Neap:

Trial Examination 2019

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

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SECTION A – CORE

Data analysis

Question 1 (8 marks)

a. Each airline has 30 days recorded. A1

b. No. The TG Airlines data is positively skewed. A1

c. TG Airlines have eight days with zero passengers, indicating that there were no flights on those days or that Supreme Airlines' data is negatively skewed. A1

d. There are 30 data points, so the median is between the 15^{th} and 16^{th} – Q_1 the median of the first 15 is the 8^{th} data point 26, and Q_3 the median of the second 15 is the 23^{rd} data point 48. The minimum value is 15 and the maximum 50.

Therefore the correct values are 15, 36, 43.5, 48 and 50. M1 A1

e. The IQR for Supreme Airlines is 18 (48 – 36 = 12, 1.5 × 12 = 18). The fences for outliers are 18 and 66 (36 – 18 = 18 and 48 + 18 = 66). The figure of 15 is lower than the lower boundary and so is an outlier. A1

f. **i.** 25% A1

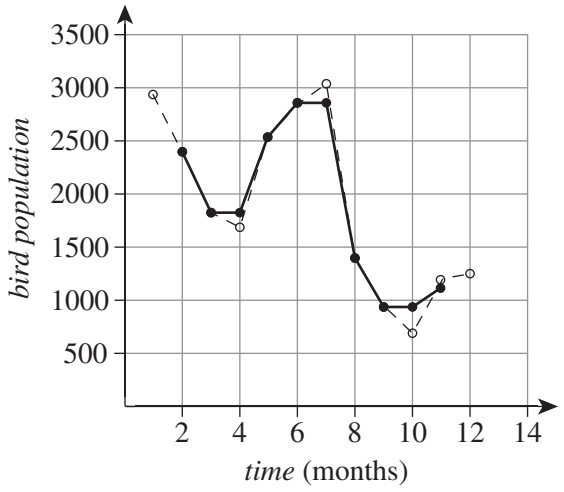
ii. $\frac{53}{2} = 26.5\%$ A1

Question 2 (6 marks)

a. It is a decreasing trend. A1

Note: There is also a seasonal element to the data, but the decreasing trend must be noted to achieve the mark.

b.



A1

c. $\frac{2400 + 1800 + 1700}{3} = 1967$ A1

d. $\dfrac{\dfrac{15\,000}{14\,250} + \dfrac{15\,500}{13\,750} + \dfrac{14\,000}{13\,875}}{3} = 1.06$ M1 A1

e. $\dfrac{13\,000}{0.85} = 15\,294$ A1

Question 3 (10 marks)

a. passengers = 229 × flights – 10 067 A1

b. $\text{flight} = \dfrac{75\,000 + 10\,067}{229}$

$= 371.5$

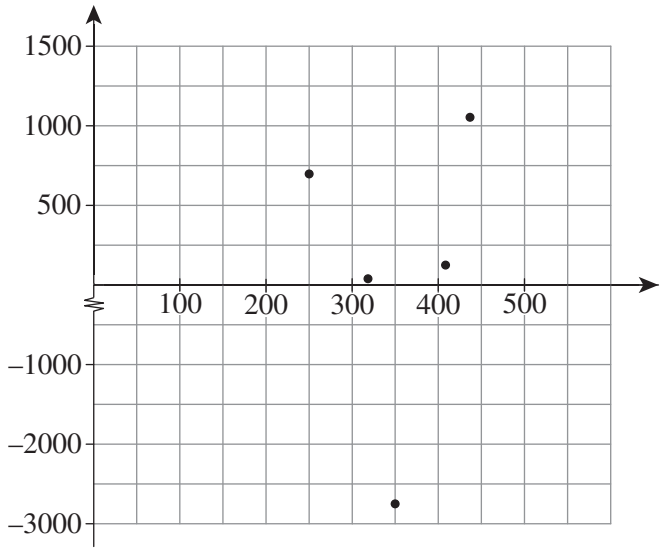
Therefore 372 flights is the minimum. A1

c.

Number of flights	250	320	350	410	430
Number of passengers (actual)	49 230	62 180	65 250	79 830	84 630
Number of passengers (predicted)	48 500	62 150	68 000	**79 700**	**83 600**
Residual	730	30	–2750	**130**	**1030**

missing passenger numbers A1
residuals A1

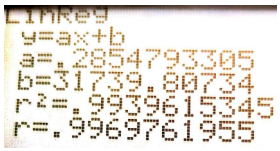
d.



A1

e. There is a pattern which does not support the idea of a linear relationship. A1

f. Enter the data into your technology and apply an x^2 transformation to the x values before doing the regression calculation.



i. $y = 0.29x^2 + 31\,740$ A1

ii. 0.997 or 1.00 A1

g. Both the linear and x^2 models have strong correlation, 0.99 and 1.00 respectively. A1

The residual plot for the linear does not support the assumption of linearity. A residual plot for the x^2 transformation would need to be done to see if that was the preferred model. A1

Note: The actual details do not need to match the answer supplied but must be accurate.

Recursion and financial modelling

Question 4 (2 marks)

$t_1 = 6$

$t_2 = 7$

$t_3 = 6 + 7 = 13$

$t_4 = 7 + 13 = 20$

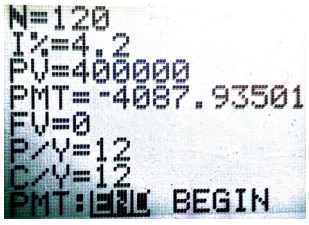
$t_5 = 13 + 20 = 33$

$6 + 7 + 20 = 33$

M1 A1

Question 5 (5 marks)

a. Enter the data into the financial application of your technology.



The monthly repayment is $4087.94. A1

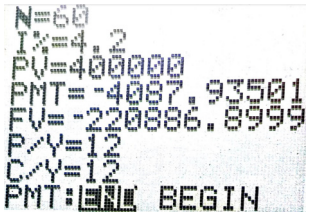
b. $P_n = 80000(1.05)^{(n-2)}$ A1

c.

Year	1	2	3	4	5	6	7
Profit	0	80 000	84 000	88 200	92 610	97 240.50	102 102.53

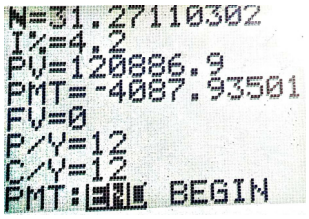
M1 A1

d. Firstly find the value after 5 years using your technology.



After 60 months (5 years) the loan is now \$120 886.90.

Substitute the value of 120 886.8999 into the present value, leave all other values the same and recalculate *N*.



N is now 31.27 months (32 months). It was 60 months, so the time saved is 28 months. A1

Question 6 (2 marks)

Method 1 $= 60\,000 \times (0.8)^3$

$= \$30\,720$ A1

Method 2 $= 60\,000 - 3 \times 10\,000$

$= \$30\,000$ A1

Question 7 (3 marks)

a. $800\,000\left(1 + \dfrac{\dfrac{3.1}{12}}{100}\right) = \$802\,066.67$

Therefore the monthly repayment is \$2066.67. A1

b.

Date	Deposit	Withdrawal	Balance
1/1/2019	**2066.67**		\$34 560
1/2/2019	**2066.67**		**\$36 626.67**
15/2/2019		\$2000	*A* **\$34 626.34**
1/3/2019	**2066.67**		**\$36 693.67**
31/3/2019	interest *B* **\$158.82**		

A is calculated by adding the income for two months and subtracting the withdrawal, which equals \$34 626.67. A1

Interest (B) is $\frac{1.8}{12}\%$ of the lowest balance each month. January's lowest balance is \$34 560, February's is \$34 626.67 and March's is \$36 693.34.

Therefore, $B = \left(\frac{\frac{1.8}{12}}{100}\right) \times (34\,560 + 34\,626.67 + 36\,693.34)$

$= \$158.82$ A1

SECTION B – MODULES

Module 1 – Matrices

Question 1 (4 marks)

a. 275 A1

b. **i.** $\begin{bmatrix} 312 & 612 & 315 \\ 405 & 513 & 275 \\ 370 & 570 & 401 \end{bmatrix} \begin{bmatrix} 15 \\ 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 26\,370 \\ 24\,585 \\ 28\,980 \end{bmatrix}$ A1

ii. R_{21} represents the total ticket revenue for week 2. A1

c. $\begin{bmatrix} 36 & 42 \\ 81 & 37 \end{bmatrix}^{-1} \begin{bmatrix} 462.60 \\ 713.10 \end{bmatrix} = \begin{bmatrix} 6.2 \\ 5.7 \end{bmatrix}$

\$6.20 A1

Question 2 (2 marks)

a. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ A1

b. $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow d$

$d + d^2 = \begin{bmatrix} 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ A1

1st	Golds
2nd	Eagles
3rd	Fires
4th	Heroes

Note: For full marks the order must be completely correct.

Question 3 (6 marks)

a. $S_0 = T^{-1} \times S_1$

$$T^{-1} \times \begin{bmatrix} 2952 \\ 1800 \\ 2050 \\ 3200 \end{bmatrix} = \begin{bmatrix} 4000 \\ 2000 \\ 1000 \\ 3000 \end{bmatrix}$$ A1

OR

Use solve function.

$$\text{solve}\left(t \cdot \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2950. \\ 1800. \\ 2050. \\ 3200. \end{bmatrix}, a, b, c, d\right)$$

$4000. \text{ and } b = 2000. \text{ and } c = 1000. \text{ and } d = 3000$

b. $2950 \times 0.1 + 1800 \times 0.5 + 2050 \times 0.1 + 3200 \times 0.1 = 1720$ A1

c. 2950 walked in June, and 20% of them will drive in July.

$0.2 \times 2950 = 590$ M1

$$S_2 = \begin{bmatrix} 2372.5 \\ 1720.0 \\ 2627.5 \\ 3280.0 \end{bmatrix}$$

$$\frac{590}{3280} \times 100 = 17.987804878$$

18% A1

d. The marketing campaign was successful, as the long-term steady state shows a clear increase in the numbers of people walking and cycling to work. A1

cycling – 1000 to 3333

walking – 3000 to 3333

Note: Figures must be used to show that the campaign was successful.

e.

$$t \times \begin{bmatrix} 2000 \\ 500 \\ 500 \\ 1000 \end{bmatrix} + \begin{bmatrix} 100 \\ 25 \\ 25 \\ 50 \end{bmatrix} = \begin{bmatrix} 1475 \\ 625 \\ 850 \\ 1250 \end{bmatrix}$$

$$t \times \begin{bmatrix} 1475 \\ 625 \\ 850 \\ 1250 \end{bmatrix} + \begin{bmatrix} 100 \\ 25 \\ 25 \\ 50 \end{bmatrix} = \begin{bmatrix} 1215 \\ 695 \\ 110 \\ 1390 \end{bmatrix}$$

695 A1

Module 2 – Networks and decision mathematics

Question 1 (3 marks)

a. 50 minutes A1

b. *For example:*

A–B–E–D–F–C–A A1

Note: Any Hamiltonian cycle that starts and ends at A is a viable answer.

c. $v = 6$

$f = 6$

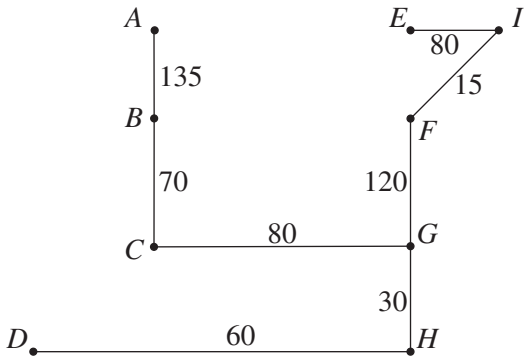
$e = 10$

$\therefore 6 + 6 = 10 + 2$ A1

Question 2 (2 marks)

a. minimum spanning tree A1

b.



\$610 A1

Question 3 (2 marks)

a. 15 + 10 + 6 + 12 = 43 L A1

b. 20 + 6 + 5 = 31 L A1

Question 4 (5 marks)

a. *G*: *C* and *F*

K: *B*, *E*, *G* and *H* A1

Note: All preceding activities must be correct for full marks.

b. 19 days A1

c. *A–B–I–J–M* A1

d. *D* and *F* A1

e. Reduce I by 2 days: $800 \times 2 = \$1600$

Reduce K by 1 day: $\$600$

$1600 + 600 = \$2200$ A1

Module 3 – Geometry and measurement

Question 1 (4 marks)

a. area of rectangle + area of semicircles = $100 \times 50 + 2(0.5 \times \pi \times 25^2)$

$= 6963.49540849$

$\therefore 6963 \text{ m}^2$ A1

b. Using Pythagoras: $\sqrt{150^2 + 40^2} = 155.241746963$

$\therefore 155.2 \text{ m}$ A1

c. Using trigonometry to find angle: $\tan^{-1}\left(\frac{40}{150}\right) = 14.9314171781$

Add angle to 90 to get bearing.

$\therefore 105°$ A1

d. surface area = curved area of cylinder + curved surfaces of hemisphere

$= 2\pi \times 21 \times 8 + \frac{1}{2} \times 4\pi \times 21^2$

$= 3826.45985207$

$\therefore 3826 \text{ m}^2$ A1

Question 2 (4 marks)

a. Using the formula: $\frac{1}{3} \times \pi \times 20^2 \times 80 = 33\,510.3216383$

$\therefore 33\,510 \text{ m}^3$ A1

b. $1 : 80 = x : 20$

$\text{solve}\left(\frac{1}{80} = \frac{x}{20}, x\right)$

$x = 0.25$

OR

$\frac{20}{80} = 0.25$

$\therefore 0.25 \text{ m}$ A1

c. arc length = 8825

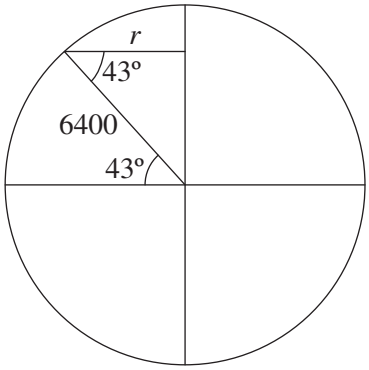
Find latitude using distance along a meridian.

$$\text{solve}\left(\frac{\pi \times 6400 \times x}{180} = 8825, x\right)$$

$x = 79.0055084692$

latitude = $79.0055084692 - 36$

≈ 43 M1



$r = 6400 \times \cos(43)$

$= 4680.24403383$

$\therefore 4700 \text{ km}$ A1

Question 3 (4 marks)

a. major sector = 290°

$$\frac{290}{360} \times \pi \times 8^2 = 161.966554585$$

$\therefore 162 \text{ m}^2$ A1

b. Use $\frac{1}{2}ab\sin(c)$.

$$\frac{1}{2} \times 40 \times 50 \times \sin(63) = 891.006524188$$

$\therefore 891 \text{ m}^2$ A1

c. Use cosine formula with correct substitution. M1

$$\sqrt{60^2 + 40^2 - 2 \times 60 \times 40 \times \cos(63)} = 54.9622197628$$

$\therefore 55 \text{ m}$ A1

Module 4 – Graphs and relations

Question 1 (2 marks)

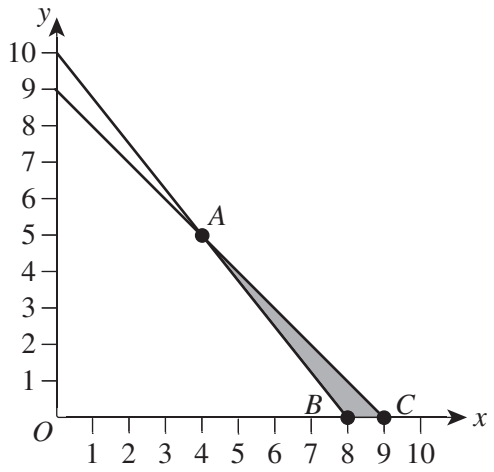
a. 6 pm is the first time the clearance is over 4.5 m. A1

b. Between 3 pm and 6 pm the clearance is less than 4.5 m, so the time is 3 hours. A1

Question 2 (6 marks)

a. There are x number of buses with 25 seats and y buses with 20 seats. The total number of seats must be at least 200. A1

b.



labels three correct points A1
shades correct area A1

c. $Z = 600x + 500y$ A1

d. The important points are (4, 5), (8, 0) and (9, 0). A1

Point	Cost
A (4, 5)	$600 \times 4 + 500 \times 5 = \$4900$
B (8, 0)	$600 \times 8 + 500 \times 0 = \$4800$
C (9, 0)	$600 \times 9 + 500 \times 0 = \$5400$

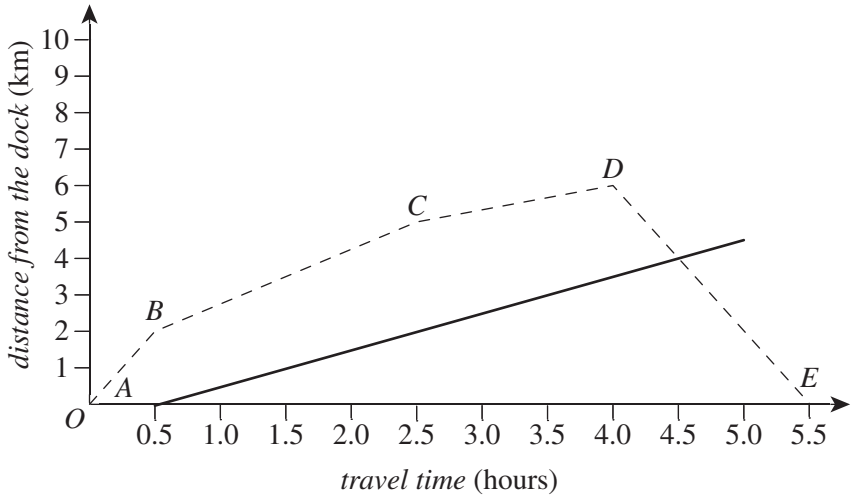
The cheapest option is to use eight 25-seater buses. A1

Question 3 (4 marks)

a. At point D Gazza's boat begins getting closer to the starting point. A1

b. In 2 hours a distance of 3 km was travelled. The average speed is $\frac{3}{2} = 1.5$ km h^{-1}. A1

c.



A1

d. 4 hours. The boats will meet 4.5 hours after Gazza sailed, which is 4 hours after the second boat sails. A1

