

# **FURTHER MATHS**

## **PRACTICE EXAM 1 SOLUTIONS 2019**

## Section A – Core

Question	Answer	Notes
1	B	The percentage can be calculated as the number of men (with a hand length under 7.2) divided by the total number of men multiplied by 100. $\frac{1 + 2 + 3}{18} \times 100 = 33\%$
2	D	The middle number for the data set lies between 7.2cm and 7.5 cm.
3	B	Using the univariate statistics function on a spreadsheet on a CAS calculator will provide the right answer.
4	D	$\frac{95}{100} \times 435 = 207 \text{ mice}$
5	E	Solve for $x$ , $-2.2 = \frac{x - 34}{2}$
6	A	A standardised score of 2 means that the mouse was 2 standard deviations above the mean. Because of this, only 2.5% of mice weigh more.
7	D	
8	C	
9	C	The graph shows that of the students who spent less than 30 minutes on social media, more received a study score of less than 25 compared to the students who spent more than 90 minutes on social media.
10	D	Both variables <i>Further Maths Study Scores (less than 25, 25-40, above 40)</i> and <i>amount of time spent on social media (less than 30 minutes, 30 minutes – 90 minutes, more than 90 minutes)</i> are variables which can be ordered from lowest to highest or vice versa.
11	C	This is because the scatterplot has a positive gradient. Hence, after square rooting the coefficient of determination, the Pearson correlation coefficient must be positive not negative.
12	B	Using the equation provided in the stem of the question: $\text{ Toenail length} = 0.9677 \times \text{ fingernail length} + 0.2104$ We can calculate the predicted toenail length: $\text{ Toenail length} = 0.9677 \times 1.5 + 0.2104 = 1.66195$ The formula needed to calculate a residual is: $\text{ Residual Value} = \text{ Actual Value} - \text{ Predicted Value}$ Hence: $\text{ Residual Value} = 1.6\text{cm} - 1.66195 = - 0.06195$
13	D	
14	C	
15	B	
16	D	

17	B	
18	C	
19	E	
20	B	A reducing balance loan will give the shape of the graph provided. There is a noticeable change in interest rate after 2 months, as seen by the gradient of the graph provided.
21	A	$3 = 25 - 350 \times \text{Depreciation when worn}$ <p><i>Depreciation when worn</i> = \$0.06, however, this is over three years, so we need to divide by three. Hence:</p> $\text{Depreciation when worn} = \$0.02$
22	E	<p>The difference in value of investment between payment 15 and 16 is \$235.70. Hence, the principal addition is \$235.70. To calculate the interest on payment 16, we need to calculate the interest rate:</p> $\frac{13.10}{10258} = 0.001277$ $0.001277 \times 10464.30 = \$13.36$ <p><i>Payment = Principal Addition – Interest</i> <i>Payment = \$235.70 – \$13.36 = \$222</i></p>
23	A	<p>Finance solver is required for this question. N: 36 (3 years, at 12 month compounding periods) I(%): 2.7 PV: -3000 (this is negative as she invests money into the bank) FV: 8870.97 – <i>this is closest to \$8871</i> PpY: 12 CpY: 12</p>
24	B	<p>This question requires the use of finance solver. N: 24 I(%): 3.6 PV: 186920.37 Pmt: -2500 FV: -138737.06 PpY: 12 CpY: 12</p> <p>N: 336 I(%): 4.2 PV: 138737.06 Pmt: -702.87 FV: 0 PpY: 12 Cpy: 12</p>

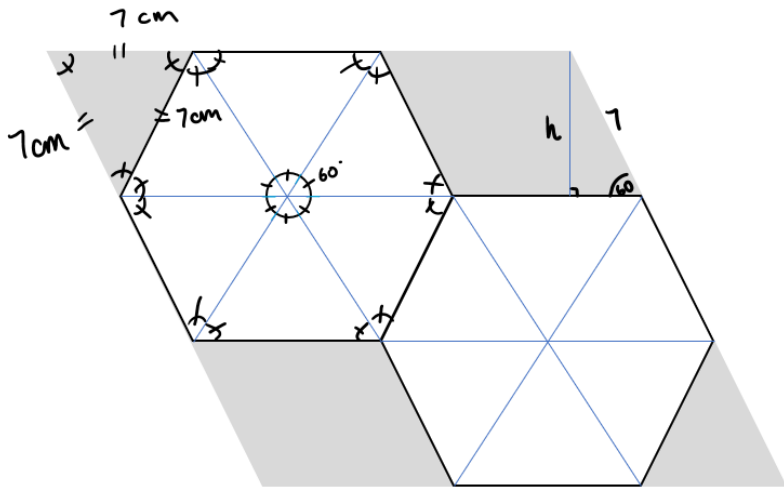
## Section B – Modules Matrices

Question	Answer	Notes
1	A	The determinant for the matrix in option A is $(3 \times 2) - 6 = 0$
2	E	The columns of the first matrix should equal the number of rows in the second matrix.
3	E	A permutation matrix is a matrix with a 1 in every row and 1 in every column.
4	D	
5	A	$7 = 6(1) + 1$ $8 = 6(1) + 2$ $13 = 6(2) + 1$ $14 = 6(2) + 2$
6	A	
7	B	
8	C	$P_2 = \begin{bmatrix} 0.9 & 0.2 & 0.5 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.4 & 0 \end{bmatrix}^2 \times \begin{bmatrix} 79 \\ 74 \\ 65 \end{bmatrix}$

## Networks and decision mathematics

Question	Answer	Notes
1	E	A loop is an edge which starts and ends at the same vertex.
2	A	A planar graph is when no edges overlap in the network.
3	A	Number of vertices – Number of edges + Number of faces = 2
4	A	
5	B	An edge is the line which connects two vertices together. There are a total of 7 in the network shown.
6	C	An adjacency matrix needed to be drawn. There is a direct connection B and C, C and D as well as D and B.
7	A	A vertex are the dots or points on a network. There are 5 of these shown in the network.
8	A	The correct definition of a minimum spanning tree is a spanning tree with the lowest weight.

## Geometry and measurement

Question	Answer	Notes
1	A	As it is a right-angle triangle, you can use sin/cos/tan. As you are given the opposite and adjacent, use tan. $\text{Angle} = \tan^{-1}\left(\frac{30}{60}\right) = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.565^\circ = 26.57^\circ$
2	D	As you are given two sides (a & b) and an included angle (C), you can use $\text{Area} = \frac{1}{2} * ab * \sin(C)$ . $\text{Area} = \frac{1}{2} * 35 * 40 * \sin(33) \approx 381.247 = 381.25 \text{ cm}^2$
3	A	Given 3 sides and an angle to solve for, use the cosine rule: $a^2 = b^2 + c^2 - 2bc * \cos(A)$ $A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$ $A = \cos^{-1}\left(\frac{27^2 + 31^2 - 23^2}{2 * 27 * 31}\right) \approx 46.09^\circ$
4	B	Given that the cities are on the same longitude. The radius is the same as the radius of the Earth, 6400 km. As the distance between the two points measures the length of the arc between the two points, use: The angle between the two points is $36 - 21 = 15^\circ$ as both are South of the equator. $\text{Arc length} = \frac{\pi r \theta}{180} = \frac{\pi * 6400 * 15}{180} \approx 1675.52 \text{ km}$
5	C	 <p>Shaded area is made of 2 parallelograms and two triangles. The sum of the two triangles areas is equivalent to the area of a parallelogram so the sum of the shaded area is equivalent to 3x the area of the parallelogram:                      Height of parallelogram = <math>\sin(60) * 7 = 6.06 \text{ cm}</math>                      Area of parallelogram = <math>b * h = 7 * 6.06 = 42.435 = 42.44 \text{ cm}^2</math>                      Total shaded area = <math>3 * 42.44 = 127.306 = 127.31 \text{ cm}^2</math></p>
6	A	OPTION A IS CORRECT. Use Pythagorean theorem twice to find the maximum length. $\text{Hypotenuse of base} = \sqrt{20^2 + 50^2} = 53.85 \text{ cm}$ $\text{Maximum length} = \sqrt{53.85^2 + 35^2} = 64.22 \text{ cm}$

7	D	<p>Difference in longitude = <math>79 + 11 = 80^\circ</math>  <math>15^\circ</math> longitude = 1 hour                      Time difference = <math>\frac{80}{15} = 6 \text{ hours}</math>                      Florence is more east than Toronto, so Florence's time is ahead of Toronto. Thus, Florence is 6 hours earlier than Toronto.</p>
8	D	<p>OPTION D IS CORRECT.                      Shaded Area = <math>\pi * r^2 * \frac{\theta}{360}</math>  <math>= \pi * 18^2 * \frac{\theta}{360} = 839.75\text{cm}^2</math>                      Thus <math>\theta = \frac{839.75 * 360}{\pi * 18^2} \approx 63^\circ</math>.</p>

## Graphs and relations

Question	Answer	Notes
1	C	Find the part of the graph with the steepest positive gradient. You can quite easily do this by sight.
2	A	It is probably the quickest to do this by trial and error, that is subbing in the x and y coordinates into the equation and checking if it is true. You will find that A is the only one that is correct.
3	B	<p>You want to first find the equation of the line, by first finding the gradient using <math>m = (y_2 - y_1) / (x_2 - x_1)</math>, and then using <math>y - y_1 = m(x - x_1)</math> find the equation of the line.</p> $M = (4 - 3) / (2 - 0) = 1/2$ $y - 3 = 1/2(x - 0) \rightarrow y = 1/2 x + 3$ <p>Now you can solve for the x intercept by making <math>y = 0</math> and solving for x, which gives you <math>x = -6</math>. Hence, point that passes through x-axis: (-6, 0)</p>
4	D	This is basically trial and error, just be careful of the signs (e.g. the > sign for option C makes it wrong, as $2 * 3 = 6 * 1$ but is not $> 6 * 1$ ).
5	B	<p>Make a small list of possible combinations:  <math>X = 1, y = 50</math>  <math>X = 2, y = 100</math>                      From this, form a relationship between y and x : <math>y = 50x</math>                      Now take into account the term "at least" and put this information in your inequality:  <math>y \leq 50x</math> (note that y (number of visitors) is less than or equal to <math>50x</math>, because you need at least 1 lifeguard for every 50 visitors. If y was more than or equal to <math>50x</math>, that would mean you have more than 50 visitors per lifeguard.)</p>
6	E	Using the sliding line method, you will realise that the objective function is parallel to the line segment BC. This means that the maximum occurs at all points between B and C.

7	A	<p>To find the relationship between <math>y</math> and <math>x</math>, you must first find <math>k</math>. Solve for <math>k</math> by substituting the point <math>(1,3)</math> into the equation <math>y = kx^2</math>; <math>3 = k(1)^2</math>, so <math>k = 3</math>. Therefore, the equation is <math>y=3x^2</math></p> <p>Now you want to find the option that also displays the relationship <math>y = 3x^2</math>. You can do this by substituting each of the given coordinates into this equation and the correct option is the one where the equation holds. You could also eliminate some options by observation; E for instance has the same axes as the given graph, and the coordinate given has the same <math>y</math> coordinate as the reference graph but a different <math>x</math> coordinate, so this cannot be correct.</p>
8	B	<p>This is a simultaneous equation question. Set up two simultaneous equations from the information given:</p> <p>Let <math>x =</math> cost per minute and Let <math>y =</math> cost per m travelled</p> $181 = 40x + 550y + 40$ $224 = 60x + 700y + 40$ <p>If you solve these equations, you will get that <math>x = 0.50</math>, and <math>y = 0.22</math></p> <p>So Celina pays: <math>50 \times 0.50 + 600 \times 0.22 + 40 = 197</math> (don't forget to add the fixed cost of \$40!)</p>

# **FURTHER MATHS**

## **PRACTICE EXAM 2 SOLUTIONS 2019**



## Section A – Core

### Question 1 a. i.

Answer:

$$34 - 3 = 31$$

### Question 1 a. ii.

Answer:

22

Notes:

9 is odd, so the middle of 9 is  $(9+1)/2 = 5$ th number  
Count to the fifth number from the smallest number: median = 22

### Question 1 b.

Answer:

Min: 3

$$Q1 : (15+5)/2 = 10$$

Median : 22

$$Q3: (29+32)/2 = 30.5$$

Max: 34

Notes:

I'd suggest if you haven't already from part b, to rewrite the set of numbers in order to make this process easier.

3, 5, 15, 18, 22, 25, 29, 32, 34

### Question 1 c.

Answer:

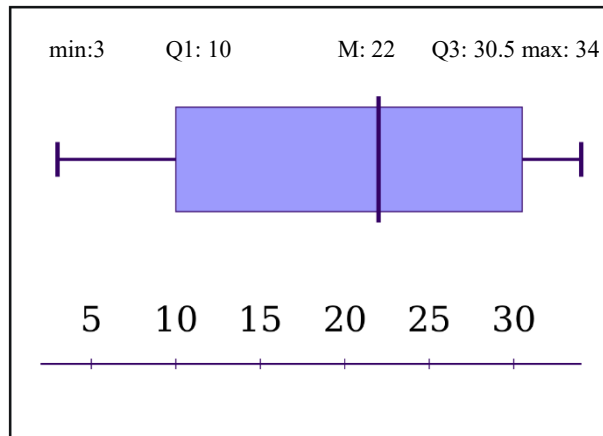
Lower fence:  $10 - (30.5-10) = -10.5$  (no number is below -10.5, so no minimum outliers)

Upper fence:  $30.5 + (30.5-10) = 51$  (no number is above 51, so no maximal outliers)

There are no outliers.

**Question 1 d.**

Answer:



Notes:

Make sure you also include some sort of scale on your axis (like the one shown above).

**Question 1 e. i.**

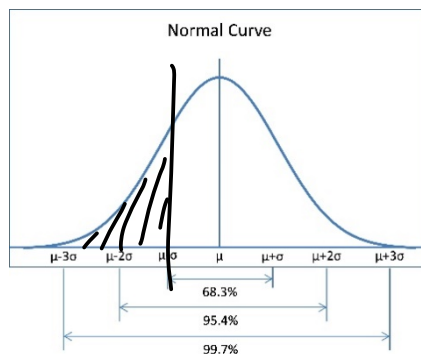
Answer:

16%

Notes:

Find out how many S.D. away from the mean 28 is:  $(35-28)/7 = 1$

28 is 1 S.D. below the mean, so you want to find the percentage that is 1 SD below the mean – I would draw a little normal distribution diagram and mark where 1 SD below the mean is:



You know 68% is within 1 SD of the mean, so the shaded area equals  $50\% - 1/2 \times 68 = 16\%$

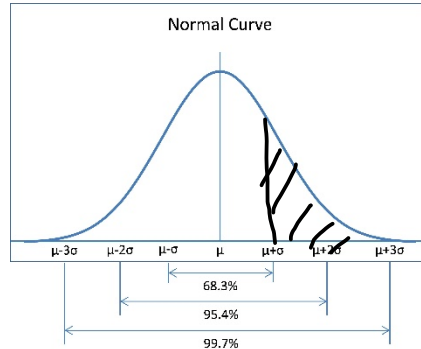
**Question 1 e. i.**

Answer:

16 children

Notes:

Find out how many S.D. 42 is away from the mean:  $42 - 35 = 1$ . I would draw another little diagram



It is helpful to realise that the percentage here is the same as in **part i.** which is why this question is worth 1 mark instead of 2.

Ensure you read what you are being asked for : number of children, not percentage.

So the answer is  $0.16 \times 100 = 16$  children

**Question 1 f.**

Answer:

$$35 - 3 \times 7 = 14$$

**Question 2 a. i.**

Answer:

8 years old (9 children, find middle age – 5th age)

**Question 2 a. ii.**

Answer:

32

**Question 2 b.**

Answer:

The explanatory variable is age.

**Question 2 c.**

Answer:

$$\text{number of lollies} = -3.72 \times 10 + 64.79 = 27.59 \approx 28$$

Notes:

Always take note of what your numbers represent: a child cannot get 0.59 of a lolly so round it off to the nearest whole number

**Question 2 d.**

Answer:

The coefficient of determination suggests that, on average, 94.22% of the variation in the number of lollies received is explained by the variation in the age of children.

Notes:

Here is another question where you have to know exactly what you are working with : given the correlation coefficient – r, you are asked to interpret the coefficient of determination –  $r^2$ . So you have to first find  $r^2 = (-0.970672)^2 = 0.9422$ , then interpret it.

**Question 2 e.**

Answer:

The association is strong, negative and linear.

Notes:

Refer to your bound reference for a table like the one below which tells you what r value gives strong, moderate or weak associations (if you don't already have it in there, put it in! or memorise it, it's actually quite intuitive).

Range	Strength of association
0	No positive association
0 to 0.25	Negligible positive association
0.25 to 0.50	Weak positive association
0.50 to 0.75	Moderate positive association
0.75 to 1	Very strong positive association
1	Perfect positive association

**Question 2 f.**

Answer:

– 1.31

Notes:

2 marks

Remember residual = actual value – predicted value. (If you always mix the order of the two things – you guessed it, put it in bound reference!)

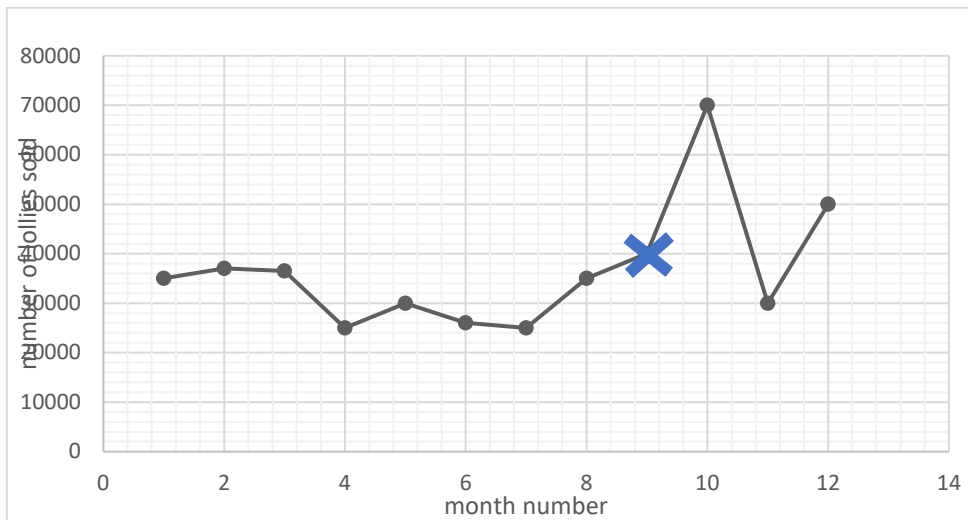
$$\text{Predicted value} = -3.72 \times 9 + 64.79 = 31.31$$

$$\text{Residual} = 30 - 31.31 = -1.31 \text{ (look at the scatterplot to get the actual value)}$$

Notice that you're asked to round to two decimal places instead of the nearest whole number : the residual value is how far a prediction is from the actual number; it is a statistical tool to see how accurate a trend line is, and does not necessarily represent a number of lollies, and hence does not have to be an integer.

### Question 3 a.

Answer:



### Question 3 b.

Answer:

$$\text{Seasonal index} = \frac{36500}{36625} = 0.9966$$

Notes:

You should have the formulas for this question ready somewhere in your bound reference:

$$\text{Seasonal index} = (\text{value for the season}) / (\text{seasonal average})$$

so first find the seasonal average by adding all the values of each month up and dividing the sum by 12. Average = 36,625

### Question 3 c.

Answer:

The value of the sales for month 3 is 99.66% of the seasonal average.

**Question 5 a.**

Answer:

\$15,000

**Question 5 b.**

Answer:

$$V_0 = 15000$$

$$V_1 = 1.03 \times 15000 = \$15,450$$

$$V_2 = 1.03 \times 15450 = \$15,913.50$$

There is \$15913.50 in Phil's account after two years.

Notes:

Make sure you show all working out including the  $V_1$  and  $V_2$  labels.

**Question 5 c.**

Answer:

3% - since n represents number of years, you can infer the annual interest rate from the multiplier of  $V_n$  in the recurrence equation.  $1.03 \times$  is also  $100\% + 3\%$ , hence the annual interest rate is 3%.

**Question 5 d. i.**

Answer:

$$V_n = 1.03^n \times 15000$$

**Question 5 d. ii.**

Answer:

$$1.03^{10} \times 15000 = \$20158.75$$

**Question 5 d. iii.**

Answer:

Use solve for this question – equate the rule to the value of the car and solve for n.

$$1.03^n \times 15000 = 367,000, \text{ solve for } n$$

$$N = 108.168 \approx 109 \text{ years}$$

**Question 6 a.**

Answer:

$$26000 - 20000 = 6000$$

$$\text{Average} = 6000/3 = \$2000$$

**Question 6 b.**

Answer:

$$2000/26000 \times 100\% = 7.69\%$$

**Question 6 c.**

Answer:

$$V_0 = 26000, V_{n+1} = 0.9231 \times V_n$$

**Question 7 a.**

Answer:

Using finance solver:

$$N: 12 \times 10 = 120$$

$$I: 4.5$$

$$PV: -15000$$

$$Pmt: -500$$

$$FV: ?$$

$$PpY/CpY: 12$$

$$\text{Solve for FV : } \$99103.928\dots$$

**Question 7 b.**

Answer:

Using Finance solver:

$$N: 120$$

$$I: 4.5$$

$$PV: -15000$$

$$Pmt: 0$$

$$FV: ?$$

$$PpY/CpY: 12$$

$$\text{Solve for FV: } \$23504.8916\dots$$

**Question 7 c.**

Answer:

You need the value of the investment to increase by another  $367,000 - 99,103.928 = 267,896$  in  $40 - 29 = 11$  years.

Using finance solver:

N:  $11 \times 12 = 132$

I: 3

PV =  $-99,103.928$ ... (remember he has already been investing for 10 years, so PV for the investment must be the value of his investment at the end of 10 years)

Pmt: ?

FV: 367000

PpY/CpY : 12

Solve for pmt: \$1200.57 a month



## Section B – Modules

### Module 1 – Matrices

#### Question 1 a.

Answer:

The order of matrix AB is  $(5 \times 5)$

#### Question 1 b.

Answer:

The order of matrix BA is  $(3 \times 3)$

#### Question 1 c.

Answer:

Matrix B and A cannot be added. To add two matrices together they must be of the same order.

#### Question 2 a.

Answer:

160 bolts packets.

#### Question 2 b.

Answer:

The number represents the number of Nail packets sold in week 2.

#### Question 2 c.

Answer:

The value of  $a$  is \$5.94

#### Question 2 d.

Answer:

Matrix S is a  $3 \times 3$  matrix.

**Question 3 a.**

Answer:

25%

Notes:

The 20% of voters who voted for Boris and would change their vote to Anthony were redistributed amongst the column once Anthony withdrew. 15% of the vote went to Claire (C) whilst the other 5% went to Boris. Hence...

$$5/20 \times 100 = 25\%$$

**Question 3 b.**

Answer:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0.75 & 0.50 & 0.45 \\ 0.25 & 0.50 & 0.55 \end{bmatrix} \times \begin{bmatrix} 5000 \\ 3459 \\ 2160 \end{bmatrix} = \begin{bmatrix} 0 \\ 6451.5 \\ 4167.5 \end{bmatrix}$$

Therefore, the amount of votes Boris is expected to receive in the election in August is 6452 votes.

## Module 1 – Networks and decision mathematics

### Question 1 a.

Answer:

Eulerian trail

Notes:

Remembering from the types of walks, a Eulerian Trail is where each edge is traversed exactly once. A key feature to look out for is if it starts and ends at an odd vertex and there are only two vertices of odd degrees.

### Question 1 b.

Answer:

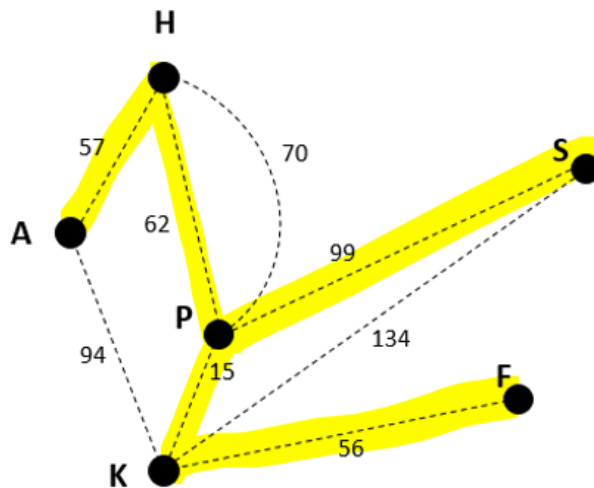
A-K-P-S

Notes:

Solve either visually or using Dijkstra's algorithm.

### Question 1 c.

Answer:



Notes:

Minimum spanning tree connects all vertices with the smallest sum possible. Use Primm's algorithm. Remember not that the minima spanning tree must have only one face.

**Question 2 a.**

Answer:

F and C

Notes:

Activity J's immediate predecessors are the activities which right occur before J. You can check how many there are by counting the number of indegrees (number of edges which are directed at activity J). The dummy line represents that C is a predecessor of both activities J and G.

**Question 2 b. i.**

Answer:

A-E-H-I-L-N

Remember that the critical path is the path which takes the longest time.

**Question 2 b. ii.**

Answer:

Activity G and K

Using backwards scanning, you can find the latest starting times for each activity. Remembers float time equal to: Latest starting time – earliest starting time.

$$225-120 = 105$$

Either activities G or K can be stalled for 105 minutes.

**Question 2 b. iii.**

Answer:

215 minutes

**Question 2 c. i.**

Answer:

275 minutes

Notes:

To decrease the minimum completion time, you need to decrease the time of activities on the critical path. E and H are both on the critical path, thus decreasing them both by 10 each will reduce the time by 20 minutes. Therefore, the minimum completion time is 275 minutes.

**Question 2 c. ii.**

Answer:

\$70

Decrease two activities, thus the cost =  $35 \times 2 = \$70$ .

**Question 3 a.**

Answer:

18

**Question 3 b.**

Answer:

$13+11+8+19+13 = 64$

Notes:

Using the Hungarian Algorithm, the tasks are assigned as so:

- Nathan- Task B
- Callum- Task D
- Rom- Task E
- Rimon- Task C
- Kenny- Task A

To find the total time, sum the time it takes for each staff member to complete their assigned tasks.

## Module 3 – Geometry and measurement

### Question 1 a.

Answer:

53.6° and 126.5°

Notes:

As you're given two sides, a known and unknown angle, you can use the sine rule. Notice that the angle given is a non-included, this means that there is a possibility of there being two angles (the ambiguous case of the sine rule).

Using the sine rule:

$$\sin \theta = \frac{\sin(42)}{9.4} * 11.3 = 0.80$$

$$\therefore \theta = \sin^{-1}(0.80) = 53.6^\circ$$

To check whether this is an ambiguous case of the sine rule, calculate the other possible angle:

$$180 - \theta = 180 - 53.6 = 126.5$$

If the sum of this new angle and the angle originally used is less than 180°, it is a valid angle. If the sum is greater than 180°, then it is not.

$$42 + 126.5 = 168.5^\circ$$

As 168.5 is less than 180, 126.5 is another possible value for  $\theta$ .

### Question 1 b. i.

Answer:

11.0 cm

Notes:

Given two sides and an included angle, you should use the cosine rule.

$$CD = \sqrt{(10.2)^2 + (11.3)^2 - 2 * 10.2 * 11.3 * \cos(61)} = 10.95 = 11.0 \text{ cm}$$

**Question 1 b. ii.**

Answer:

50.55 cm<sup>2</sup>

Notes:

Using Heron's formula:

$$s = \frac{a + b + c}{2} = \frac{10.2 + 11.3 + 11.0}{2} = 16.25$$

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

$$= \sqrt{16.25(16.25 - 10.2)(16.25 - 11.3)(16.25 - 11.0)} = 50.55 \text{ cm}^2$$

**Question 2 a.**

Answer:

14 hours and 45 minutes

Notes:

As Singapore is 6 hours ahead of Luxembourg City. 11:17 a.m. Luxembourg time is 5:17 p.m. on the same day in Singapore time (add 6 hours).

The time difference between 2:32 a.m. and 5:17 p.m. is therefore the flight time, 14 hours and 45 minutes.

**Question 2 b. i.**

Answer:

4:38 p.m. the next day

**Question 2 b. ii.**

Answer:

30°N

Notes:

The angle between Chengdu and Singapore is the difference between the latitudes:  $31 - 1 = 30^\circ N$

**Question 2 b. iii.**

Answer:

3351.03 km

Notes:

The distance is found by calculating the length of the arc

$$\text{Distance} = 2 * \pi * 6400 * \left(\frac{30}{360}\right) \approx 3351.03 \text{ km}$$

**Question 3 a.**

Answer:

45°

Notes:

As each section is equal, the angles must be equal for each section.

$$\alpha = \frac{360}{8} = 45^\circ$$

**Question 3 b.**

Answer:

The shaded area can be calculated multiple ways.

One way is to split the watch into the even sections, calculate the shaded area of one section and then multiply it to get the whole watch face. The shaded area of one section can be calculated by subtracting the unshaded area by the area of the sector: Area of sector – Area of unshaded triangle

$$\left(\pi * 1.5^2 * \frac{45}{360}\right) - \left(\frac{1}{2} * 1.5^2 * \sin(45)\right) = 0.0880 \dots \text{cm}^2$$

Multiply by 8 to account for the whole watch face:

$$0.0880 * 8 \approx 0.70 \text{ cm}^2$$

**Question 3 c.**

Answer:

0.04 cm<sup>3</sup>

Notes:

Volume of a prism is calculated by: Area \* Height

The area is the same as calculated earlier, 0.0880... cm<sup>2</sup> per section.

Divide 5 by 10 to convert mm to cm: 5 mm = 0.5 cm

Thus, the amount of gold required for one section of the watch face is equal to:

$$0.5 * 0.0880 = 0.04 \text{ cm}^3$$



## Module 4 – Graphs and relations

### Question 1 a.

Answer:

\$21

### Question 1 b.

Answer:

5

### Question 2 a.

Answer:

Profits =  $15.5 \times 15 - 102 = \$130.50$

### Question 2 b.

Answer:

\$102

Notes:

(Recall that profits = revenue - costs; the only term that is subtracted in the equation is 102, so this is the fixed cost. The cost of buying yarn for to make each scarf is accounted for in the  $15.5 \times$  scarves sold part of the equation)

### Question 2 c. i.

Answer:

Solve  $15.5 \times \text{scarves sold} - 102 = 0$  for scarves sold

scarves sold:  $x = 6.58$  ;  $x$ -intercept: (6.58,0)

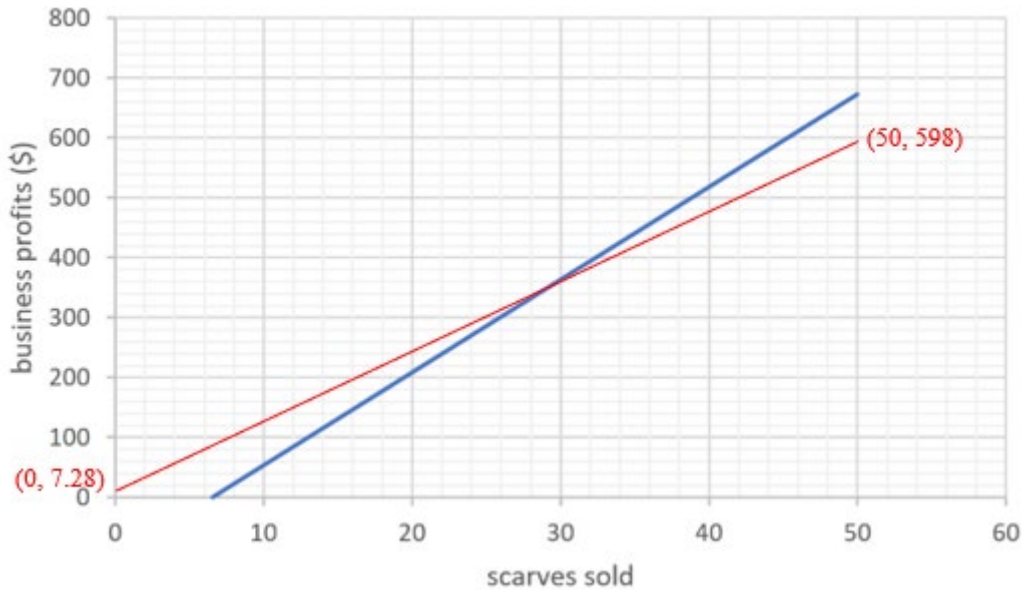
### Question 2 c. ii.

Answer:

Jasmine must sell at least 7 scarves to make a profit OR If Jasmine sells less than 7 scarves, she will make a loss. (remember you can't have 6.58 scarves so you must round the number up. In this case, you round up and not down because selling 6 scarves would still result in a loss).

**Question 3 a.**

Answer:



Notes:

Again, profits = revenue – costs

Revenue =  $23x$  (where  $x$  is the number of scarves sold)

Costs =  $9x + 102$  (cost per scarf + fixed cost)

Profits =  $23x - (9x + 102) = 14x - 102$

To find 2 coordinates to plot the graph, I would find the  $y/x$  intercept, and the point for the largest  $x$  value in the other graph (where  $x = 50$  in the example). If you solved for the  $x$  intercept you would realise that it is negative which doesn't work in the example of tangible scarves, which should signal for you to solve for the  $y$  intercept instead.

**Question 3 b.**

Answer:

At the point of intersection, the number of scarves sold yield the same profits for both months.

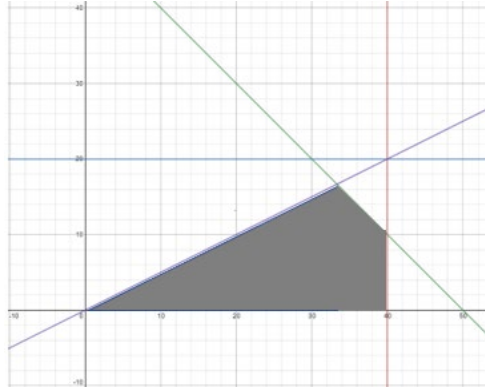
**Question 4 a.**

Answer:

The number of scarves Jasmine produces per month is no more than half the number of beanies she produces per month.

**Question 4 b.**

Answer:



**Question 4 c.**

Answer:

This is best done using the sliding line method.

Gradient of objective function =  $-15/12 = -5/4$ ; draw in a line with this gradient on the graph, use your ruler to slide it to the right to find that the maximum is at the point (40,10)

So max profit =  $15(40) + 12(10) = \$720$

**Question 4 d.**

Answer:

This question was inspired by the flinks and jinks question from the 2016 exam which I found pretty hard (plus only 7% of students got the full two marks!). The trick here was to recognise that 1. This is a simultaneous equation question, and 2. The fact that they told you where the maximum profit lies gives you 2 pieces of information.

$$850 = 36m + 14n - (1)$$

(This is probably the equation most people would be able to produce)

The point (36,14) lies on the line  $x + y = 50$  (look at the graph on the previous page), so the gradient of the objective function must be the same as the gradient of  $x + y = 50$ , which is -1 (rearrange the equation into the form  $y = mx + c$  to get this).

Gradient of objective function =  $\frac{-m}{n}$  (rearrange  $Q = mx + ny$  into the form  $y = mx + c$ )

$$\frac{-m}{n} = -1$$

$m = n$  - (2) (this is the equation that takes a little bit more understanding of linear graphs/ feasible region)

sub (2) into (1):

$$850 = 36m + 14m \text{ (or } 850 = 36n + 14n)$$

$$850 = 50m$$

$$m = n = 850/50 = \$17$$