.11 \\ 0.10 & 0.03 & 0.75 & 0.06 \\ 0.06 & 0.86 & 0.12 & 0.05 \\ 0.11 & 0.04 & 0.08 & 0.78 \end{bmatrix} \begin{matrix} N \\ P \\ K \\ Z \end{matrix} \end{array} \text{ next year}$$

Fran purchases more of each chemical depending on how much of each is used in the fertiliser mix.

a. What percentage of the weight of nitrogen purchased in 2021 was transferred to the weight of other chemicals purchased in 2022? 1 mark

b. What was the weight, in grams, of zinc purchased in 2022? 1 mark

The weight of the chemicals, in grams, purchased in the nth year is given by

$$S_{n+1} = T \times S_n$$

c. Determine the state matrix, S_3, for the weight of the chemicals purchased in 2024.

Give your answers correct to the nearest whole number. 1 mark

$$S_3 = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{matrix} N \\ P \\ K \\ Z \end{matrix}$$

d. Find the steady state matrix for the weight of the chemicals purchased in the long term.

Give your answer correct to the nearest whole number. 1 mark

e. In 2025, Fran is planning to expand the number of crops he grows. Fran increases the weight of the chemicals he purchases in 2024 to make a larger amount of fertiliser mix.

The weight of the chemicals purchased in 2024 can be calculated using the matrix equation $A_{n+1} = T \times A_n + G$, where G is a single column matrix and:

$$A_0 = \begin{bmatrix} 4200 \\ 4200 \\ 4200 \\ 4200 \end{bmatrix} \begin{matrix} N \\ P \\ K \\ Z \end{matrix} \qquad T = \begin{matrix} \\ N \\ P \\ K \\ Z \end{matrix} \begin{matrix} \text{this year} \\ \begin{matrix} N & P & K & Z \end{matrix} \\ \begin{bmatrix} 0.73 & 0.07 & 0.05 & 0.11 \\ 0.10 & 0.03 & 0.75 & 0.06 \\ 0.06 & 0.86 & 0.12 & 0.05 \\ 0.11 & 0.04 & 0.08 & 0.78 \end{bmatrix} \end{matrix} \begin{matrix} N \\ P \\ K \\ Z \end{matrix} \text{ next year}$$

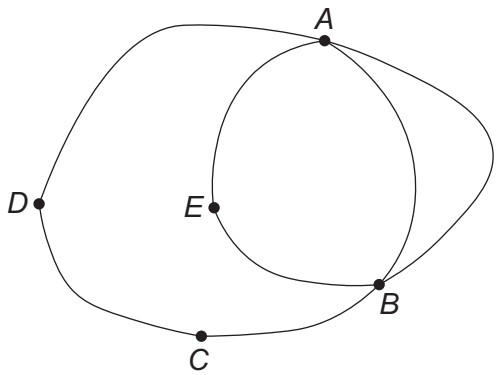
Fran decides that the amount of each chemical he uses will remain consistent in the future.

Complete matrix G. 1 mark

$$G = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{matrix} N \\ P \\ K \\ Z \end{matrix}$$

Networks and decision mathematics

Question 11 (5 marks)
Xian and his friends each live a different town. The following graph shows the roads that link the five towns.



a. Is an Eulerian trail possible in this graph? Explain your answer. 2 marks

b. Xian starts in town *B* and walks along each road exactly once.

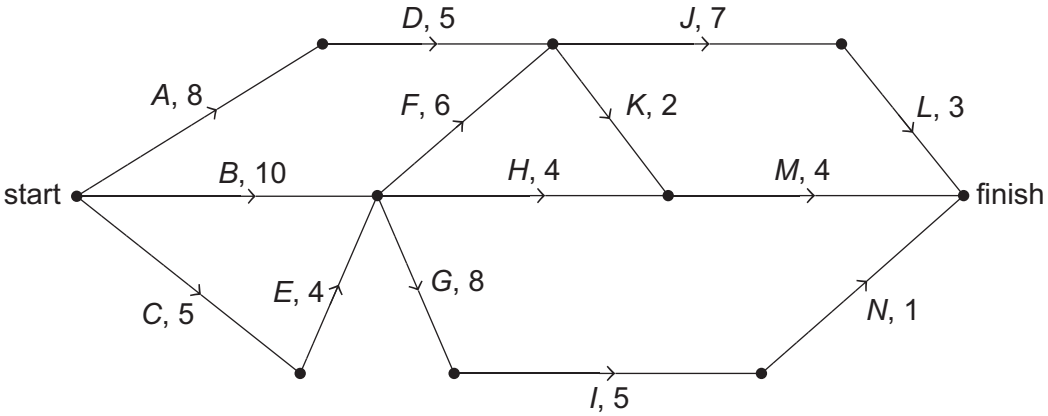
i. If Xian returns to town *B*, what is the term given to the path he takes? 1 mark

ii. Write the path that Xian takes in **part b.i.** 1 mark

c. Write a Hamiltonian path that allows Xian to visit all of his friends before returning to town *B*. 1 mark

Question 12 (5 marks)

In preparation for the grand opening of a new restaurant, several activities need to be completed. The directed network below shows the activities and their completion times, in hours.



a. What is the minimum completion time of the project? 1 mark

b. Find the critical path of the project. 1 mark

c. What is the float time for activity *K*? 1 mark

The completion times for activities *F*, *G*, *I* and *J* could be reduced by one hour.

The cost of reducing the completion time for each of these activities by one hour is shown in the table below.

Activity	Cost ($)
F	4000
G	5000
I	2000
J	3000

d. What is the minimum cost to reduce the completion time of the project by one hour? 1 mark

e. The project manager wants to reduce the minimum completion time of the project by two hours to avoid paying overtime.

Assume that any activity can have its completion time reduced and the cost of a one-hour reduction in completion time will be the same for any activity.

To fulfil the project manager's request, which activities' completion times can be reduced and by how many hours? 1 mark

Question 13 (2 marks)

Four students – Amy, Ben, Carol and Dino – are completing a group assignment for their English class. They decide to split the assignment into four tasks – planning, researching, writing and editing. They plan to assign one group member to each task, based on who is the most efficient at each task. The time, in hours, it takes each member to complete each task is shown in the table below.

	Planning	Researching	Writing	Editing
Amy	4	4	2	4
Ben	4	3	4	5
Carol	2	5	6	3
Dino	8	7	4	6

The students used the Hungarian method to determine how the tasks should be allocated to minimise the completion time of the assignment. The outcome of the calculations is shown in the table below.

	Planning	Researching	Writing	Editing
Amy	1	1	0	0
Ben	1	0	2	1
Carol	0	3	5	0
Dino	3	2	0	0

a. Complete the table below to assign each task to the most appropriate student. 1 mark

Member	Task
Amy	
Ben	
Carol	
Dino	

b. Find the minimum completion time, in hours, that it will take the students to complete the assignment. 1 mark

End of examination questions

