The Mathematical Association of Victoria

Trial Examination 2023

GENERAL MATHEMATICS

Written Examination 1

MULTIPLE CHOICE QUESTION BOOK

STUDENT NAME: _____

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
40	40	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference (which may be annotated), one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 23 pages
- Formula sheet.
- Answer sheet for multiple choice questions
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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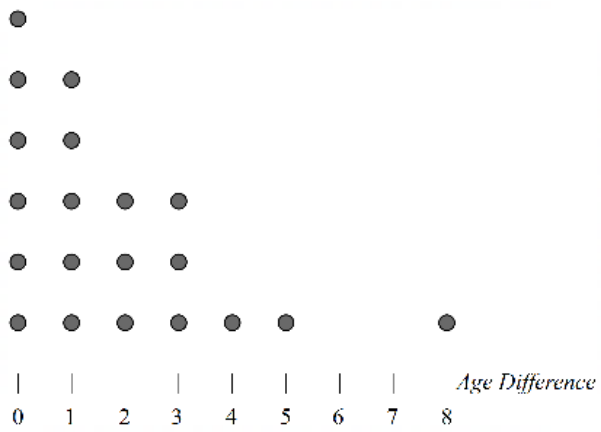
Instructions

Answer **all** questions in pencil on the answer sheet provided for multiple – choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Use the following information to answer Questions 1,2 and 3:

The dot plot below displays the *Age Difference* (in years) between the parents, for a sample of 20 newborn children.



Question 1

The percentage of parents with an *Age Difference* of more than 4 years is closest to

A. 2.5%
B. 5%
C. 10%
D. 16%
E. 32%

Question 2

The mean and sample standard deviation, correct to two decimal places, for this data set are

A. mean: 3.33, standard deviation: 2.69.
B. mean: 3.33, standard deviation: 2.49.
C. mean: 2.86, standard deviation: 2.04.
D. mean: 1.85, standard deviation: 2.06.
E. mean: 1.85, standard deviation: 2.01.

TURN OVER

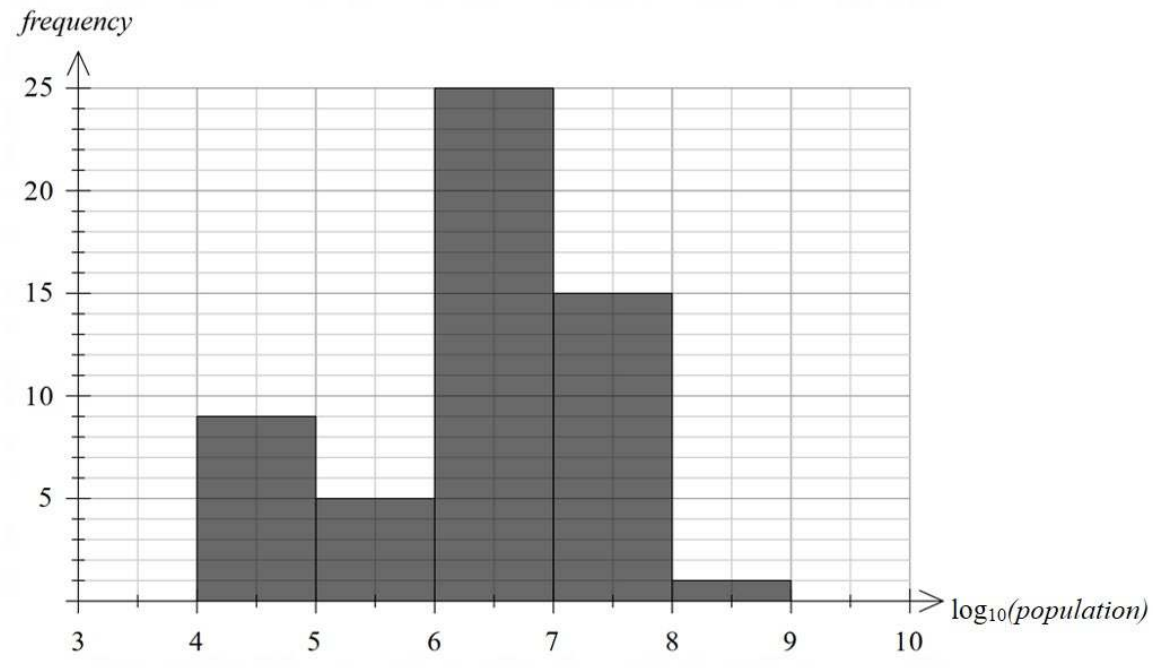
Question 3

The shape of this data distribution is best described as being

A. Symmetric with an outlier
B. Positively skewed with an outlier
C. Negatively skewed with an outlier
D. Negatively skewed
E. Positively skewed

Question 4

The histogram below displays the distribution of *population* for a sample of 55 European countries in 2021. The histogram has a $\log_{10}$ scale.



The percentage of countries with a 2021 *population* greater than 100 000 people is closest to

A. 9%
B. 14%
C. 16%
D. 46%
E. 84%

Question 5

Data was collected to investigate the association between the following two variables:

- opening time of the school canteen (before school, recess, lunchtime)
- the year level of the students visiting the canteen at that time (years 7, 8, 9, 10, 11 & 12)

A display that could be used to show the survey's results is

A. A set of parallel boxplots
B. A back-to-back stem plot
C. A set of percentaged segmented bar charts
D. A histogram
E. A scatterplot

Question 6

The stem plot displays the number of people who visit a cinema on Saturdays over the course of a 52-week year.

Number of people visiting a cinema

18	1 3 3 4 5 7
19	2 3 4 4 6 7 8 8 8 8
20	0 0 2 3 3 3 4 5 5 7 8 9
21	0 1 2 2 3 5 6 6 8
22	1 2 2 3 4 7
23	3 4 5 6
24	4 8
25	1 4
26	
27	0

Key 181 is 18|1

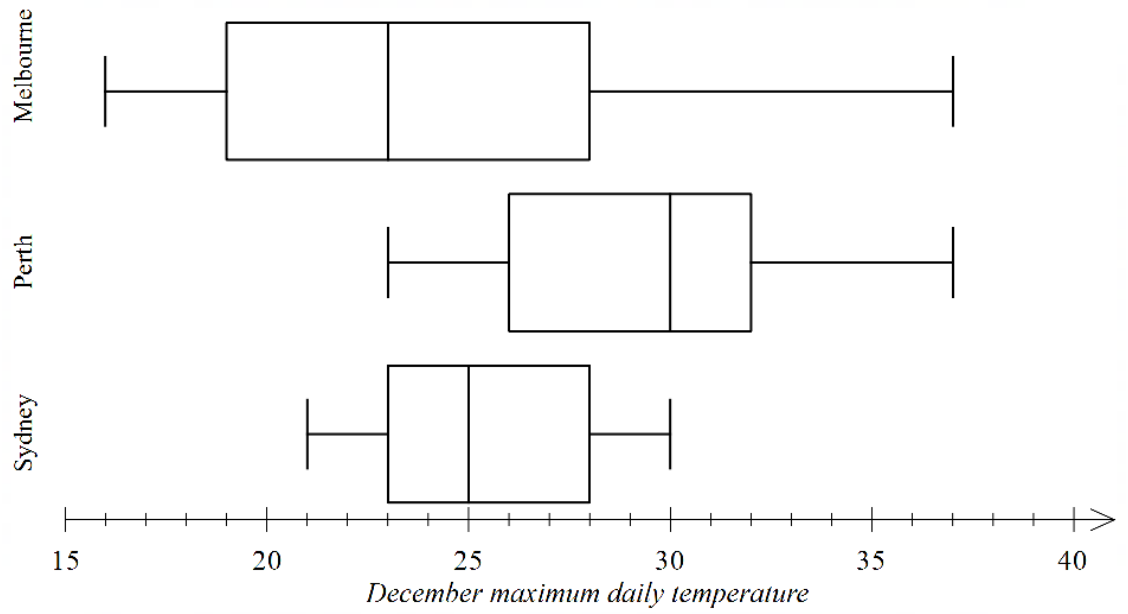
The cinema suspects that the week where 270 people visited the cinema was unusual and was an outlier. The **upper** fence used to determine outliers in this distribution is

A. 222
B. 234
C. 248
D. 254
E. 258

TURN OVER

Use the following information to answer Questions 7 and 8:

The parallel boxplots below summarise the distribution of *December maximum daily temperature* during 2022, for Melbourne, Perth and Sydney



Question 7

Which one of the following statements is **not** true?

A. The lowest *December maximum daily temperature* for Perth in greater than the lowest *December maximum daily temperatures* for both Melbourne and Sydney.
B. There is more variation in the *December maximum daily temperature* in Perth than for *December maximum daily temperature* in Sydney.
C. *December maximum daily temperature* is, on average, lower in Sydney than in the other two cities.
D. The highest *December maximum daily temperature* in Sydney is the same as the median for the *December maximum daily temperature* in Perth.
E. More than 75% of *December maximum daily temperature* for Perth were hotter than the median *December maximum daily temperature for* Sydney and Melbourne.

Question 8

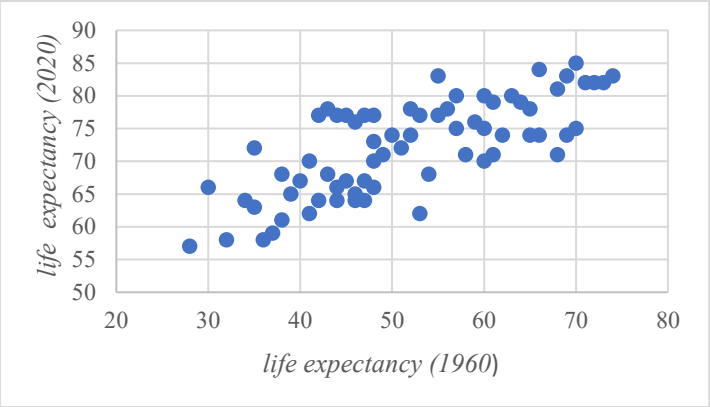
For each of the three cities, there were no days in the 31 days of December where the maximum temperature was 23°C for more than one day.

The total number of days with a *December maximum daily temperature* of 23°C or above, across all three cities is closest to

A. 58
B. 59
C. 60
D. 68
E. 71

Use the information below to answer Questions 9 and 10:

The scatterplot below shows the *life expectancy* at birth for a sample of 70 countries for 1960 and 2020:



The r correlation coefficient is 0.745.

The mean and standard deviation for the data is given in the table below.

	life expectancy (1960)	*life expectancy (2020)*
Mean	51.93	72.27
Standard Deviation	11.89	7.22

Question 9

What percentage of the variation in *life expectancy (2020)* cannot be explained by the variation in *life expectancy (1960)*?

A. 25.5%
B. 44.5%
C. 55.5%
D. 74.5%
E. 86.3%

Question 10

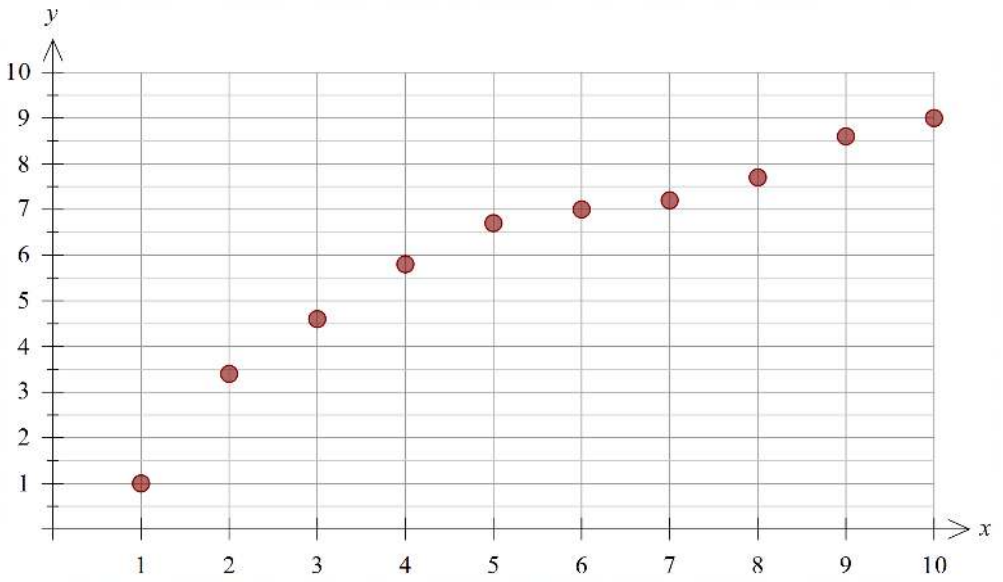
Using the summary statistics given, the equation of the least squares regression line that can be used to predict the *life expectancy (2020)* from the *life expectancy (1960)* is closest to

A. $life\,expectancy(2020) = -37 + 1.23 \times life\,expectancy(1960)$
B. $life\,expectancy(2020) = 49 + 0.45 \times life\,expectancy(1960)$
C. $life\,expectancy(2020) = 49 + 1.23 \times life\,expectancy(1960)$
D. $life\,expectancy(2020) = 14 + 0.45 \times life\,expectancy(1960)$
E. $life\,expectancy(2020) = 9 + 1.23 \times life\,expectancy(1960)$

TURN OVER

Question 11

Consider the non-linear scatterplot below:



To linearise this scatterplot, it would be best to plot

A. $\log_{10}(y)$ against x

B. $\frac{1}{y}$ against x

C. y against $\log_{10}(x)$

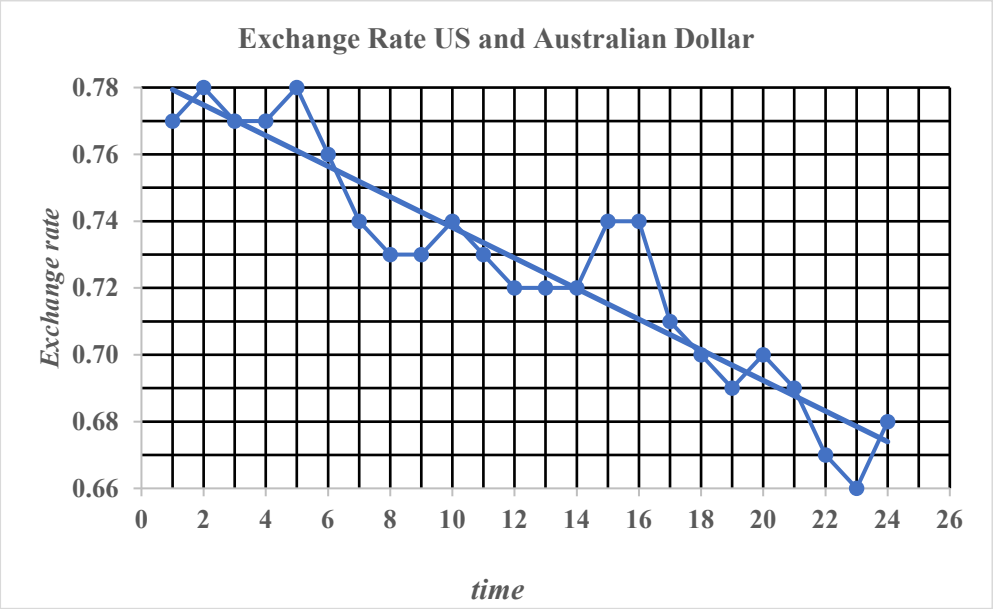
D. y against x^2

E. y against x

Use the following information to answer Questions 12, 13 and 14:

The time series plot below shows the *average monthly exchange rate* for the Australian dollar (AUD) to the US dollar (USD) from January 2021 ($t = 1$) to December 2022 ($t = 24$).

A least squares regression trendline has been added.



Question 12

The time series is best described as having

A. a decreasing trend only
B. irregular fluctuations only
C. seasonality with irregular fluctuations
D. a decreasing trend with irregular fluctuations.
E. decreasing trend with seasonality

Question 13

This time series plot is to be smoothed using **seven-median** smoothing.

The seven-median smoothed *Exchange Rate* for April 2022 is closest to

A. 0.69
B. 0.70
C. 0.71
D. 0.72
E. 0.73

Question 14

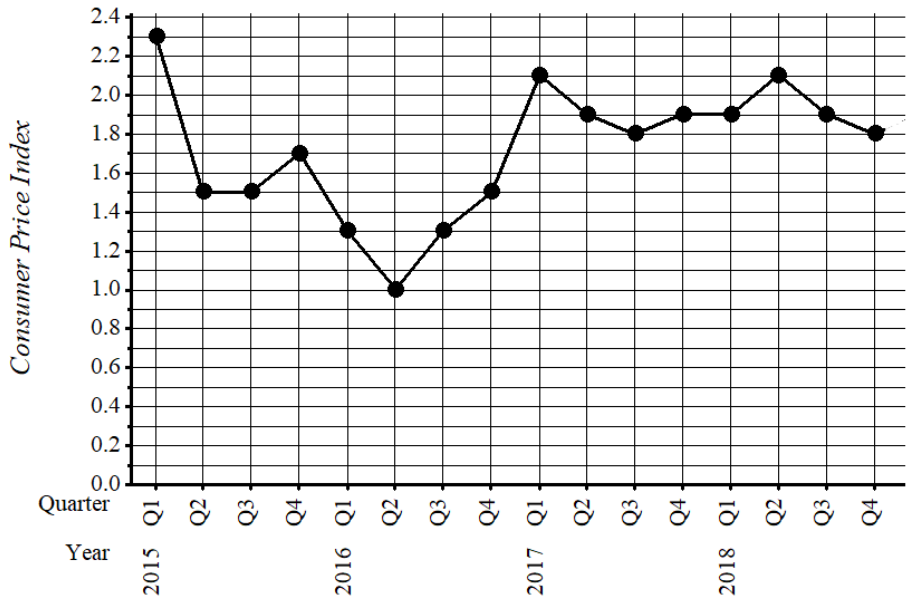
The equation of the least squares regression line for the time series data is closest to

A. $time = 0.784 - 0.0046 \times Exchange\ Rate$
B. $Exchange\ Rate = 0.774 - 0.0039 \times time$
C. $Exchange\ Rate = 0.784 - 0.0046 \times time$
D. $Exchange\ Rate = 0.0046 - 0.784 \times time$
E. $time = 21 - 16 \times Exchange\ Rate$

TURN OVER

Use the following information to answer Questions 15 and 16:

The time series plot and table below show the quarterly *Consumer Price Index* for the four years from 2015 to 2018 inclusive. On the graph and in the table, quarters 1, 2, 3 and 4 are represented by Q1, Q2, Q3 and Q4 respectively.



	2015	2016	2017	2018	Average
Q1	2.3	1.3	2.1	1.9	1.9
Q2	1.5	1	1.9	2.1	1.625
Q3	1.5	1.3	1.8	1.9	1.625
Q4	1.7	1.5	1.9	1.8	1.725
Average	1.75	1.275	1.925	1.925	

Question 15

Using the data for the given 4 years, the seasonal index for Quarter 4, correct to two decimal places, is

A. 0.97
B. 1.02
C. 1.03
D. 1.73
E. 1.75

Question 16

If the data is smoothed using a 4-point centred moving mean, the smoothed figure for Quarter 3, 2016 would be

A. 1.275
B. 1.375
C. 1.475
D. 1.625
E. 1.65

Recursion and financial modelling

Question 17

A printing press was purchased for $43 500.

The printing press is depreciated using the flat rate method at 6% per annum.

After three years the printing press is valued at

A. $35 670
B. $7830
C. $36 130
D. $9396
E. $40 890

Question 18

A recurrence relation is defined as $A_0 = 5, \quad A_{n+1} = -3A_n + k$, where k is an integer.

For what value of k will $A_{n+1} = A_n$?

A. 5
B. 15
C. 20
D. -20
E. -5

Question 19

Peter buys a van for his delivery business. The van costs $50 000.

He can depreciate his van using the unit cost method or using reducing balance depreciation.

If he uses the unit cost method, the van will be depreciated at 12.7 cents per kilometre travelled. He will travel a total of 66 000 km in the van over the next two years.

The annual percentage rate of reducing balance depreciation that would result in the same value for the van after two years would be closest to

A. 16.8%
B. 8.8%
C. 8.4%
D. 30.7%
E. 20.1%

Question 20

A recurrence relation showing the balance for an account is shown below:

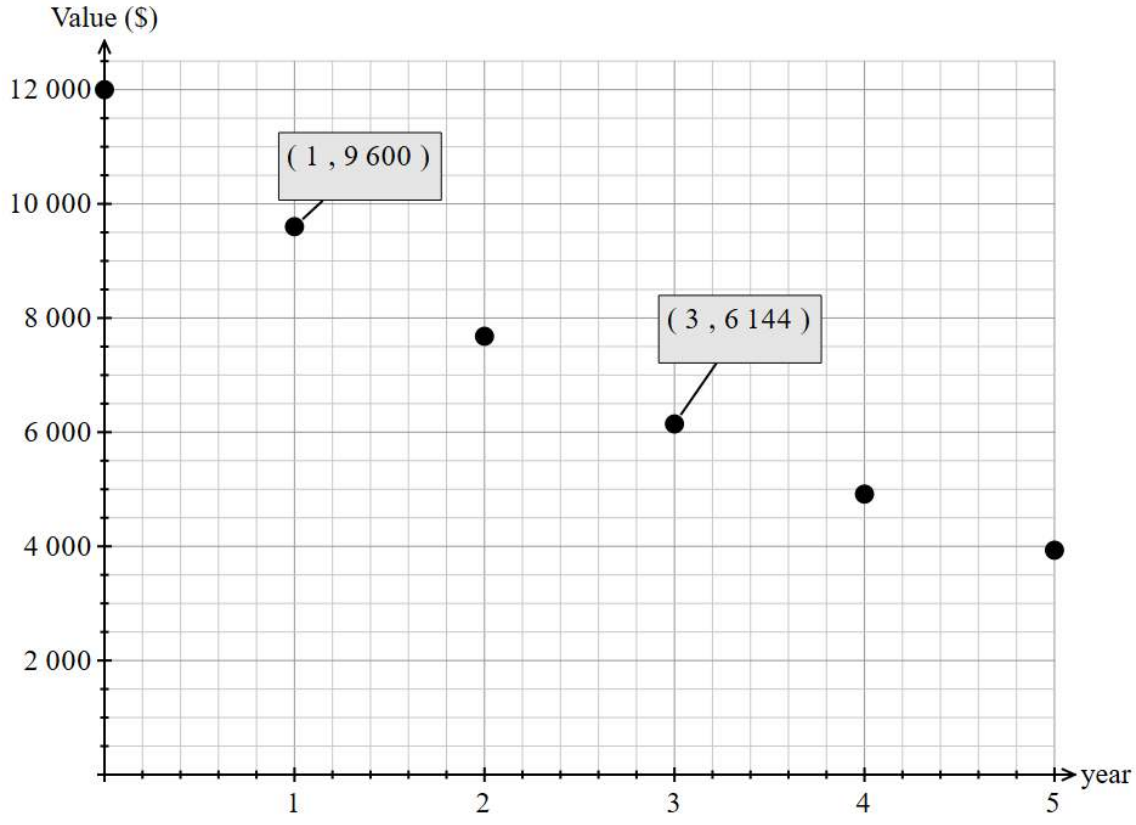
$$G_0 = 100\,000, \quad G_{n+1} = 1.0075 \times G_n - 800$$

This recurrence relation could **not** represent the balance of

A. A reducing balance loan at 9% pa compounding monthly with monthly payments of $800.
B. An annuity at 9% pa compounding monthly with monthly payments of $800.
C. An annuity at 7.5% pa compounding annually with yearly payments of $800.
D. A reducing balance loan at 3% pa compounding quarterly with quarterly payments of $800.
E. An annuity at 1.5% pa compounding twice per year with half-yearly payments of $800.

Question 21

The graph below shows the value of a computer system after each of the first five years of a reducing balance depreciation schedule:



Assuming that the depreciation continues to follow the same rule, the amount of the depreciation during the seventh year would be closest to:

A. $2517
B. $1728
C. $786
D. $629
E. $503

Use the following information to answer Questions 22 and 23:

The amortisation table for a reducing balance loan after 20, 21 and 22 months is shown below:

Month number	Payment made	Interest Charged	Principal Reduction	Balance
20	\$3046.51	\$491.65	\$2554.86	\$106 700.00
21	\$3046.51	\$480.15	\$2566.36	\$104 133.64
22	\$3046.51			

Question 22

The balance of this loan at the end of month 22 would be closest to

A. \$101 010.23
B. \$101 087.13
C. \$101 567.28
D. \$101 578.78
E. \$101 555.73

Question 23

The original amount borrowed at the start of the loan is closest to

A. \$166 147
B. \$159 576
C. \$155 680
D. \$155 678
E. \$166 594

Question 24

Tony is due to retire in five years. He currently has \$250 000 in his superannuation account. Every month for the next five years he will add \$1200 to the account and the account earns 3.2% per annum compounding monthly.

Upon retirement, Tony will invest his account balance into a perpetuity earning 4.8% per annum.

He plans to leave the money in the perpetuity for five years and then he will transfer the money into an annuity also at 4.8% per annum compounding monthly.

He will then take a monthly payment so that the annuity will last another 30 years.

A recurrence relation that would give the balance of his 30-year annuity account after n months is

A. $T_0 = 282\,174.85, \quad T_{n+1} = 1.004 \times T_n - 1480.47$
B. $T_0 = 371\,282.70, \quad T_{n+1} = 1.048 \times T_n - 1947.99$
C. $T_0 = 282\,174.85, \quad T_{n+1} = 1.048 \times T_n - 1480.47$
D. $T_0 = 371\,282.70, \quad T_{n+1} = 1.004 \times T_n - 1947.99$
E. $T_0 = 371\,282.70, \quad T_{n+1} = 1.004 \times T_n - 1826.73$

TURN OVER

Matrices

Question 25

The transpose of matrix $B = \begin{bmatrix} 3 & 6 \\ 1 & -8 \\ 0 & 2 \end{bmatrix}$ is

A. $\begin{bmatrix} 6 & 3 \\ -8 & 1 \\ 2 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 2 \\ 1 & -8 \\ 3 & 6 \end{bmatrix}$

C. $\begin{bmatrix} 6 & -8 & 2 \\ 3 & 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 1 & 0 \\ 6 & -8 & 2 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 & 3 \\ 2 & -8 & 6 \end{bmatrix}$

Question 26

A 4×1 column matrix, A, has a series of numerical elements in ascending order.

The same elements must be arranged in the opposite order in a 4×1 matrix, D, so that the elements are in descending order.

The following matrix equation can be used to achieve this change: $D = P \times A$.

The matrix P could be

A. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

TURN OVER

The following information is used in Questions 27 and 28:

At a martial arts club, 60 members train on a Tuesday night.

During training members can do floor exercises (F), combat exercises (C) or general fitness (G).

On the **second Tuesday night** 16 members did floor exercises, 18 members did combat exercises and 26 members did general fitness.

The members move each week between activities according to the transition matrix below:

$$T = \begin{array}{c} \\ F \\ C \\ G \end{array} \begin{array}{c} \begin{array}{ccc} \textit{this week} & & \\ F & C & G \end{array} \\ \begin{bmatrix} 0.5 & 0.1 & 0.2 \\ 0.2 & 0.6 & 0.1 \\ 0.3 & 0.3 & 0.7 \end{bmatrix} \end{array} \begin{array}{l} \\ F \\ C \\ G \end{array} \quad \textit{next week}$$

Question 27

Which of the following statements is <u>not</u> true?

A. In the long run half of the members will do general fitness every week.
B. In the long run the same number of members will do floor exercises and combat exercises every week.
C. In the long run three members will change from general fitness to floor exercises from one week to the next.
D. In the long run nine members will stay in combat exercises from one week to the next.
E. In the long run 21 members will stay in general fitness from one week to the next.

Question 28

The number of members who changed activity from the first week to the second week is

A. 12
B. 23
C. 24
D. 36
E. 37

Question 29

The element in row *i* and column *j* of matrix, *M*, is m_{ij}. The elements in matrix *M* are determined by the rule $m_{ij} = 4i - 8j$.

The element in row *i* and column *j* of matrix, *N*, is n_{ij}. The elements in matrix *N* are determined by the rule $n_{ij} = 5i + 2j$.

Matrix $L = M - N$. The elements in matrix *L* are determined by the rule

A. $l_{ij} = 9i - 10j$
B. $l_{ij} = -i - 10j$
C. $l_{ij} = -i - 6j$
D. $l_{ij} = i - 10j$
E. $l_{ij} = i - 6j$

Question 30

Matrix *A* is a 3×5 matrix. Matrix *B* is a 4×3 matrix. Matrix *C* is a 5×4 matrix.

Which one of the following matrix expressions could **not** be defined?

A. $(BAC)^2$
B. $(AC)^T - B$
C. $(CB)^T + A$
D. $CBA + CA$
E. $(CBA)^2$

TURN OVER

Question 31

A school organises a pizza lunch for their Year 12 students. Students can order a meat-based pizza (M) or a vegetarian pizza (V) and students can also choose between a small (S) or a large (L) pizza.

The number of students ordering each type of pizza is given in the matrix below:

$$P = \begin{matrix} \\ S \\ L \end{matrix} \begin{matrix} \begin{matrix} M & V \end{matrix} \\ \begin{bmatrix} 23 & 16 \\ 43 & 5 \end{bmatrix} \end{matrix}$$

Small pizzas cost \$6 each and large pizzas cost \$8 each.

Because the school is ordering a large number of pizzas, the shop offers a discount of 10% on small pizzas and 20% on large pizzas.

A matrix equation that could calculate the total cost to the school of all ordered pizzas is

A. $\begin{bmatrix} 0.9 & 0.8 \end{bmatrix} \times P \times \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

B. $\begin{bmatrix} 0.9 & 0.8 \end{bmatrix} \times P^T \times \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

C. $\begin{bmatrix} 6 \\ 8 \end{bmatrix} \times P^T \times \begin{bmatrix} 0.9 & 0.8 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 \end{bmatrix} \times P \times \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix} \times \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 \end{bmatrix} \times P^T \times \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix} \times \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

Question 32

A Leslie matrix, L, is used to predict the numbers of female eggs (E), caterpillars (C), pupae (P) and butterflies (B) from month to month in a breeding colony of butterflies.

The number of female insects at each stage is given by $K_{n+1} = L \times K_n$ where L is shown below:

$$L = \begin{array}{c} \\ E \\ C \\ P \\ B \end{array} \begin{array}{c} \begin{array}{cccc} E & C & P & B \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 10 \\ 0.2 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix} \end{array} \begin{array}{l} \text{this month} \\ \\ \\ \\ \\ \text{next month} \end{array}$$

Initially there are 200 female eggs, 100 female caterpillars, 100 female pupae and 100 female butterflies.

Which of the following statements about the colony is **not** true?

A. Only 30% of the female pupae survive to become female butterflies.
B. After the first month there are never more than 30 female butterflies in the colony.
C. Every fourth month, the predicted numbers of female caterpillars, pupae and butterflies are equal.
D. The greatest number of female caterpillars at any time is 300.
E. Eventually the female colony is predicted to become extinct.

TURN OVER

Networks and decision mathematics

Question 33

An icosahedron is a three dimensional object that has 20 triangular faces and 12 vertices.

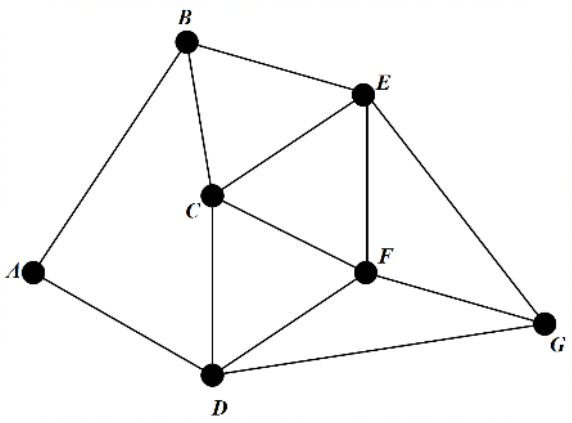
A planar graph can be drawn to represent the vertices and edges of an icosahedron.

The number of edges the icosahedron must have is

A. 28
B. 30
C. 32
D. 36
E. 60

Question 34

The following network shows the connections between 7 towns.

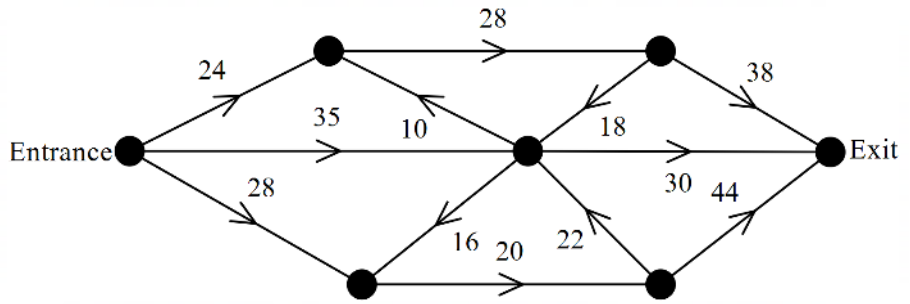


An Eulerian trail for this network is

A. A-B-C-D-E-F-G
B. A-D-C-B-E-G-F
C. A-D-C-B-E-C-F-D-G-E-F
D. B-A-D-C-B-E-C-F-D-G-F-E-G
E. B-A-D-C-F-E-F-D-G-E-F-G

Question 35

The network shows the maximum flow of people through a gallery.

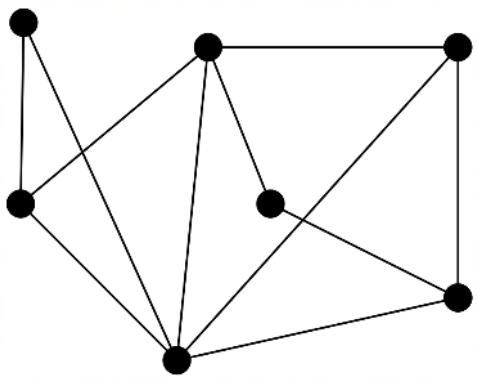


The maximum flow from the entrance to the exit is

A. 57
B. 66
C. 78
D. 87
E. 112

Question 36

The graph shown below is planar.



How many faces does this graph have?

A. 5
B. 6
C. 7
D. 8
E. 9

TURN OVER

Question 37

Allan, Brianna, Caroline and Daisy are four workers at a company. Each worker will perform one activity. Caroline is unable to complete activity 3.

The time for each worker to complete activities, in minutes, is shown in the table below:

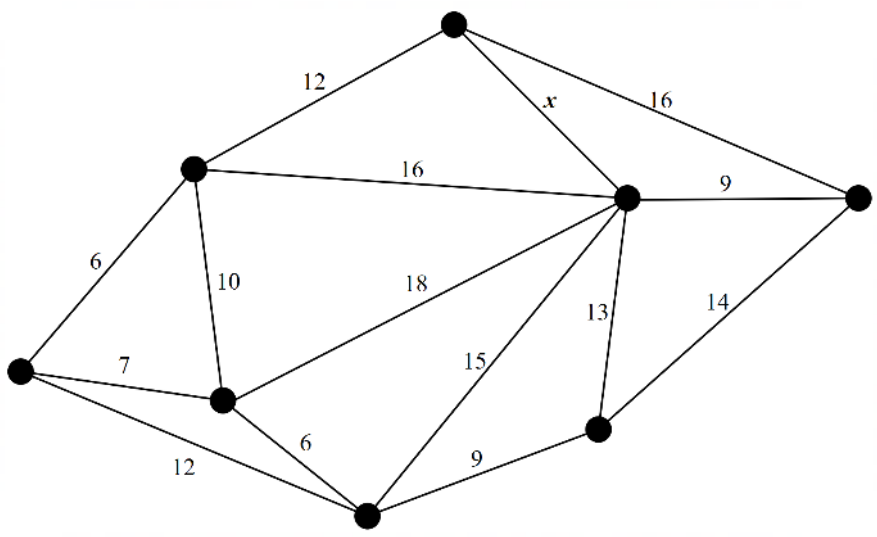
	Activity 1	Activity 2	Activity 3	Activity 4
Allan	6	9	12	10
Brianna	7	10	14	8
Caroline	6	8	X	9
Daisy	9	8	16	8

If Daisy is able to reduce her time for activity 3 to 12 minutes, the minimum total time to complete all four activities would

A. Stay the same.
B. reduce by 1 minute.
C. reduce by 2 minutes.
D. reduce by 3 minutes.
E. reduce by 4 minutes.

Question 38

The weights on the edges of the network below show the cost, in millions of dollars, of building gas pipes, between eight towns.



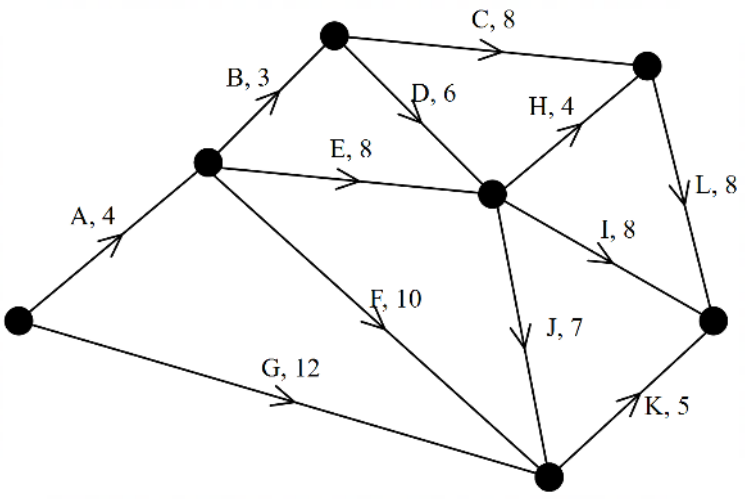
A minimum spanning tree is drawn that includes the edge labelled x.

The **maximum** weight of the edge labelled x is

A. 10
B. 12
C. 13
D. 15
E. 16

The following information is used in Questions 39 and 40:

The activity network below shows the the time it takes for activities, given in days, for a project.



Question 39

The float time for activity I is

A. 0
B. 2
C. 4
D. 13
E. 17

Question 40

The time to complete the project must be reduced.

Which one of the following would yield the maximum change in the expected time for completing the project?

A. Reduce activity A by 2 days.
B. Reduce activity D by 3 days.
C. Reduce activity E by 3 days.
D. Reduce activity I by 2 days.
E. Reduce activity L by 2 days.

END OF MULTIPLE - CHOICE QUESTION BOOKLET