The Mathematical Association of Victoria

Trial Examination 2024

GENERAL MATHEMATICS

Written Examination 2

STUDENT NAME: _____

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Content Area	*Number of questions*	*Number of questions to be answered*	*Number of mars*
Data analysis	6	6	24
Recursion and financial modelling	4	4	12
Matrices	3	3	12
Networks and decision mathematics	3	3	12
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- One bound reference (which may be annotated)
- One approved technology with numerical, graphical, symbolic, financial and statistical functionality and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 25 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer all questions in the spaces provided.

In all questions where a numerical answer is required, you should only round your answer when instructed to do so.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Data analysis

Question 1 (6 marks)

The *lifespan*, in years, of a sample of 18 Common **Brushtail** possums is given below.

3, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 7, 7, 10

a. What percentage of this sample of possums had a *lifespan* of less than 7 years?
Round your answer to one decimal place. 1 mark

b. Show, with calculations, that the possum with a *lifespan* of 10 years is an outlier.
2 marks

Possums can have different *tail types*.

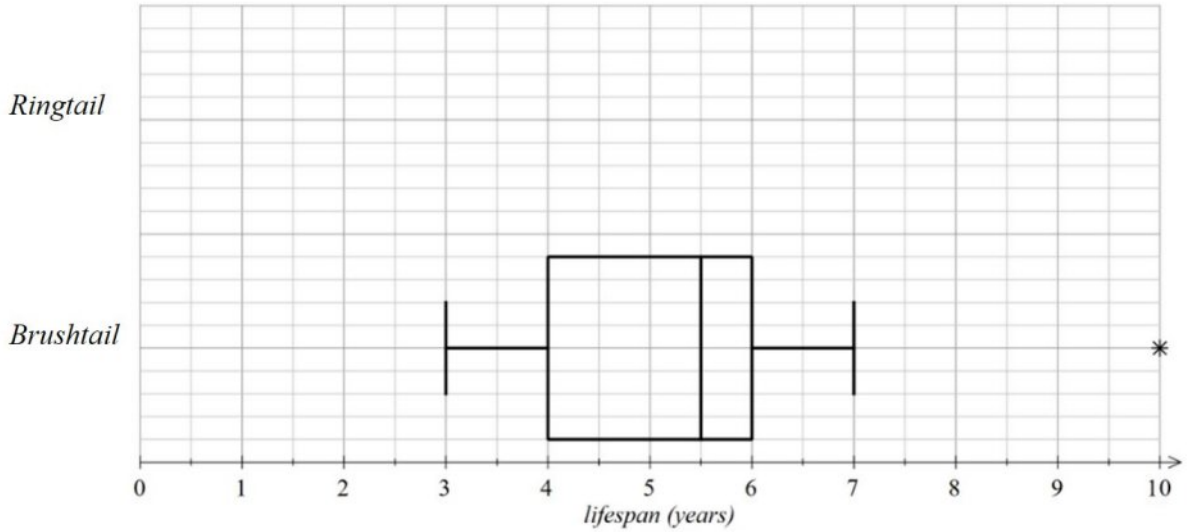
The five-number summary below was determined for the *lifespan*, in years, of a sample of 20 Common **Ringtail** possums.

	***lifespan* (years)**
minimum	1
first quartile	3
median	4
third quartile	4.5
maximum	6

c. The boxplot for the distribution of *lifespan*, in years, of the Common **Brushtail** possums is given below.

Add the boxplot for the distribution of *lifespan*, in years, of the Common **Ringtail** possums to the graph below.

1 mark



d. Do the boxplots support the contention that *lifespan* is associated with *tail type*, Brushtail or Ringtail? Refer to the values of an appropriate statistic in your response.

2 marks

Question 2 (2 marks)

Table 1, below, gives the *average weight,* in grams, as well as the log_{10} *(average weight)* for a sample of 16 breeds of Australian possums.

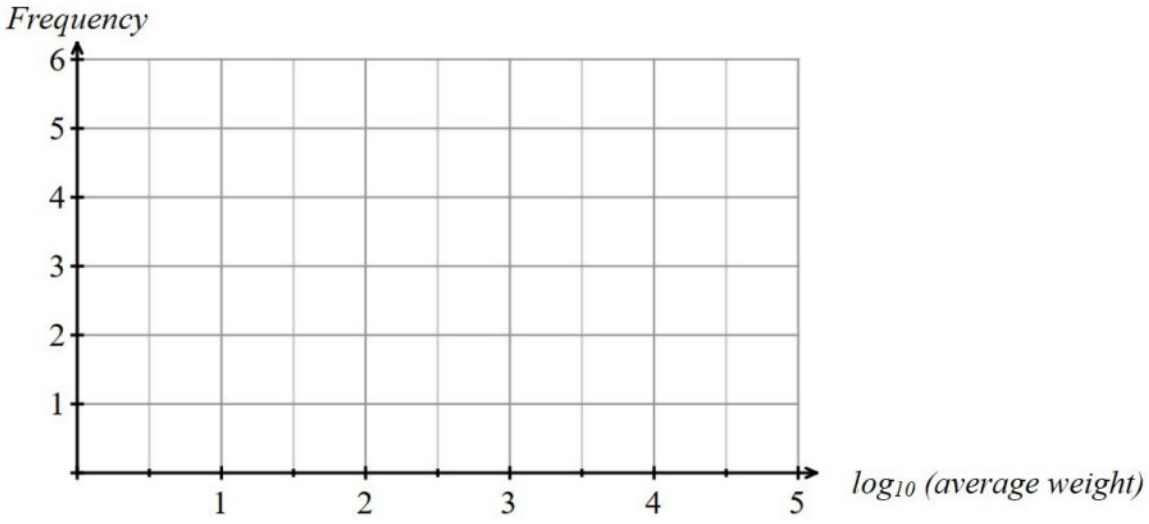
a. Complete the two shaded cells in the table below.
Give your logarithmic answer correct to 2 decimal places. 1 mark

Table 1

species name	*average weight* (g)	log_{10} *(average weight)*
Common Brushtail	2400	3.38
Common Ringtail	850	2.93
Mountain Pygmy	43	1.63
Western Pygmy	16	1.20
Daintree River Ringtail	950	
Rock Haunting ringtail	1750	3.24
Lemuroid Ringtail	980	2.99
Herbert River	1150	3.06
Green Ringtail	1080	3.03
Leadbeater's	135	2.13
Mountain Brushtail	3500	3.54
Scaly-tailed	1500	3.18
Honey		1.00
Eastern Pygmy	40	1.60
Long tailed Pygmy	30	1.48
Striped	420	2.62

b. Complete the histogram below for the distribution of log_{10} *(average weight)* of Australian possums using a log_{10} *(average weight)* class interval of 0.5.

1 mark



Question 3 (7 marks)

An environmental researcher collected more data to investigate the 16 different species of possum found in Australia. The data collected is displayed in Table 2.

The variables in this study were:

- *species name:* the common name used for that species.
- *natural habitat:* the most common habitat where the species of possum is found.
- *tail type:* brush, ring or scaly.
- *conservation status:* 1 = least concern, 2 = near threatened or 3 = critically endangered.
- *litter size:* average number of offspring at one time.
- *body length:* average length from head to the tail in centimetres.
- *tail length:* average length of the tail in centimetres.

Table 2

species name	*natural habitat*	*tail type*	*conservation status*	*litter size*	*Body length* (cm)	*tail length* (cm)
Common Brushtail	Forest	Brush	1	1	45	32
Common Ringtail	Forest	Ring	1	2	33	33
Mountain Pygmy	Rocky	Ring	3	4	11	14
Western Pygmy	Bushland	Scaly	1	5	8	8
Daintree River Ringtail	Forest	Ring	2	2	35	30
Rock Haunting Ringtail	Rocky	Brush	1	2	37	22
Lemuroid Ringtail	Forest	Brush	2	1	34	32
Herbert River	Forest	Ring	1	2	35	38
Green Ringtail	Forest	Ring	2	1	18	19
Leadbeater's	Forest	Brush	3	2	16	20
Honey	Bushland	Ring	1	3	8	10
Mountain Brushtail	Bushland	Brush	1	1	38	40
Scaly-tailed	Rocky	Scaly	2	1	39	30
Eastern Pygmy	Forest	Ring	1	5	9	10
Long-tailed Pygmy	Forest	Ring	1	3	10	15
Striped	Bushland	Brush	1	2	26	33

a. Write down the number of ordinal variables in Table 2. 1 mark

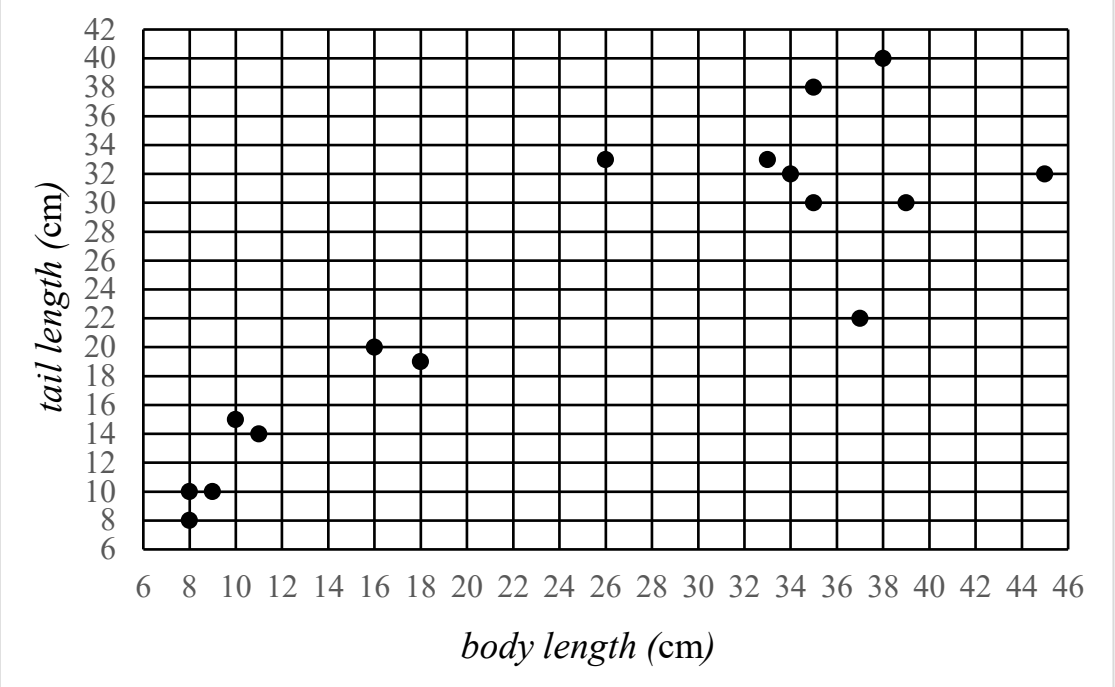
_____

b. The environmental researcher believes that the smaller possum species, based on *body length,* tend to have a larger *litter size*.

Using the information in Table 2 explain why the environmental researcher came to this conclusion. Refer to the value of an appropriate statistic in your response.

1 mark

The scatterplot below shows the *tail length*, in cm, plotted against *body length,* in cm, for the 16 species of possum.



The least squares line equation is found to be: *tail length* = 6.4 + 0.70 × *body length*

The correlation coefficient, *r*, is 0.8816

c. Add the least squares line to the scatterplot. 1 mark

ANSWER ON SCATTERPLOT ABOVE

d. **i.** Determine the value of the coefficient of determination.
Round your answer to three decimal places. 1 mark

ii. Interpret the coefficient of determination in terms of the variables *tail length* and *body length*. 1 mark

The rock haunting ringtail possum with an average *body length* of 37cm has an average *tail length* of 22cm.

e. Calculate the residual when the least squares line is used to predict the average *tail length* of the rock haunting ringtail possum.
Round your answer to one decimal place. 2 marks

Question 4 (3 marks)

Table 3, below, is an incomplete two-way table showing the *habitat* and *conservation status* for the 16 species of possum.

Table 3

		habitat			
		Bushland	Forest	Rocky	Total
conservation status	Least concern	4	5	1	10
	Near threatened	0		1	4
	Critically endangered	0	1	1	2
	Total	4	9	3	

a. Write down the number of species whose *habitat* is forest and have the *conservation status* of near threatened. 1 mark

_____

The environmental researcher believes that the possums' *conservation status* is associated with their *habitat*.

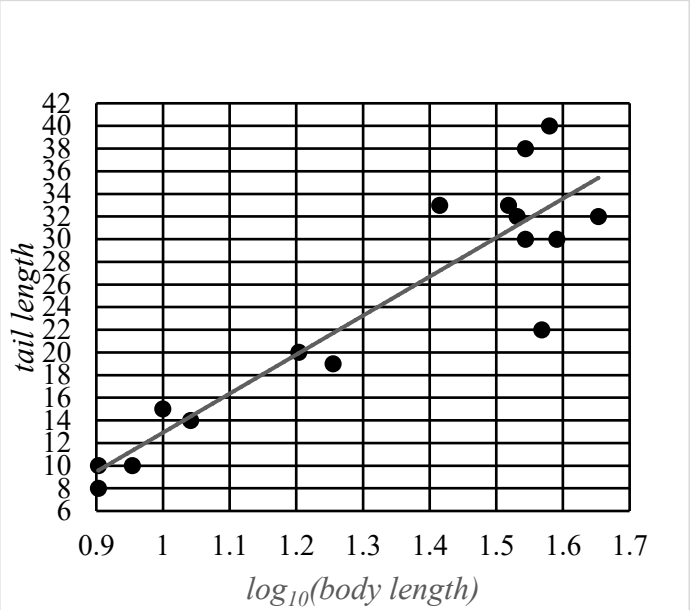
b. Does the information in Table 3 support the environmental researchers belief?
Explain your conclusion by comparing appropriate percentages.
Round these percentages to the nearest whole number. 2 marks

Question 5 (4 marks)

Another environmental researcher suggested that the *tail length* of a possum could be predicted using the least squares regression line found after a $\log_{10}$ transformation of the *body length*.

The scatterplot and least squares regression line is shown below:

species name	*body length* (cm)	*tail length* (cm)
Common Brushtail	45	32
Common Ringtail	33	33
Mountain Pygmy	11	14
Western Pygmy	8	8
Daintree River Ringtail	35	30
Rock Haunting Ringtail	37	22
Lemuroid Ringtail	34	32
Herbert River	35	38
Green Ringtail	18	19
Leadbeater's	16	20
Honey	8	10
Mountain Brushtail	38	40
Scaly-tailed	39	30
Eastern Pygmy	9	10
Long-tailed Pygmy	10	15
Striped	26	33



a. Find the vertical intercept of the transformed least squares regression line.
Round your answer to three significant figures. 1 mark

tail length = ☐ + 34.4 × log_{10} *(body length)*

b. Interpret the slope of the least squares line in terms of the variables *tail length* and $\log_{10}$(*body length)*. 1 mark

The short-eared possum has an average body length of 35cm.

c. Using the least squares regression line calculated in part(a), show a calculation to predict the *tail length* is 31.6 cm, correct to one decimal place.

1 mark

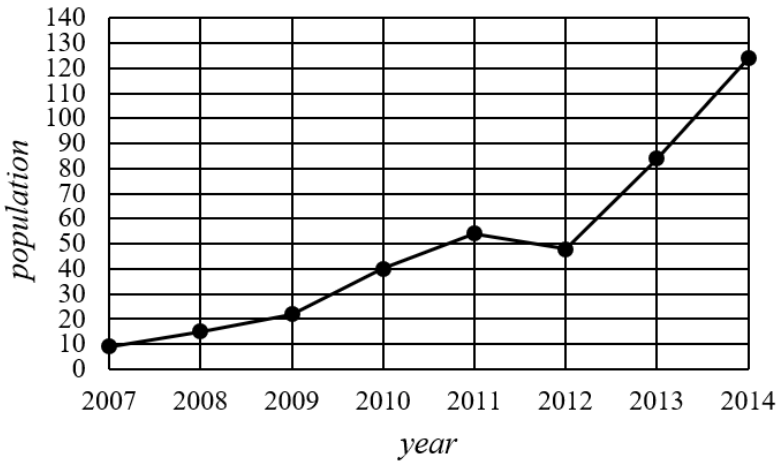
The Tasmanian pygmy possum with a body length of 6.7cm is predicted to have a tail length of 6.9cm, correct to one decimal place.

d. Compare the reliability of this prediction to the prediction made in part **c.** 1 mark

Question 6 (2 marks)

The time series plot below shows the *population* of Mountain Pygmy possums in a Victorian mountain area, recorded over a period of eight *years*.

year	*population*
2007	9
2008	15
2009	22
2010	40
2011	54
2012	48
2013	84
2014	124



a. Describe two features of the time series from 2007 to 2014. 1 mark

b. Calculate the five-mean smoothed population for 2011.

Round your answer correct to one decimal place. 1 mark

Recursion and financial modelling

Question 7 (3 marks)

Rayshan has a mobile coffee van. He takes his van to events where he supplies takeaway coffee for customers.

As part of his set-up Rayshan purchases a commercial coffee machine for $12 800.

The value of Rayshan's coffee machine, in dollars, after *n* coffees are made, C_n, is given by the rule $C_n = 12800 - 0.0975n$.

a. What is the value, in dollars, of Rayshan's coffee machine after 10 000 coffees have been made?
1 mark

b. How many coffees will Rayshan make with his coffee machine before the value first falls below $6000?
1 mark

c. What type of depreciation is being used to determine the value of Rayshan's coffee machine?
1 mark

Question 8 (5 marks)

Rayshan took out a reducing balance loan for $30 000 when he first bought his van and coffee machine. The interest rate for the loan was 7.2% per annum compounding monthly, with monthly payments of $600.

a. Show that the monthly interest rate is 0.6%. 1 mark

An incomplete amortisation table for the first three months of Rayshan's loan is shown below:

Payment number	Payment ($)	Interest ($)	Principal Reduction ($)	Balance ($)
0	0.00	0.00	0.00	30 000.00
1	600.00	180.00	420.00	29 580.00
2	600.00	177.48	422.52	29 157.48
3				

b. Using the information in the table complete the shaded cells in the amortisation table.

Write your answers in the spaces provided in the table above.

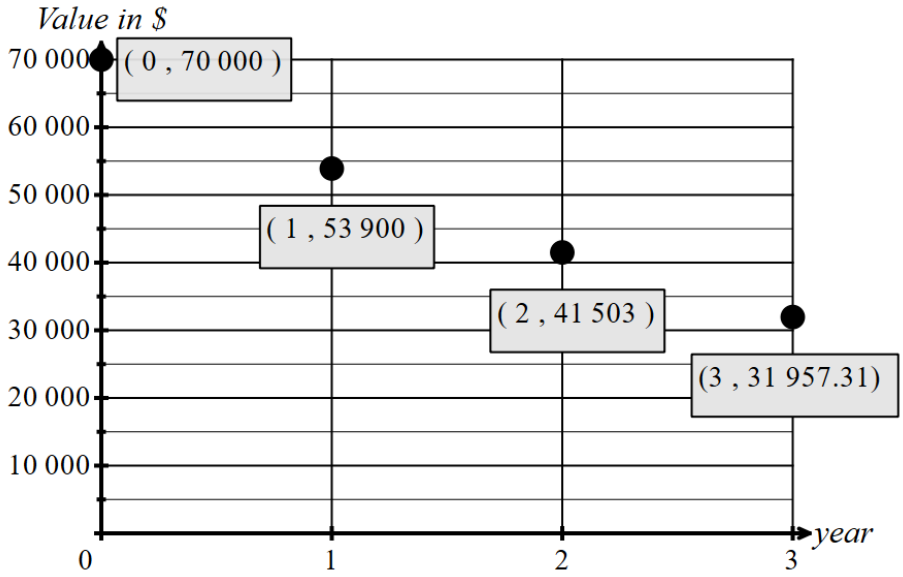
Round all values to the nearest cent. 1 mark

c. Let R_n be the balance of Rayshan's loan, in dollars, after n months.
Write a recurrence relation in terms of R_0, R_{n+1} and R_n that can model the value of the loan from month to month. 1 mark

d. How much interest will Rayshan have paid, in total, two years after he started this loan?
Round your answer to the nearest cent. 2 marks

Question 9 (2 marks)

Rayshan purchased the van for $70 000. He depreciates the van using reducing balance depreciation. A series of points representing the value of his van after each of the first three years is shown below:



a. Graphs of reducing balance depreciation follow a curve.
Explain why this graph, and other graphs of values using reducing balance depreciation, display a curve. 1 mark

b. Determine the annual rate of reducing balance depreciation for Rayshan's van. 1 mark

Question 10 (2 marks)

Rayshan starts a new savings account with a balance of $500 at 6.9% per annum interest compounding monthly, when he starts his business. Every month he adds $990 to the account.

After five years, the interest rate increases to 7.2% per annum compounding monthly and Rayshan increases his monthly payments to $1190.

After another five years, Rayshan withdraws all of the money in the account.

What percentage of the final balance was interest?

Round your answer correct to one decimal place.

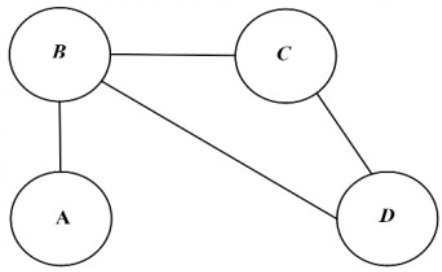
Matrices

Question 11 (3 marks)

A scientist is breeding critically endangered Australian frogs in captivity. A system of four breeding ponds, *A, B, C* and *D*, are connected by pipes, as shown in the diagram below.

The matrix *P* is used to represent the information in the diagram showing all the direct connections between the four ponds.

- The 1 in row 2, column 1 indicates that a pipe directly connects pond B to pond A.
- The 0 in row 4, column 1 indicates that pond D is not directly connected to pond A by a pipe.



$$P = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a. Explain in terms of the breeding ponds what the sum of the elements in row 3 of matrix *P* represents. 1 mark

The matrix P^2 is shown below but some of the elements are missing.

$$P^2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 3 & 1 & 1 \\ 1 & 1 & x & 1 \\ y & 1 & 1 & 2 \end{bmatrix}$$

b. State the value of elements x and y.

$x =$ _____ $y =$ _____ 1 mark

c. Explain what the element in row 3, column 2 of matrix P^2 represents.

Give examples to illustrate your answer. 1 mark

Question 12 (4 marks)

The frog has a life span of 9 years and have been categorised into three stages, 1, 2 and 3 which are age groups 0−3 years, 3−6 years, 6−9 years respectively.

Table 1 shows the birth rate for each of the three stages, that is the average number of female offspring born to each female at each stage, and the survival rate of females, given as the average percentage of female offspring to survive to the next age group.

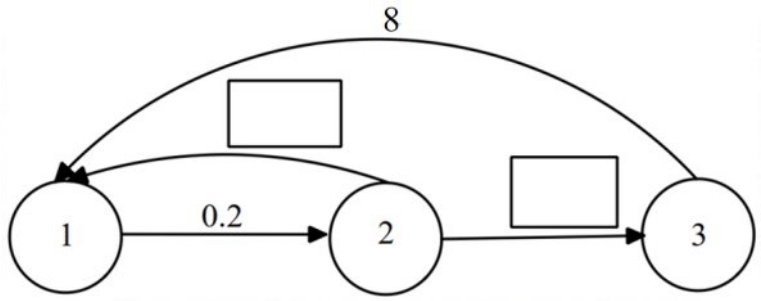
Table 1

Stage	1	2	3
age group	0-3 years	3-6 years	6-9 years
birth rate	0	15	8
survival rate	20%	50%	0

The Leslie matrix, L, that models the breeding patterns for this population is as follows.

$$L = \begin{bmatrix} 0 & 15 & 8 \\ 0.2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}$$

a. The same information is presented in the transition diagram below.



Use the Leslie matrix to complete the diagram. 1 mark

A breeding program starts with 6 adult female frogs, aged 3-6 years.

The initial state matrix for this female frog population, S_0, can be written $S_0 = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$.

The recurrence relation to model the breeding patterns of this marsupial population is

$$S_{n+1} = LS_n$$

b. By calculating S_1, determine the percentage of female frogs that are aged 6-9 years after one three-year time period. Round your answer to one decimal place. 1 mark

After two three-year time periods, $S_2 = \begin{bmatrix} 24 \\ 18 \\ 0 \end{bmatrix}$.

c. Show, with a calculation that the expected number of frogs ages 6-9 years, after three three-year periods is 9. 1 mark

d. The population growth will reach a point where the rates of growth of the different age groups of the population are constant.

Find the rate of growth per time period at this point.

Write your answer as a percentage and round your answer to the nearest whole number.

1 mark

Question 13 (5 marks)

In the wild the survival rate of the frogs is lower than it is in captivity. This is due to changes in habitat and fungal disease.

In a surveyed wilderness area, it was found that there were

- 120 adult female frogs (A), that is frogs aged 3-6 years.
- 20 mature female frogs (M), that is frogs aged 6-9 years.

The matrix W_0 displays the number of adult female frogs (A) and the number of mature female frogs (M), that were initially in the wilderness area.

$$W_0 = \begin{bmatrix} 120 \\ 20 \end{bmatrix} \begin{matrix} A \\ M \end{matrix}$$

The expected number of adult female frogs (A) and mature female frogs (M) after **one** year is determined by the matrix equation $W_1 = GW_0$ where G is given below.

$$G = \begin{matrix} A & M \\ \begin{bmatrix} 0.20 & 0 \\ 0.35 & 0.15 \end{bmatrix} & \begin{matrix} A \\ M \end{matrix} \end{matrix}$$

a. How many of the initial group of mature frogs are still alive after one year? 1 mark

The survival pattern continues using the relation $W_{n+1} = GW_n$ where $W_0 = \begin{bmatrix} 120 \\ 20 \end{bmatrix} \begin{matrix} A \\ M \end{matrix}$.

b. How many frogs, in total, will still be alive after 3 years?
Give your answer correct to the nearest whole number. 1 mark

After five years, there are no frogs living in the wilderness area and scientists attempt to re-introduce some frogs back into the wilderness area.

A recurrence relation for determining the number of frogs is

$$C_{n+1} = GC_n + B \text{ where } C_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 10 \end{bmatrix}.$$

c. Using the given recurrence relation, in the long term, how many adult female frogs would the scientist expect to find in the wilderness area? 1 mark

After two years the scientists want to ensure that there are at least 100 frogs from the third year onwards.

d. **i.** What is the minimum number of adult frogs that need to be introduced after the first two years to ensure that the adult population is at least 100 after three years. 1 mark

The number of adult frogs calculated in **d.i.** is introduced each year after the first two years, along with ten mature frogs.

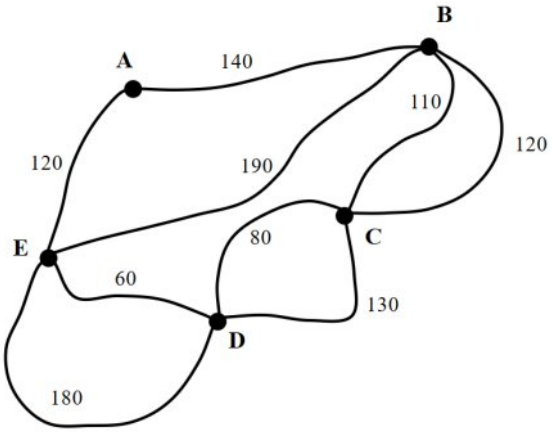
ii. What is the long-term effect on the number of mature frogs? 1 mark

Networks and decision mathematics

Question 14 (3 marks)

Vera is organising a fete in a local park that has five pergolas (A, B, C, D and E) connected by a number of walking tracks.

A map of the park showing each of the pergolas and the tracks between them, along with the distances in metres for each track is shown below.



a. What is the shortest distance, in metres, between pergola C and pergola A? 1 mark

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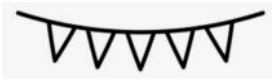
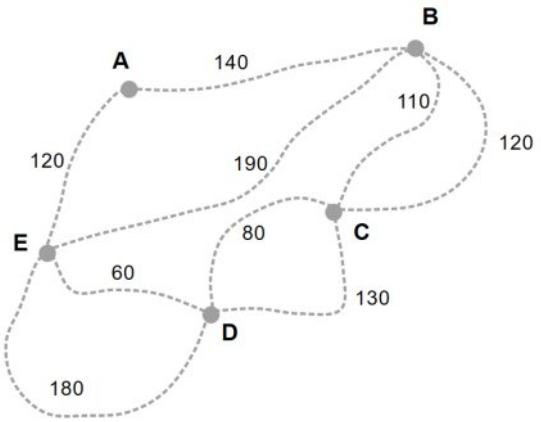
One of the sponsors of the fete has some advertising bunting that they want to use to connect all of the pergolas in the park. Bunting is a string of small flags as shown in the picture below:



The sponsor wants to use the minimum length of bunting that would connect all of the pergolas, by placing the bunting along the tracks.

b. Draw in the tracks that should be used so that the sponsor could use the minimum length of bunting on the diagram below. 1 mark



Vera needs to check that all the tracks in the park are safe for customers at the fete.

She will start and finish at pergola A and walk along every track at least once, taking the minimum possible distance.

c. What is the mathematical name of the route that Vera should take? 1 mark

Question 15 (3 marks)

Vera needs to hire food suppliers for the fete. She contacts four different catering companies, Abbey's, Barnaby's, Carla's and Deeka's to determine whether they can provide hamburgers, sandwiches, kebabs or donuts.

Abbey's can provide hamburgers and kebabs.

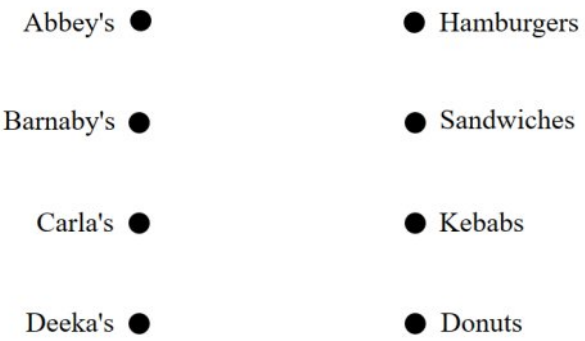
Barnaby's can provide donuts or sandwiches.

Carla's can provide hamburgers or donuts.

Deeka's can provide sandwiches or donuts.

a. Complete the bipartite graph below showing which caterer can provide each food type.

1 mark



b. Vera must choose a different caterer for each task. How many different ways can she do this?

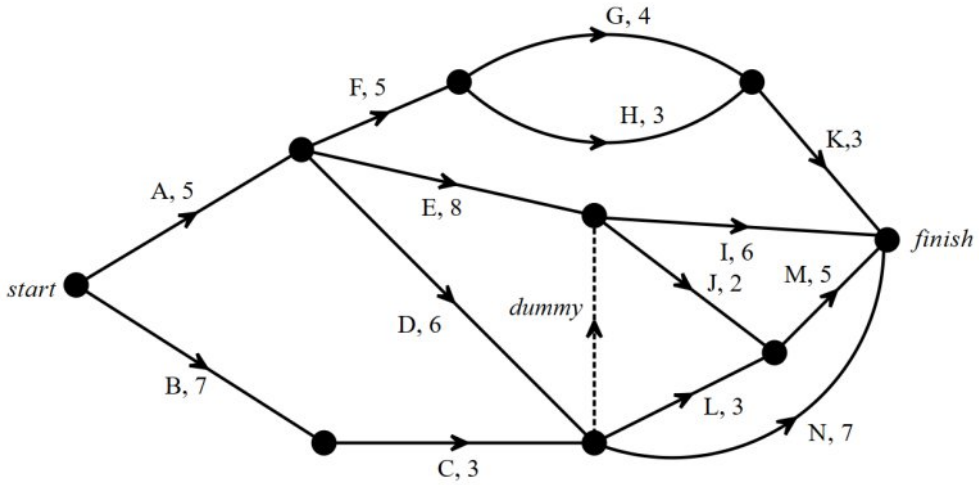
1 mark

c. Is the bipartite graph planar? Give a reason for your answer. 1 mark

Question 16 (6 marks)

The project of preparing for the fete involves 14 activities, A to N.

The directed network below shows these activities and their completion times, in days.



a. Which activities have activity D as an immediate predecessor? 1 mark

b. What is the earliest finishing time for activity L? 1 mark

c. What is the minimum number of days in which this project can be completed? 1 mark

d. Which activity has the greatest float time? 1 mark

Vera wants to reduce the overall project time. She can hire extra workers for five days in order to reduce time.

e. Vera considers using the extra workers to reduce the time for activity E by five days, from eight days to three days.

How many days would Vera reduce the project by, in total, if she reduces activity E by five days?
1 mark

Vera is advised that she may be better to use the extra workers on a variety of activities.

She can now reduce any activity by one or more days, but she can only use a total of five days reduction overall. No activity can be reduced below one day duration.

f. What is the maximum reduction in time that Vera can achieve? 1 mark

END OF QUESTION AND ANSWER BOOK