Neap:

Trial Examination 2019

VCE Further Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	7	7	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

Students are to write in blue or black pen.
Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 29 pages
Formula sheet
Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – CORE

Instructions for Section A

Answer **all** questions in the spaces provided.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Data analysis

Question 1 (8 marks)

Landhere Airport is serviced by two small regional airlines, TG Airlines and Supreme Airlines. The number of passengers carried per day by each of the small airlines is recorded in the back to back stem plot below.

TG Airlines	*stem*	*Supreme Airlines*
0 0 0 0 0 0 0 0	0	
9 9 8 8 7	1	5
5 5 5 4 4 4 4 4 3 3 3 2 2 1 1 1 0	2	8 8 9
	3	2 4 6 6 8 8 9
	4	2 2 2 3 4 5 6 6 6 7 7 8 8 9
	5	0 0 0 0 0

2|1 = 21 passengers

a. For how many days were the results recorded? 1 mark

_____

b. Are the data sets symmetrical? Explain your answer. 1 mark

_____

_____

c. One of the airlines operates flights only five days per week.
Use the numbers in the stem plot to identify this airline and explain your answer. 1 mark

_____

_____

d. Find the five-figure summary for Supreme Airlines. 2 marks

_____

_____

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e. Are there any outliers in the data for Supreme Airlines? Use a calculation to support your answer. 1 mark

_____

_____

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_____

f. Two hundred passengers are surveyed about the number of safety checks they need to pass through at the airport. The results are recorded in the following table.

	Frequent flyer	**Occasional flyer**
Too many	42	11
About right	55	64
Need more	3	25

i. What percentage of occasional flyers believe that more safety checks are required? 1 mark

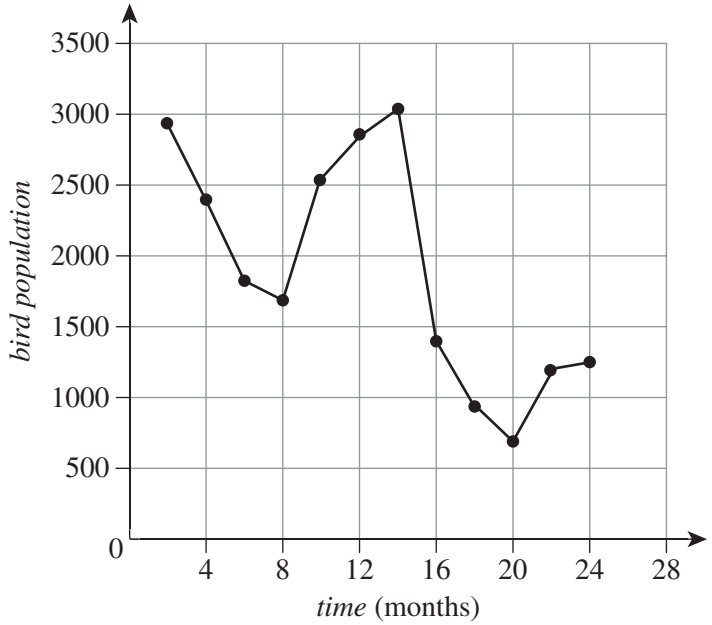
_____

ii. What percentage of all flyers believe that there are too many safety checks? 1 mark

_____

Question 2 (6 marks)

The population of birds that live along the flight path of landing planes can be a problem both for the bird population and the planes if not properly managed. The time series below shows the recorded bird population near Landhere Airport every two months over a two-year period.



a. Describe the trend of the time series of the bird population. 1 mark

_____

_____

b. On the time series above, apply a three-point moving median to smooth the curve. 1 mark

c. The coordinates for the first year are shown in the table below.

Month	2	4	6	8	10	12
Bird population	2952	2400	1800	1700	2515	2820

Find the three-point moving average centred on month 6.1 mark

_____

_____

In areas unaffected by the flight path, the bird population is seasonal. The following data is collected over a three-year period in a location well away from the airport.

	Summer	**Autumn**	**Winter**	**Spring**	**Total**	**Mean**
2016	15 000	14 000	12 000	16 000	57 000	14 250
2017	15 500	13 500	11 500	14 500	55 000	13 750
2018	14 000	13 500	12 000	16 000	55 500	13 875
Total	44 500	41 000	35 500	46 500	167 500	

d. Use the above information to find the summer seasonal index. Give your answer to two decimal places. 2 marks

_____

_____

_____

_____

e. The seasonal index for winter is 0.85.

If the raw population number is 13 000, then what is the de-seasonalised figure? 1 mark

_____

_____

Question 3 (10 marks)

Considering the departing flights only, the passenger numbers and the number of flights have been recorded for random weeks.

Flights	380	410	240	310	460	680
Passengers	76 060	82 000	46 500	60 450	95 400	146 200

a. Find the equation of the least squares line.

Round all values to the nearest whole number. 1 mark

_____

_____

b. What is the minimum number of flights required to carry 75 000 passengers in one week? 1 mark

_____

_____

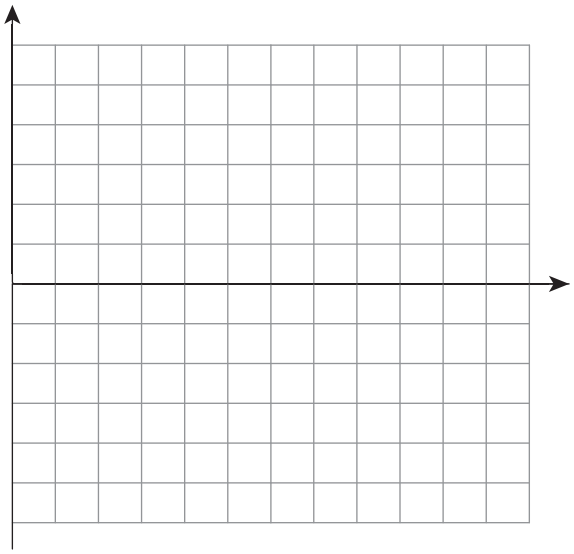
The equation of the least square line for a different set of weeks is shown below.

number of passengers = 195 × number of flights – 250, with $r = 0.99$

c. Complete the table of residuals below. 2 marks

Number of flights	250	320	350	410	430
Number of passengers (actual)	49 230	62 180	65 250	79 830	84 630
Number of passengers (predicted)	48 500	62 150	68 000		
Residual	730	30	–2750		

d. Draw a residual plot on the axes below. 1 mark



e. Comment on what the residual plot shows. 1 mark

_____

f. Perform an x^2 transformation on this data.

i. Write the equation of the line of least squares. Give your answer to two decimal places and/or round to the nearest ten as needed. 1 mark

_____

_____

ii. Find the correlation coefficient. Give your answer to two decimal places. 1 mark

_____

_____

g. Using all the information you have gathered, does the linear or the x^2 transformation best describe the data supplied? Refer to your previous answers to justify your answer. Do **not** do any new maths. 2 marks

_____

_____

_____

_____

Recursion and financial modelling

Question 4 (2 marks)

Anika is researching some ideas for her new business, which sells mathematical puzzles. As she loves maths, her business is named Fibonacci after the Italian mathematician born in 1170. He is famous for studying recursion series. An example of a Fibonacci series is shown below.

$$t_{n+2} = t_n + t_{n+1};\ t_1 = 6 \text{ and } t_2 = 7$$

By finding the values of t_4 and t_5, show that $t_5 = t_1 + t_2 + t_4$.

Question 5 (5 marks)

Anika borrows $400 000 to set the Fibonacci business up. The interest on the loan is fixed at 4.2% per annum compounding monthly. The loan will be repaid over a ten-year period.

a. Find Anika's monthly repayments. 1 mark

_____

_____

The financial prediction for the business is to break even in the first year and make a profit of $80 000 in the second year, increasing by 5% annually for the next eight years until the loan is repaid.

b. Write an expression that can be used to find the profit in year n, where $3 \leq n \leq 10$. 1 mark

_____

c. If the predictions are correct, in what year will the profit first pass $100 000? 2 marks

_____

_____

_____

_____

d. Year five turns out to be very good year, making a profit of $200 000. Anika decides to make a one-off repayment of $100 000 off the loan.

If she continues to make the same repayments, how much time is taken off the length of the loan? 1 mark

_____

_____

_____

_____

Question 6 (2 marks)

The $60 000 copier/printer that Anika purchases for the business can be depreciated using two different methods.

Method 1: A 20% depreciation on the reducing balance per year is used.

Method 2: A unit cost depreciation of 1 cent per print is used.

The business prints an average of 1 000 000 pages per year.

Find the value of the copier/printer under both methods after three years.

_____

_____

_____

_____

Question 7 (3 marks)

After six years Anika sells the business for a profit of $800 000, which she invests in a perpetuity account earning 3.1% per annum. The interest is calculated and paid monthly.

a. What monthly payment will Anika receive? 1 mark

_____

_____

b. Anika places her monthly payments into her working bank account on the first of each month. A portion of an account statement is shown below.

Date	Deposit	Withdrawal	Balance
1/1/2019			$34 560
1/2/2019			
15/2/2019		$2000	*A*
1/3/2019			
31/3/2019	interest *B*		

Interest is added at the end of each three-month period. Interest is calculated monthly at 1.8% per annum on the lowest balance for each month.

Find *A* and *B*. 2 marks

_____

_____

_____

_____

END OF SECTION A

SECTION B – MODULES

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Question 1 (4 marks)

The matrix M shows how many tickets were sold by the Badgers Netball Club over the last three weeks. Tickets are available in either Bronze (B), Silver (S) or Gold (G) categories.

$$M = \begin{array}{c} \begin{matrix} B & S & G \end{matrix} \\ \begin{bmatrix} 312 & 612 & 315 \\ 405 & 513 & 275 \\ 370 & 570 & 401 \end{bmatrix} \end{array} \begin{matrix} W_1 \\ W_2 \\ W_3 \end{matrix}$$

a. How many Gold tickets were sold in week 2? 1 mark

_____

_____

b. The following matrix, Q, contains the prices for Bronze, Silver and Gold tickets:

$$Q = \begin{bmatrix} 15 \\ 20 \\ 30 \end{bmatrix}$$

i. Calculate the matrix product $R = M \times Q$. 1 mark

_____

_____

_____

_____

ii. What does R_{21} represent? 1 mark

_____

_____

c. Aside from tickets, the Badgers Netball Club also sells food at their games to raise revenue. Consider the matrix equation below where a is the cost of a steak pie and b is the cost of a spinach and ricotta roll.

$$\begin{bmatrix} 36 & 42 \\ 81 & 37 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 462.60 \\ 713.10 \end{bmatrix}$$

What is the cost of a steak pie? 1 mark

_____

_____

Question 2 (2 marks)

In the pre-season warm-up competition four teams were placed in a round-robin pool together: the Anteaters (*A*), the Badgers (*B*), the Camels (*C*) and the Dingoes (*D*).

Over the course of the competition:

- the Anteaters beat the Badgers;
- the Badgers beat the Camels and the Dingoes;
- the Camels beat the Anteaters, and;
- the Dingoes beat the Anteaters and the Camels.

a. Represent this information in a dominance matrix. 1 mark

b. Another pool featured another four teams: the Eagles (*E*), the Fires (*F*), the Golds (*G*) and the Heroes (*H*). The following matrix *Z* represents the matches between the teams.

$$Z = \begin{array}{c} \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{array}{cccc} losers & & & \\ E & F & G & H \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ winners \\ \\ \\ \end{array}$$

Using the sum of both one-step and two-step dominances, rank the teams from 1st to 4th. 1 mark

1st	
2nd	
3rd	
4th	

Question 3 (6 marks)

As part of a healthy living campaign, 10 000 netball players were surveyed to see how they travel to work following a marketing campaign. The marketing campaign was intended to encourage people to either cycle or walk to work. Participants either drove (D), took public transport (P), cycled (C) or walked (W).

The transition matrix (T) shown below shows how participants were expected to travel to work from one month to the next.

$$T = \begin{array}{c} \\ D \\ P \\ C \\ W \end{array} \begin{array}{c} \begin{array}{cccc} D & P & C & W \end{array} \\ \begin{bmatrix} 0.6 & 0.1 & 0.05 & 0.1 \\ 0.1 & 0.5 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.65 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.6 \end{bmatrix} \end{array} \begin{array}{l} \\ D \\ P \\ C \\ W \end{array}$$

this month (columns) / *next month* (rows)

a. Matrix S_1 shows the number of participants and how they travelled to work in June.

$$S_1 = \begin{bmatrix} 2950 \\ 1800 \\ 2050 \\ 3200 \end{bmatrix} \begin{array}{l} D \\ P \\ C \\ W \end{array}$$

Complete matrix S_0 to show how participants travelled to work in May. 1 mark

$$S_0 = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{array}{l} D \\ P \\ C \\ W \end{array}$$

_____

_____

_____

_____

b. Write a calculation that shows 1720 participants took public transport to work in July. 1 mark

_____

_____

c. Of the participants that walked to work in July, what percentage drove to work in June? Give your answer to the nearest whole number. 2 marks

_____

_____

d. Was the marketing campaign successful? Use appropriate figures to support your answer. 1 mark

_____

_____

_____

_____

e. The following year, survey organisers chose to increase the number of participants by inviting football players from the local league to join each month.

Matrix R_0 shows the initial number surveyed in May of the following year.

$$R_0 = \begin{bmatrix} 2000 \\ 500 \\ 500 \\ 1000 \end{bmatrix}$$

The matrix to predict how participants are expected to travel to work can be modelled as shown in the equation below.

$$R_{n+1} = TR_n + \begin{bmatrix} 100 \\ 25 \\ 25 \\ 50 \end{bmatrix}$$

How many participants are expected to use public transport in July? 1 mark

_____

_____

_____

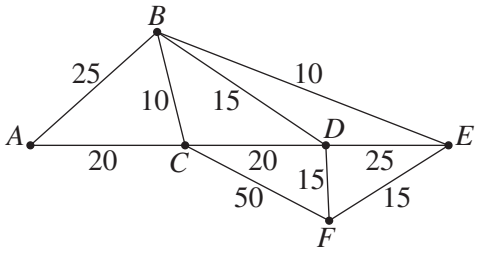
_____

END OF MODULE 1

Module 2 – Networks and decision mathematics

Question 1 (3 marks)

The following diagram shows the time, in minutes, that it takes to travel between various branches of Fresh Value, a chain of convenience stores.



a. How long would it take to travel from *A* to *F* if the quickest route was taken? 1 mark

_____

_____

b. The area manager departs shop *A* and follows a Hamiltonian cycle.

Write down a route that the manager could take. 1 mark

_____

_____

c. Show that Euler's formula ($v + f = e + 2$) holds for the diagram above. 1 mark

_____

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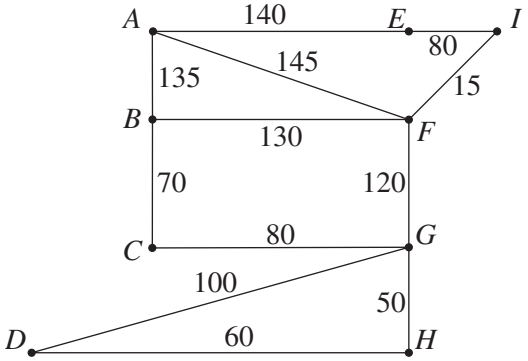
Question 2 (2 marks)

The diagram below shows a computer network in a distribution warehouse. The numbers on the edges indicate the cost, in dollars, of connecting each computer.



The computers will be connected via network cables, which will form a connected graph. The minimum cost of connecting these computers needs to be found.

a. What is the mathematical term that would represent these network cables? 1 mark

_____

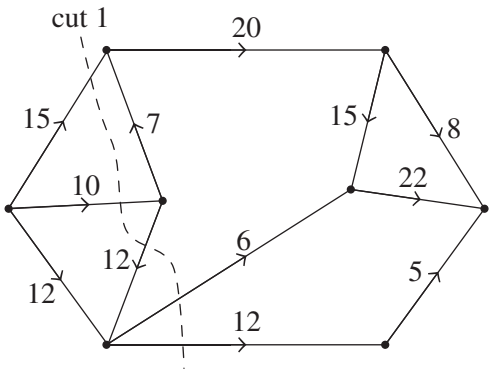
b. What is the minimum cost of connecting the computers? 1 mark

_____

_____

Question 3 (2 marks)

The diagram below shows the capacity, in litres, of various pipes that distribute water in a production facility.



a. Write down the capacity of cut 1. 1 mark

_____

_____

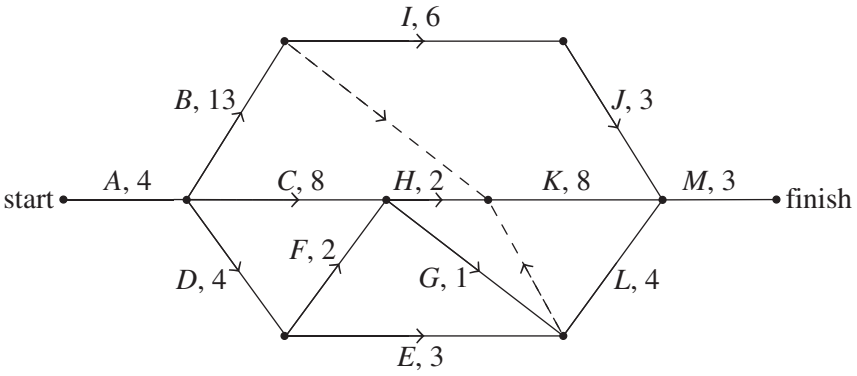
b. What is the maximum volume of water, in litres, that can flow through the network of pipes? 1 mark

_____

_____

Question 4 (5 marks)

A new store is planned. The project to construct the new shop involves activities *A* to *M*. The directed network below shows these activities and their completion times in days.



a. Complete the precedence table for the network above. 1 mark

Activity	Preceding activities
A	–
B	*A*
C	*A*
D	*A*
E	*D*
F	*D*
G	
H	*C, F*
I	*B*
J	*I*
K	
L	*E, G*
M	*J, K, L*

b. Determine the latest starting time, in days, for activity *E*. 1 mark

_____

c. Determine the critical path for this network. 1 mark

_____

d. Two of the activities have a float time of six hours.

State the two activities with this float time. 1 mark

_____

e. The completion times for activities *I*, *K* and *L* can each be reduced. The cost of reducing the completion time by one day for these activities is shown in the table below.

Activity	Maximum reduction (days)	Cost per day ($)
I	2	800
K	2	600
L	4	1000

What is the minimum cost to complete the project in the shortest possible time? 1 mark

_____

_____

_____

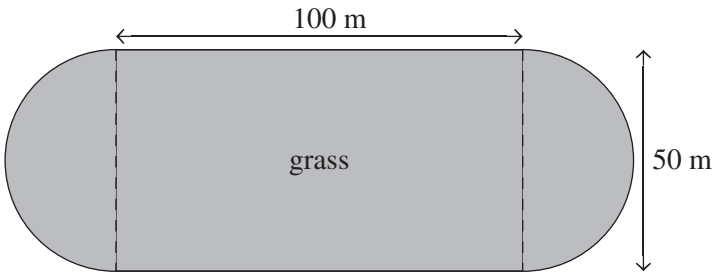
_____

END OF MODULE 2

Module 3 – Geometry and measurement

Question 1 (4 marks)

The Regional Building Authority is planning a new town in Victoria. They plan to install an athletics venue, as shown in the diagram below.

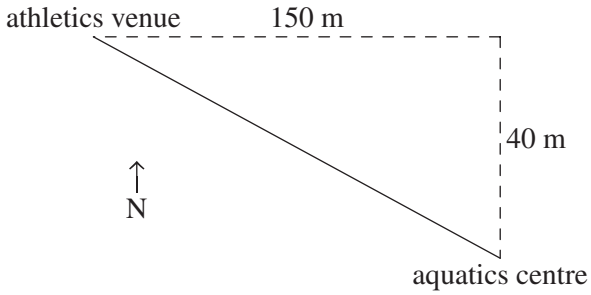


a. Calculate the area of the above section of the venue. Give your answer to the nearest metre squared. 1 mark

_____

_____

Rita has swimming training straight after athletics training. In order to travel from the athletics venue to the aquatics centre, she must travel across the leisure complex.



b. If Rita walks in a straight line from the athletics venue to the aquatics centre, what distance will she walk? Give your answer, in metres, to one decimal place. 1 mark

_____

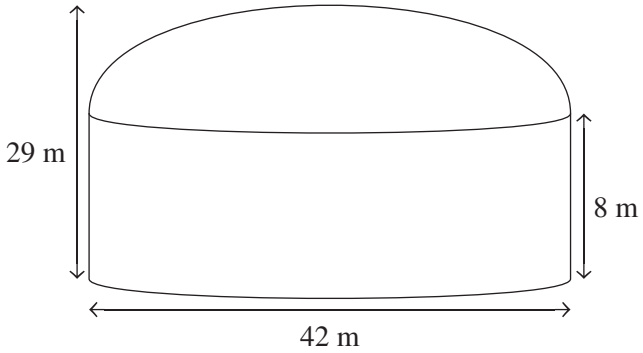
_____

c. What is the three-figure bearing that Rita must walk on to get to the aquatics centre from the athletics venue? Give your answer to the nearest degree. 1 mark

_____

_____

d. The aquatics centre is to be designed as a cylindrical building with a hemispherical dome for the roof, as shown in the diagram below.



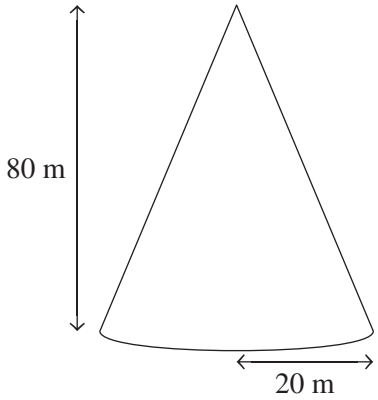
Calculate the surface area of the exterior of the aquatics centre. Give your answer to the nearest metre squared. 1 mark

_____

_____

Question 2 (4 marks)

Inspired by a cathedral in Brazil, the town planners are planning to build a conical-shaped community centre. The original design uses a cone with a height of 80 m and a radius of 20 m as shown in the diagram below.



The planners need to calculate the volume as part of the engineering report.

a. Calculate the volume of the cone building. Round your answer to the nearest cubic metre. 1 mark

_____

_____

b. A scale model of the building is to be made. It is mathematically similar. The scale model is to be 1 m tall.

Calculate the radius of the base of the scale cone model. 1 mark

_____

_____

c. The new town is due to be built at a latitude of 36°S and a longitude of 141°E. A team of investors from Sapporo, Japan are due to visit. Sapporo is also located at a longitude of 141°E.

Assume the radius of Earth is 6400 km. The shortest great circle distance from Sapporo to the new town is approximately 8825 km.

What is the radius of the small circle of Earth at Sapporo? Give your answer to the closest 100 km. 2 marks

_____

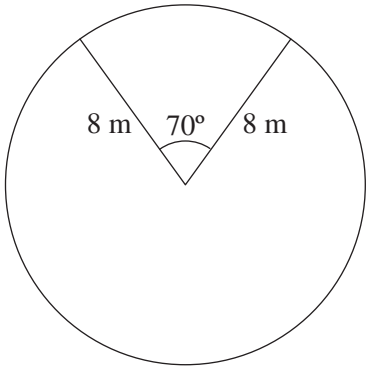
_____

_____

_____

Question 3 (4 marks)

A circular flower bed in the proposed parkland is set to be divided into two sectors, as shown in the diagram below.



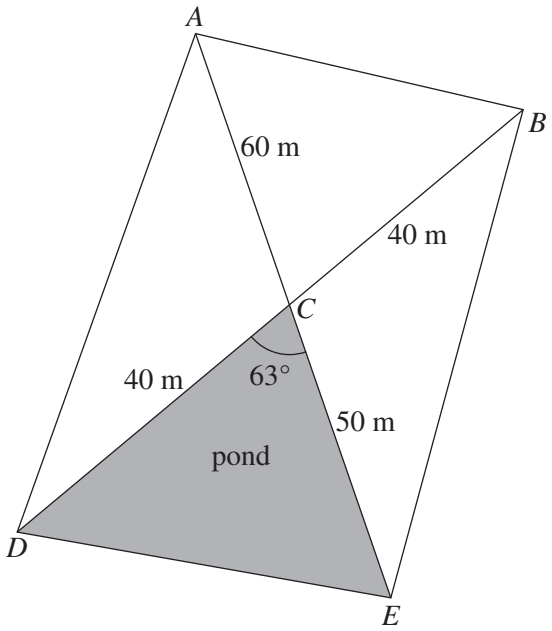
Blue flowers will occupy the major sector and yellow flowers will occupy the minor sector.

a. Calculate the area of the major sector. Give your answer to the nearest square metre. 1 mark

_____

_____

A section of parkland is bounded by four paths. It is also crossed by two paths, as shown in the diagram below.



b. Calculate the area of the pond (ΔCDE). Give your answer to the nearest square metre. 1 mark

_____

_____

c. Calculate the length of path *AB*. Give your answer to the nearest metre. 2 marks

_____

_____

_____

_____

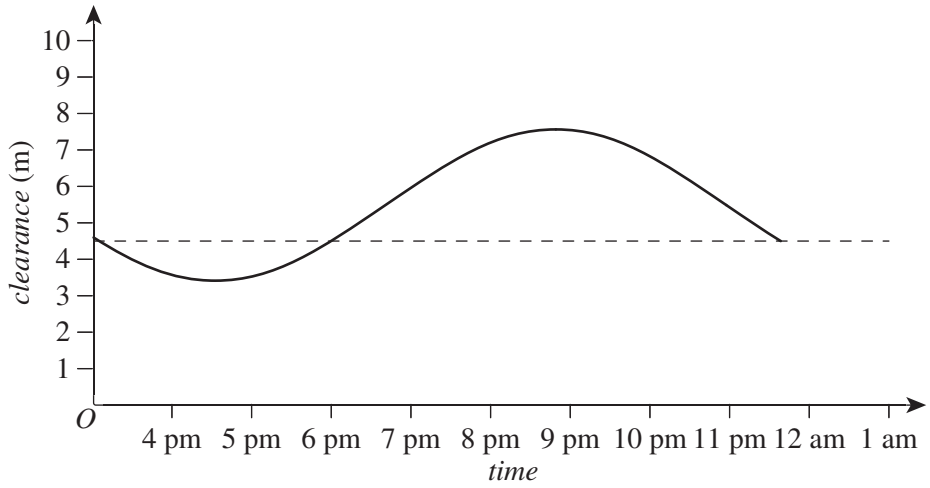
END OF MODULE 3

Module 4 – Graphs and relations

Question 1 (2 marks)

Most evenings Gazza's party boat travels up and down the river, taking groups of up to 200 passengers for a dinner cruise. The only obstacle is a bridge to be passed under. At high tide Gazza's boat cannot pass under the bridge. Gazza needs at least 4.5 m of clearance between the bottom of the bridge and the water's surface to safely pass.

Gazza has a booking for next Wednesday and he is checking the tidal map below to see what times he will be unable to pass under the bridges.



a. When should Gazza plan for the cruise to leave? 1 mark

_____

_____

b. Reading from the graph, to the nearest hour, for how long is it **not** possible to pass under the bridge? 1 mark

_____

_____

Question 2 (6 marks)

A big company books Gazza for a group of 200. Gazza will provide the buses to transport the group to and from their place of work. There are 10 buses with 25 seats available and 8 buses with 20 seats. There are only 9 drivers in total.

Let x = the number of 25-seater buses used.

Let y = the number of 20-seater buses used.

Constraint 1: $x + y \leq 9$

Constraint 2: $x \geq 0$

Constraint 3: $y \geq 0$

a. There is one other constraint for the values of x and y:

Constraint 4: $25x + 20y \geq 200$

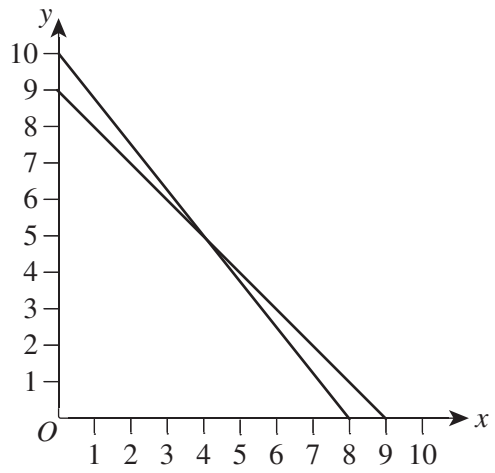
Explain how this constraint is arrived at. 1 mark

_____

_____

b. The four constraints are graphed on the grid below.

Shade the required area and label potential solutions (A, B and so on). 2 marks



c. Gazza wants to minimise the cost of hiring the buses and so will use the cheapest combination of 25-seater and 20-seater buses that will transport all 200 people. 25-seater buses cost $600 each and 20-seater buses cost $500 each.

Write the objective function. 1 mark

_____

d. State the minimum cost and the combination of buses required for this cost. 2 marks

_____

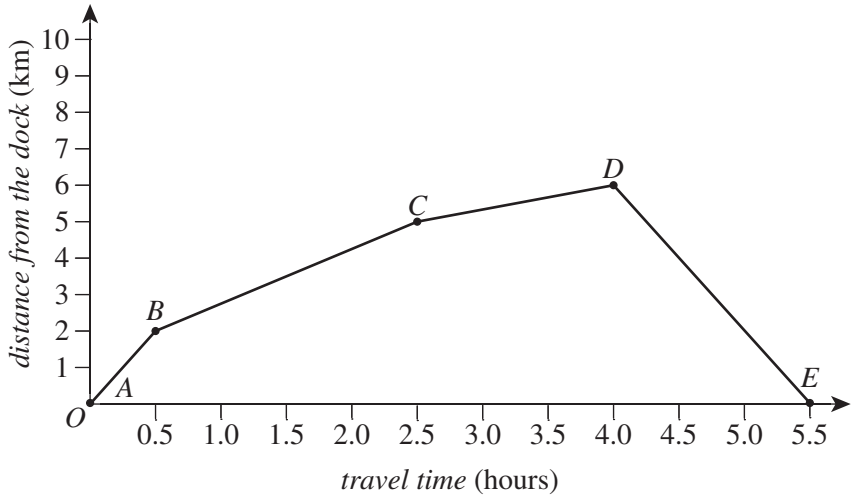
_____

_____

_____

Question 3 (4 marks)

The following distance–time graph shows the trip of Gazza's boat.



a. At what point(s) does the boat change direction? 1 mark

_____

_____

b. At what speed is the boat travelling between *B* and *C*? 1 mark

_____

_____

c. Another party boat leaves half an hour after Gazza's boat and travels at a constant speed of 1 km per hour in the same initial direction as Gazza.

Add this line to the graph above. 1 mark

d. How long after the second boat leaves will the two boats meet? 1 mark

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END OF QUESTION AND ANSWER BOOKLET