

The Mathematical Association of Victoria

Trial Examination 2018

FURTHER MATHEMATICS

Written Examination 2

STUDENT NAME: \_\_\_\_\_

Reading time: 15 minutes  
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Section A - Core	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
	8	8	36
Section B - Modules	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

- Question and answer book of 36 pages
- Formula sheet.
- Working space is provided throughout the book.

**Instructions**

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Section A – CORE****Instructions for Section A**

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example,  $\pi$ , surds or fractions.

In ‘Recursion and financial modelling’, all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale

**Data Analysis****Question 1** (9 marks)

The stem plot below shows the age in years, correct to the nearest whole year, at which Australia’s 29 Prime Ministers first took office.

Stem	leaf
3	
3	7
4	0 4
4	5 6 7 7 8 9
5	0 2 2 3 3 3 3
5	5 6 6 6 7 7 7 9 9
6	0 1 3
6	8

Key : 3 | 7 represents 37

- (a) What is the modal age of Australian Prime Ministers shown in the stem plot above?

1 mark

- (b) Complete the following table.

1 mark

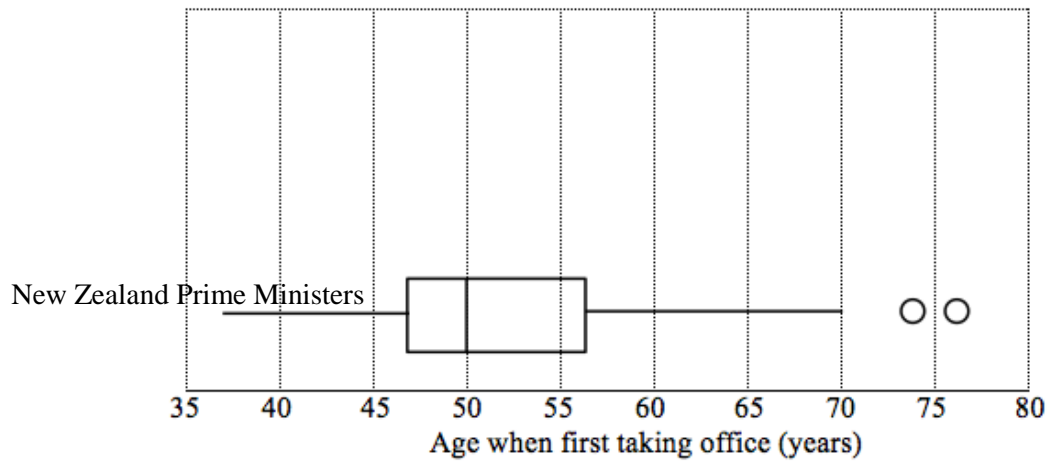
Minimum	Q1	Median	Q3	Maximum
37	47.5		57	68

- (c) Chris Watson was just 37 when he became Australia’s third Prime Minister in 1904. Show the mathematical calculations that explain why his age is NOT an outlier to this data set.

2 marks

**SECTION A – continued  
TURN OVER**

The box plot below shows the age in years, correct to the nearest whole year, at which New Zealand's 40 Prime Ministers first took office.



- (d) Describe the shape of the distribution of Ages for the New Zealand Prime Ministers. 1 mark
- (e) Using the data in the stem plot, construct a boxplot of the data for Australian Prime Ministers **above** the New Zealand boxplot. 1 mark
- (f) The mean age of New Zealand Prime Ministers when first taking office is 51.7 years. Using the data as displayed in the completed boxplot above, explain why, in this case, the median age of first taking office is a better measure of centre of the distribution than the mean. 2 marks

**SECTION A** – continued

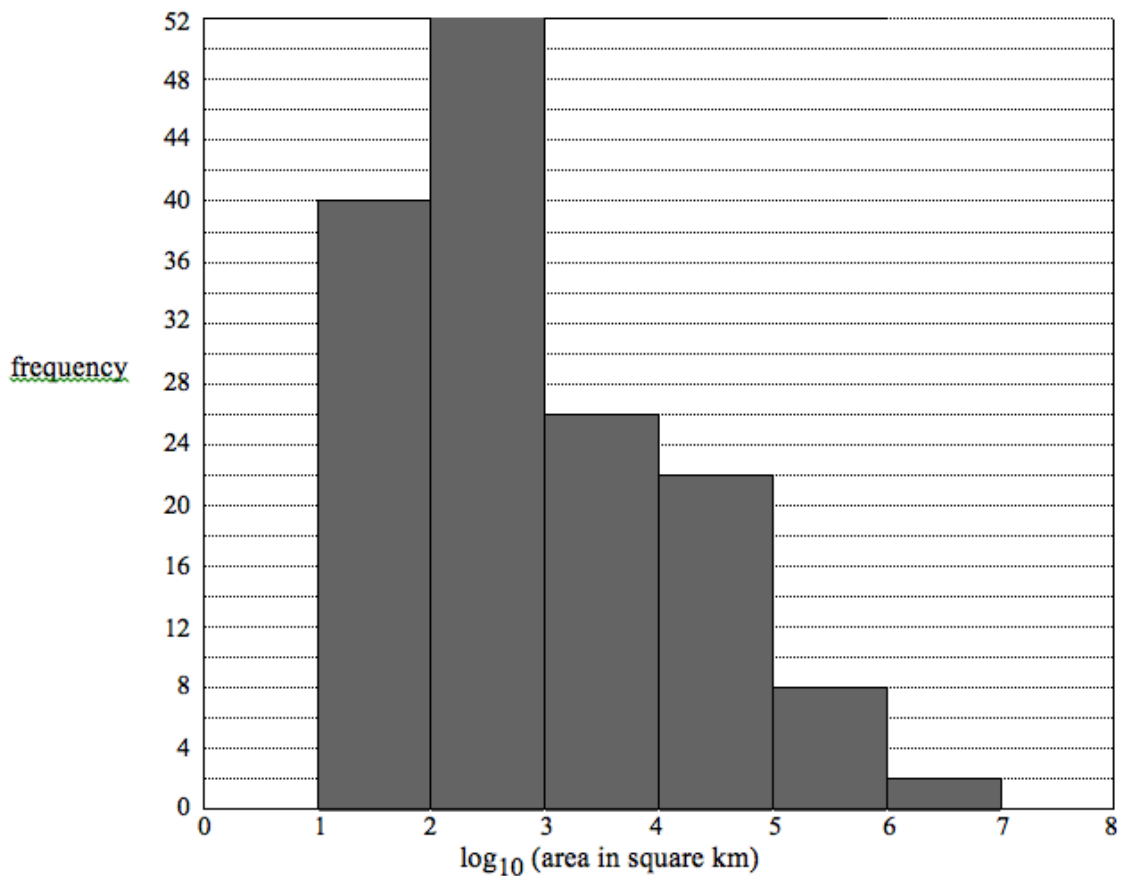
At the start of 2018, the mean of the ages of the 42 ministers in the Australian Government was 48.7 years, correct to one decimal place. The standard deviation of the ministers' ages, correct to one decimal place, was 7.7 years.

- (g) Wyatt Roy is the youngest minister in the government at 25 years old.  
Calculate the  $z$ -score of Wyatt Roy's age correct to two decimal places. 1 mark

**SECTION A – continued**  
**TURN OVER**

**Question 2** (3 marks)

Australian electorates vary in area greatly as they are each based on a similar number of people rather than area. The histogram below shows the distribution of the logarithm of the area of each of the 150 federal electorates.



(a) Complete the following:

The histogram shows that no federal electorate is less than \_\_\_\_\_ square kilometres.

1 mark

(b) Describe the shape of the histogram of  $\log$  (area in square km).

1 mark

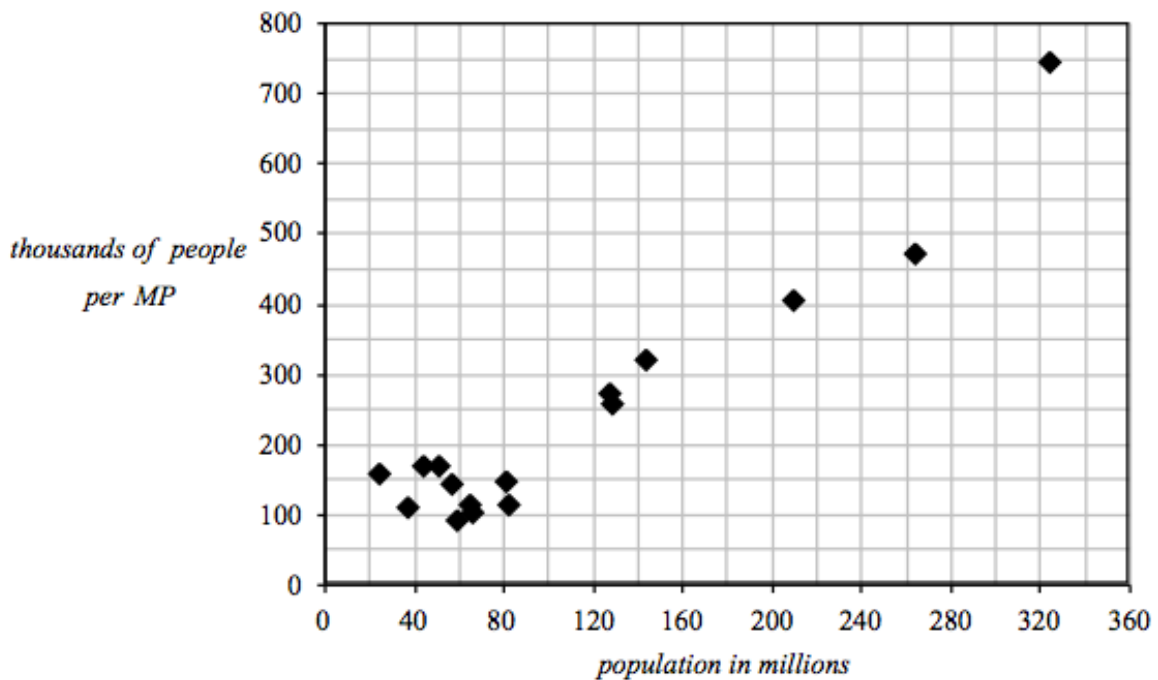
(c) The median size of a federal electorate is between \_\_\_\_\_ and \_\_\_\_\_ square kilometres.

1 mark

**SECTION A – continued**

**Question 3** (5 marks)

The scatterplot below shows the *thousands of people per MP* (member of parliament) versus the *population in millions* for sixteen countries.



The equation of the least square regression line that best fits this data is

$$\text{Thousands of people per MP} = 22.8 + 1.95 \times \text{population}$$

- (a) Draw this regression line on the graph. 1 mark
- (b) Interpret the slope of this least squares regression equation in terms of the variable *people per MP* and *population*. 1 mark
- (c) The correlation coefficient,  $r$ , is equal to 0.925.  
 What percentage of the variation in the number of people per MP is explained by the variation in population?  
 Write your answer as a percentage correct to one decimal place. 1 mark

**SECTION A – continued**  
**TURN OVER**

- (d) Australia has a population of 24 000 000 and the lower house has 150 members, meaning that Australia has 160 000 people per member of parliament.  
If the number of people per member of parliament for Australia was on this regression line, how many members of parliament should Australia have (correct to the nearest integer) ?  
2 marks

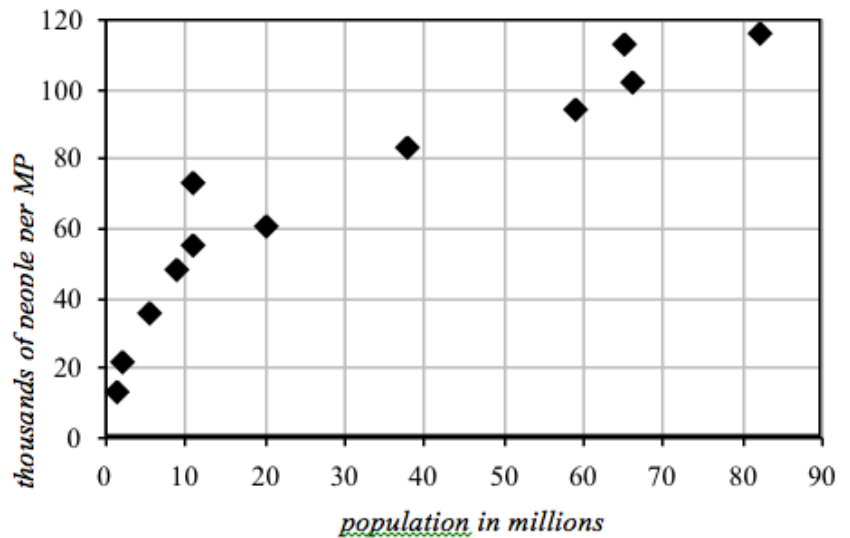
**SECTION A – continued**



**Question 4** (3 marks)

The data in the table below shows a sample of *populations in millions* and the *thousands of people per MP* (member of parliament) for twelve countries of the European Union. A scatterplot of the data is also shown.

<i>population (millions)</i>	<i>thousands of people per MP</i>
8.7	48
11	55
1.3	13
65	113
82	116
59	94
38	83
20	61
5.4	36
2	22
66	102
11	73



For these countries, the relationship between the *population in millions* and the *thousands of people per MP* is non-linear.

A log transformation can be applied to the variable *population in millions* to linearise the data.

- (a) Apply the log transformation to the data and determine the equation of the least squares regression line that allows the *thousands of people per MP* of a country to be predicted from the logarithm of its *population in millions*.

Write the slope and intercept of this regression line in the boxes provided below.

Write your answers, correct to two significant figures.

2 marks

$$\text{Thousands of people per MP} = \boxed{\phantom{000}} + \boxed{\phantom{000}} \times \log_{10}(\text{population in millions})$$

- (b) Use this regression equation to predict the number of people per MP for Hungary with a population of 9.7 million.

Write your answer, correct to the nearest one thousand people.

1 mark

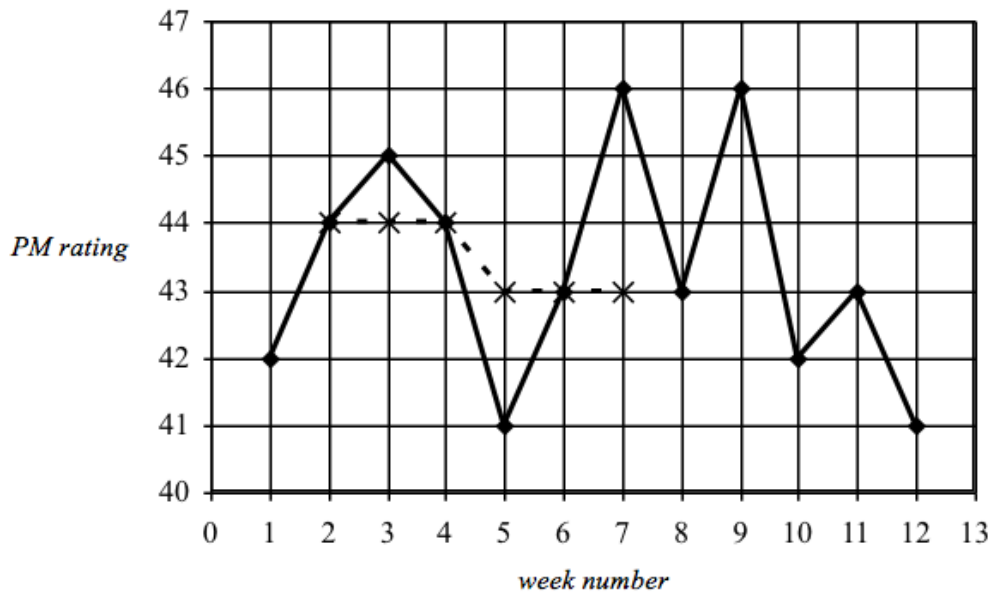
**SECTION A – continued  
TURN OVER**

**Question 5** (4 marks)

The time series plot below shows a prime minister’s approval rating (*PM rating*) plotted against the *week number*.

The rating is calculated as a percentage of the 2000 individuals polled by telephone each week.

The data was collected over a period of twelve consecutive weeks.



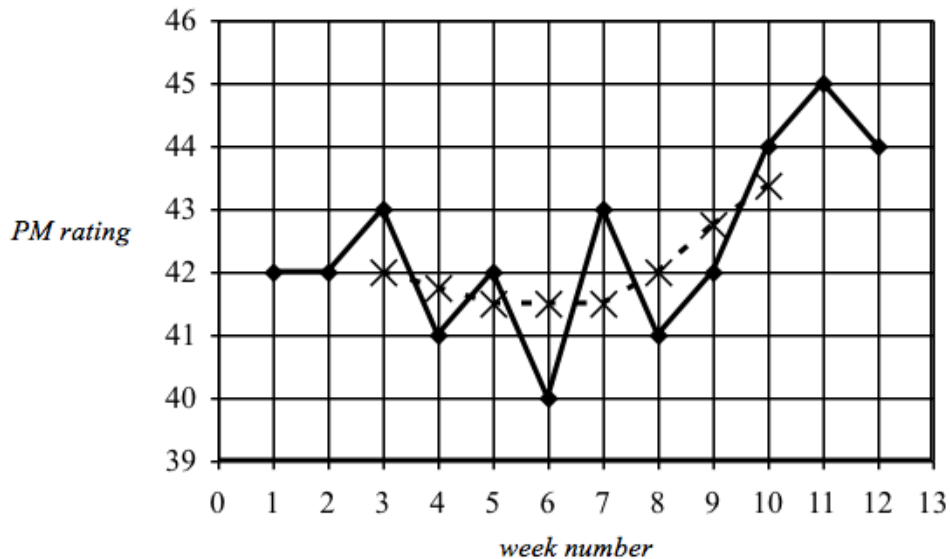
- (a) Three-median smoothing has been used to smooth the time series plot above. The first six smoothed points are shown as crosses (×). Complete the three-median smoothing by marking smoothed values with crosses (×) on the time series plot above. 2 marks

*(Answer on the time series plot above.)*

The table below shows a prime minister's approval rating (*PM rating*) each week for a different twelve-week period.

<i>Week</i>	1	2	3	4	5	6	7	8	9	10	11	12
<i>PM Rating</i>	42	42	43	41	42	40	43	41	42	44	45	44

The data in the table has been used to plot *PM rating* against *week number* in the time series plot below.



- (b) Four-mean smoothing with centring has been used to smooth the time series plot above. The smoothed values are marked with crosses (×). Using the data given in the table, show that the four-mean smoothed PM rating centred on week 8 is 42.0%. 2 marks

**SECTION A – continued  
TURN OVER**

**Recursion and financial modelling****Question 6** (5 marks)

Kevin has been investing his money in a new cryptocurrency called Ripcoin. Kevin can buy one Ripcoin for \$450 with a predicted increase in value according to the recurrence relation

$$R_0 = 450, \quad R_{n+1} = 1.26R_n$$

where  $R_n$  is the value of each Ripcoin after  $n$  months.

- (a) What is the predicted monthly percentage increase in the price of the Ripcoin? 1 mark
- (b) Using the recurrence relation, show the calculations that predict the value of the Ripcoin after two months. 2 marks
- (c) What is the predicted value of Kevin's Ripcoin after nine months? 1 mark

Unfortunately after reaching a high value of \$15 000 each, Kevin's Ripcoin starts to depreciate by 32% per month.

- (d) Write a recurrence relation in terms of  $D_{n+1}$  and  $D_n$  that would give the balance of Kevin's Ripcoin  $n$  months after reaching the value of \$15 000. 1 mark

**SECTION A** – continued

**Question 7** (5 marks)

Kevin has decided to invest his money in a bank instead.

He has \$3000 to invest and he invests this money at 3.9% pa compounding monthly. Every month he will add \$150 to the investment.

- (a) Write a recurrence relation in terms of  $K_{n+1}$  and  $K_n$  that would give the balance of Kevin's account after  $n$  months. 1 mark
- (b) What is the effective rate of interest that Kevin is earning, correct to two decimal places? 1 mark
- (c) How much interest will Kevin have earned in his account after 18 months? 1 mark

Kevin's wife Penny also invested \$3000 in an account at the same time as Kevin started his account, but her account compounds weekly and every week she adds \$35 to her investment. After five years her account has exactly the same amount of money as Kevin's account.

- (d) What is the per annum interest rate in Penny's account? Give your answer correct to two decimal places. 2 marks

**SECTION A – continued**  
**TURN OVER**

**Question 8** (2 marks)

When Kevin has \$15 000 saved he decides to buy a car that will cost him \$40 000. He is offered a loan at 6.5% pa compounding monthly with equal monthly payments over 4 years. Kevin can borrow the full \$40 000 or he can put his \$15 000 towards the car and only borrow \$25 000.

How much extra interest would Kevin pay if he borrows the full \$40 000 for his car?  
Give your answer correct to the nearest dollar.

2 marks

**END OF SECTION A**

**SECTION B – Modules****Instructions for Section B**

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example,  $\pi$ , surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale

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**SECTION B – continued**  
**TURN OVER**

**Module 1 – Matrices****Question 1** (6 marks)

DaVinci's Delicatessen sells three types of chicken pieces – breast fillet ( $B$ ), drumsticks ( $D$ ) and thigh fillet ( $T$ ). The price per kilogram that they charge for each type of chicken piece is given in matrix  $C$  below.

$$C = \begin{bmatrix} 12 \\ 9 \\ 11 \end{bmatrix} \begin{matrix} B \\ D \\ T \end{matrix}$$

- (a) Write down the order of matrix  $C$ . 1 mark
- (b) Genna buys 2 kilograms of breast fillet, 3 kilograms of drumsticks and 4 kilograms of thigh fillets. Write these values in a row matrix,  $G$ . 1 mark
- (c) Use matrix multiplication to show that the total cost of Genna's purchases should be \$95.00. 1 mark
- (d) Marina bought whole kilogram quantities of two types of chicken pieces and paid a total of \$40.00.  
Using matrix  $M$  to represent the quantities purchased by Marina, write down a detailed matrix calculation (of the form  $M \times C = [40]$ ) that accurately shows this transaction. 1 mark

**SECTION B – Module 1 – continued**



On “Market Thursday” last week, DaVinci’s cut the prices of all of their chicken pieces. Three shoppers took advantage of this. Anna bought 3 kg of breast fillets and 2.5 kg thigh fillets for \$48.25, Bella bought 2.5 kg drumsticks and 3 kg thigh fillets for \$44.25, and Clara bought 2 kg breast fillets and 3.5 kg drumsticks for \$44.25.

- (e) Using  $b$ ,  $d$  and  $t$  to represent the cost of breast fillets, drumsticks and thigh fillets respectively, fill in the gaps in the matrix equation below to represent this information. 1 mark

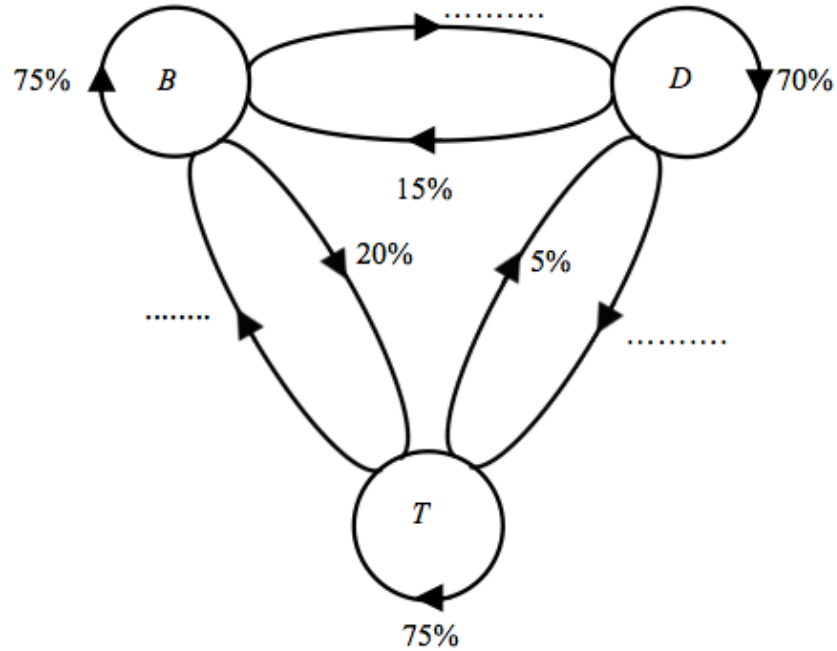
$$\begin{bmatrix} \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \end{bmatrix} \times \begin{bmatrix} b \\ d \\ t \end{bmatrix} = \begin{bmatrix} 48.25 \\ 44.25 \\ 44.25 \end{bmatrix}$$

- (f) What price per kilogram was DaVinci’s charging for drumsticks on “Market Thursday”? 1 mark

**SECTION B – Module 1 – continued**  
**TURN OVER**

**Question 2** (2 marks)

From examining the details of chicken piece purchases by a large number of regular customers over the past month, DaVinci's was able to see how their customers changed their chicken piece preferences each week. These are represented in the partially complete transition diagram below.



- (a) Write the missing values onto the diagram. 1 mark

Last week the sales of each type of chicken piece (in kilograms) were as shown in matrix  $S_0$  below.

$$S_0 = \begin{bmatrix} 275 \\ 265 \\ 230 \end{bmatrix} \begin{matrix} B \\ D \\ T \end{matrix}$$

- (b) Calculate the expected sales of each type of chicken piece for this week, writing your answer correct to the nearest kilogram. 1 mark

**SECTION B – Module 1 – continued**

**Question 3** (2 marks)

DaVinci's chicken suppliers have designated the first two weeks of July as "Chicken Special" weeks and will be supporting DaVinci's to sell their chicken pieces at lower prices. DaVinci's believes that this price support will enable them to sell more chicken pieces than usual.

The matrix relation  $S_{n+1} = TS_n + C$  can describe the situation.

The transition matrix,  $T$ , that will apply to this period, the state matrix for the week prior to the "Chicken Specials",  $S_0$  and the additional matrix  $C$  are given below.  $S_n$  represents the weight of chicken pieces sold, in kilograms, during the  $n$ th "Chicken Special" week.

$$T = \begin{array}{c} \begin{array}{ccc} & \textit{this week} & \\ & B & D & T \\ \begin{bmatrix} 0.80 & 0.10 & 0.15 \\ 0.05 & 0.75 & 0.05 \\ 0.15 & 0.15 & 0.80 \end{bmatrix} & \begin{array}{l} B \\ D \\ T \end{array} & \textit{next week} \end{array} \end{array} \quad S_0 = \begin{bmatrix} 375 \\ 345 \\ 370 \end{bmatrix} \begin{array}{l} B \\ D \\ T \end{array}, \quad C = \begin{bmatrix} 75 \\ 65 \\ 80 \end{bmatrix} \begin{array}{l} B \\ D \\ T \end{array}$$

Find the quantity of chicken thighs sold in the second of the "Chicken Special" weeks, correct to the nearest whole kilogram. 2 marks

**SECTION B – Module 1 – continued**  
**TURN OVER**

**Question 4** (2 marks)

DaVinci's Delicatessen sponsors a bocce competition and the four teams involved have played each other once. Unfortunately, the results keeper has had some problems. He knows the results of the first two games, and has a  $d^2$  matrix of the results when every team has played each of the other teams once, but has lost the details of who won each of the other four games.

Teams : Kitty (K), Leo (L), Max (M) and Nona (N)

$$d^2 = \begin{array}{c} \text{winner} \\ \begin{array}{c} \mathbf{K} \\ \mathbf{L} \\ \mathbf{M} \\ \mathbf{N} \end{array} \end{array} \begin{array}{c} \text{loser} \\ \begin{array}{c} \mathbf{K} \quad \mathbf{L} \quad \mathbf{M} \quad \mathbf{N} \end{array} \end{array} \left[ \begin{array}{cccc} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{array} \right]$$

In the first two games, Max defeated Kitty and Leo defeated Nona.

Write down the results of the other four games.

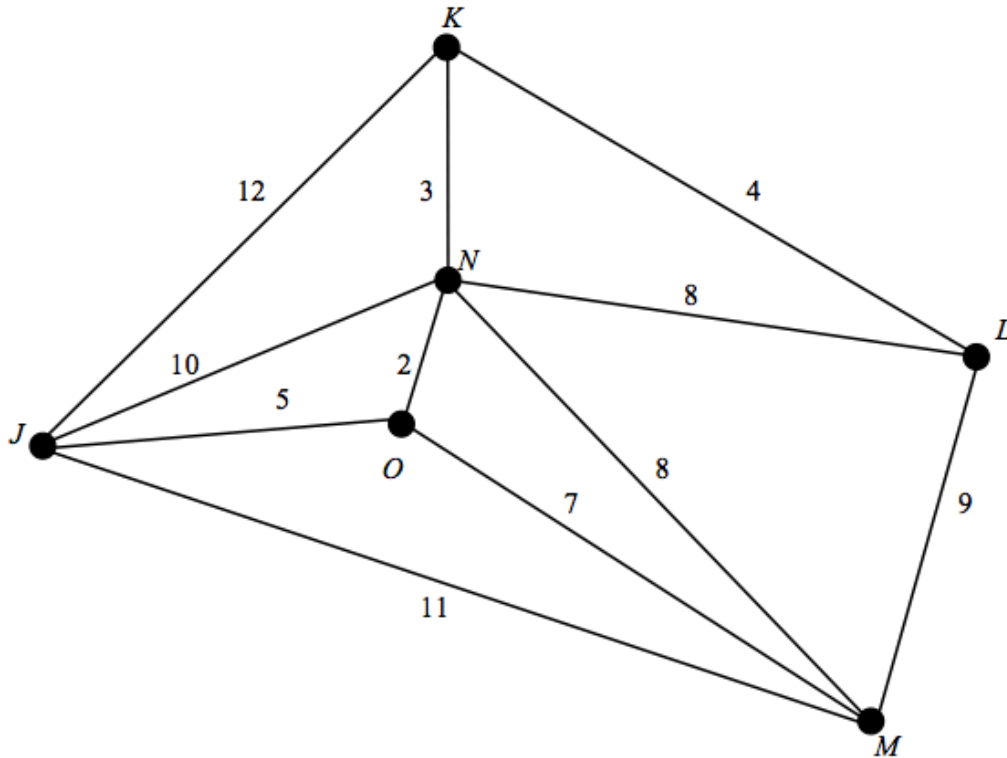
2 marks

**END OF MODULE 1 – SECTION B – continued**

## Module 2 – Networks and decision mathematics

### Question 1 (5 marks)

A small island off the coast of Victoria is being used as an exclusive holiday retreat. There are six small resorts on the island, Jordan ( $J$ ), Kingsley ( $K$ ), Latrobe ( $L$ ), Milsom ( $M$ ), Nelson ( $N$ ) and Oldham ( $O$ ). The resorts are connected by roads shown in the network below. The values on each edge represent the distance in km between each resort.



(a) What is the shortest distance from Jordan to Latrobe? 1 mark

(b) Using Euler's formula  $v + f = e + 2$  to show that the network is planar. 1 mark

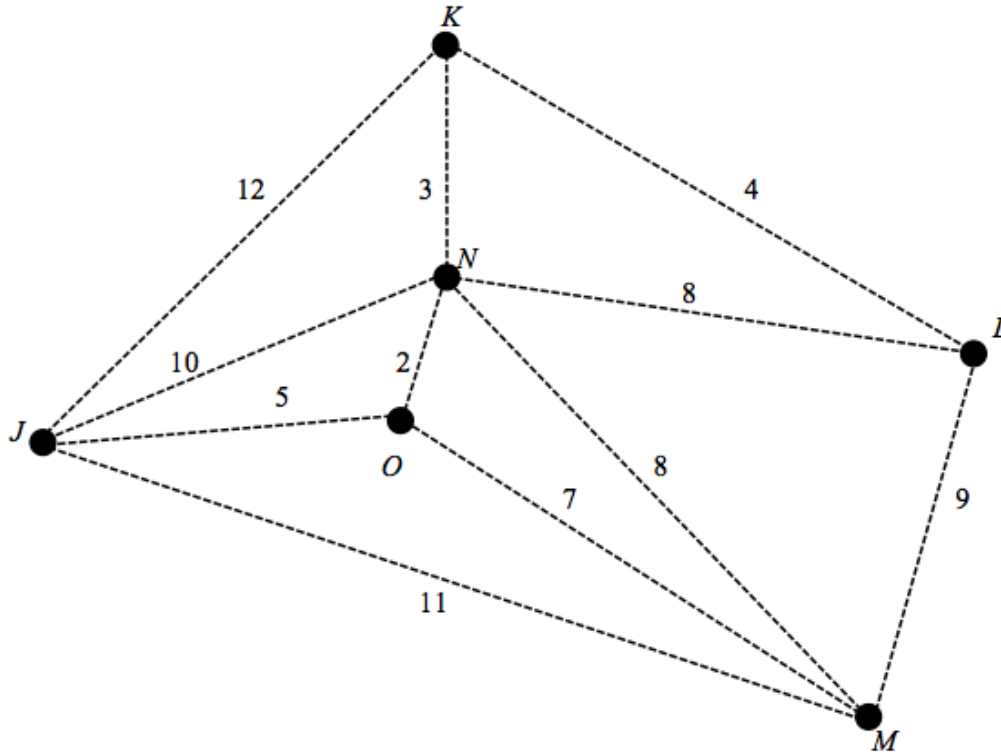
Alice cycles around the island, starting at Jordan, cycling to Nelson, Kingsley, Latrobe, Milsom, Oldham and then back to Jordan.

(c) What is the mathematical name given to the route taken by Alice? 1 mark

**SECTION B – Module 2 – continued**  
**TURN OVER**

The island is remote and so they cannot get the internet other than at a tower at Jordan. The operators of the holiday retreat want to connect the six resorts using cables so that all people can use technology during their stay. To do this they use a minimum spanning tree.

- (d) Add a minimum spanning tree to the copy of the network below. 1 mark



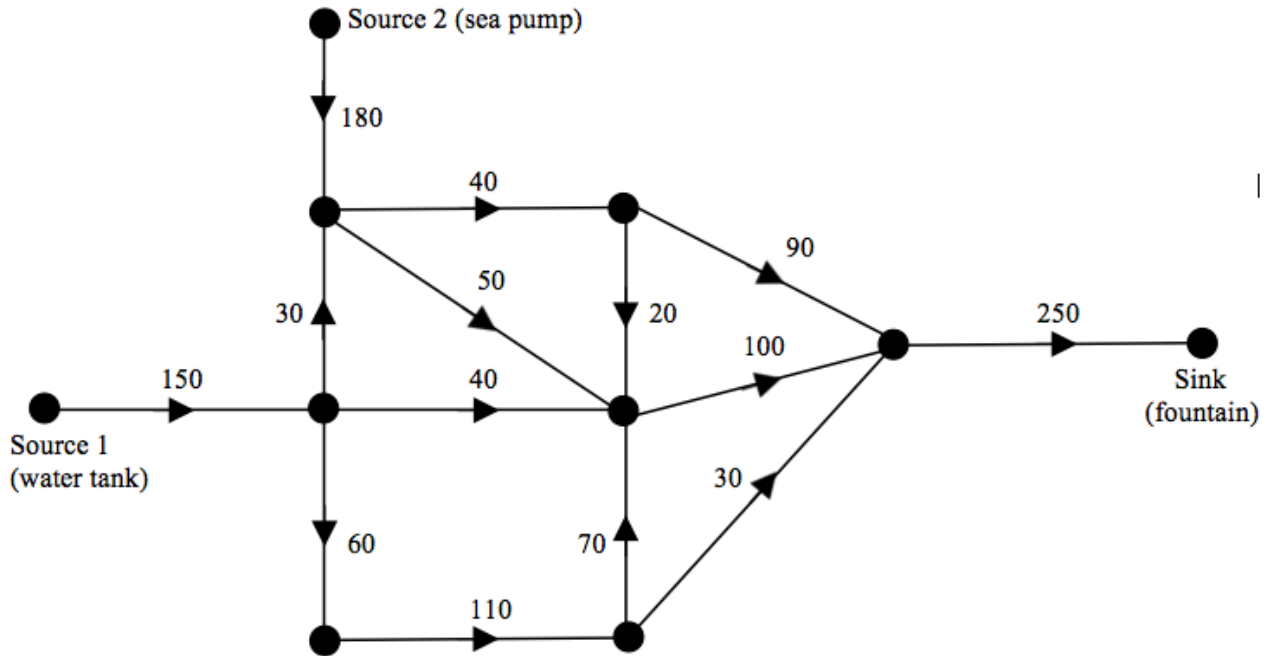
Alice is training for a cycling event. The rules of the event are that the riders have to start at Jordan and travel along every road at least once, returning to Jordan.

- (e) Given that the total length of all roads on the island is 79 kilometres, what is the minimum distance that a rider could travel to complete this race? 1 mark

**SECTION B – Module 2 – continued**

**Question 2** (3 marks)

There is a water fountain feature at Oldham. The fountain receives water from a water tank as well as from a pump that draws water from the sea. The water then passes through a number of pipes to the fountain. The network below shows the flow through these pipes to the fountain. The values on each edge represent the flow capacities of each pipe in litres per minute:

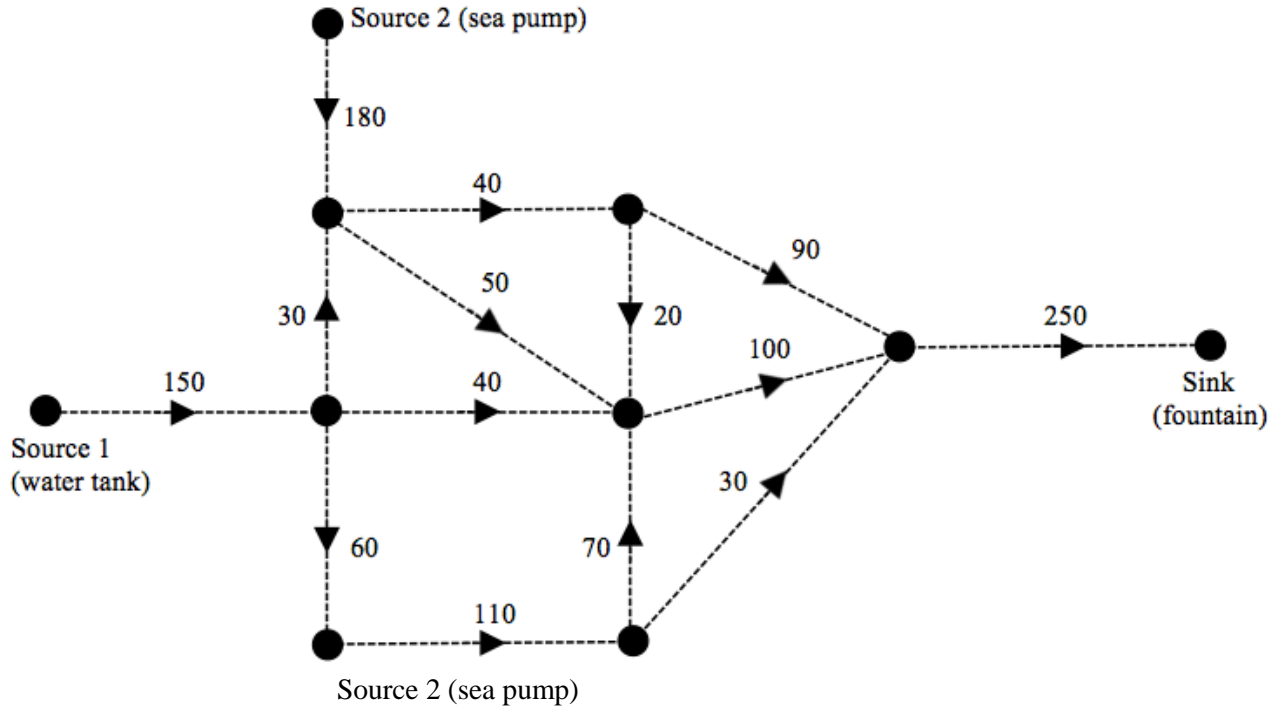


- (a) The maximum amount of water in litres that can reach the fountain per minute given this flow pattern is 170 litres. On the diagram above add the cut that would have a capacity of 170 litres. 1 mark

**SECTION B – Module 2 – continued  
TURN OVER**

The fountain needs a minimum of 200 litres of water per minute if it is to operate properly. One of pipes is to be increased in capacity in order to increase flow.

- (b) On the diagram below indicate, by making the edge solid, which pipe should be replaced in order to increase flow. 1 mark



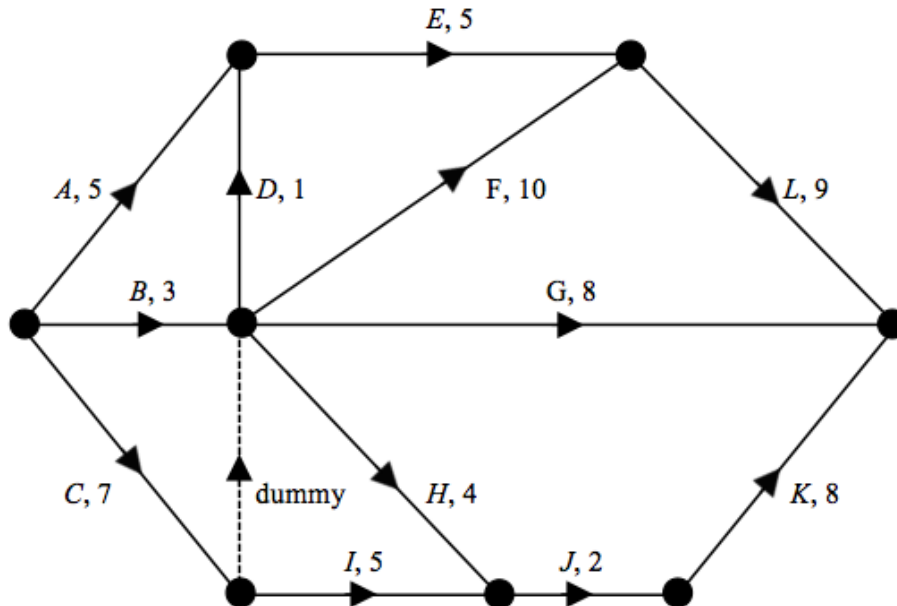
- (c) Given that the pipe can be replaced by a pipe of any capacity, what is the maximum flow in litres per minute achievable at the fountain? 1 mark

**SECTION B – Module 2 – continued**



**Question 3** (4 marks)

The resort workers are planning to upgrade some of the facilities at Nelson. The project manager has produced an activity network to assist the workers in completing the project. The activity network consists of 12 activities (A to L) and it is shown below with times in days for each activity:



- (a) What is the minimum time in which this project could be completed? 1 mark
  
- (b) Activity *D* needs to be delayed. What is the maximum time that activity *D* could be delayed without affecting the minimum completion time? 1 mark
  
- (c) By hiring extra workers activity *F* could be reduced from 10 days to 4 days duration. If this is done, what would the new minimum completion time for this project be? 1 mark

**SECTION B – Module 2 – continued**  
**TURN OVER**

*F* resumes its original duration of 10 days and a different strategy to save time is employed.

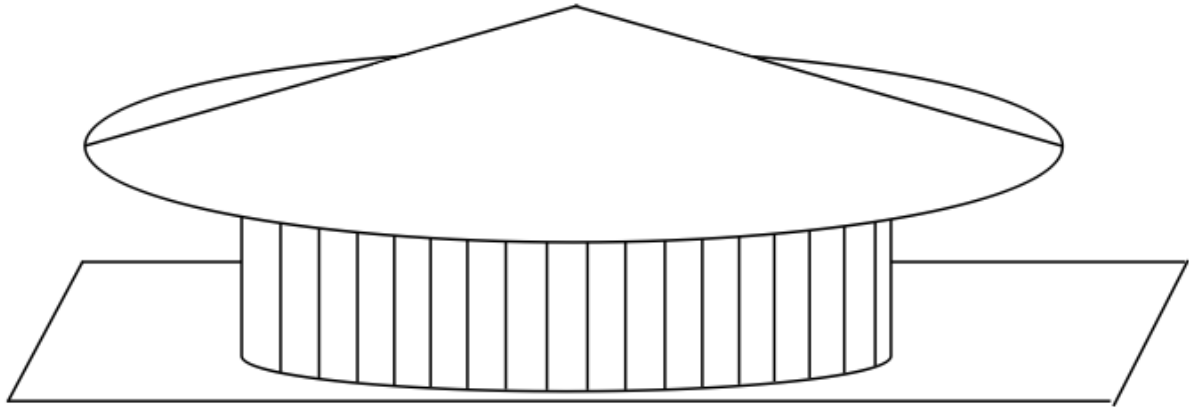
Extra workers can be employed at \$200 per day to reduce the length of any activity other than *A*, *B*, *C*, *D* or *J*. No activity can be reduced to less than 2 days in duration. The project manager wants to complete the project in three weeks (21 days).

- (d) What is the minimum cost that would achieve a completion time of 21 days under these circumstances? 1 mark

**END OF MODULE 2 – SECTION B – continued**  
**TURN OVER**

**Module 3 – Geometry and measurement****Question 1** (4 marks)

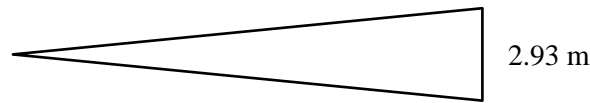
The ground-level entrance lobby to a new underground lecture theatre consists of 45 vertical rectangular glass panels arranged as a glass-walled cylinder with a conical roof overhead. This is shown in the figure below.



- (a) Each glass panel is 2.93 m wide by 5.00 m high.  
Calculate the area of each glass panel, in  $\text{m}^2$  correct to two decimal places. 1 mark
- (b) Each glass panel is 9.5 mm thick.  
The glass that it is made of weighs 3000 kg per  $\text{m}^3$ .  
Calculate the weight of each glass panel, in kilograms correct to one decimal place. 1 mark

**SECTION B – Module 3 – continued**  
**TURN OVER**

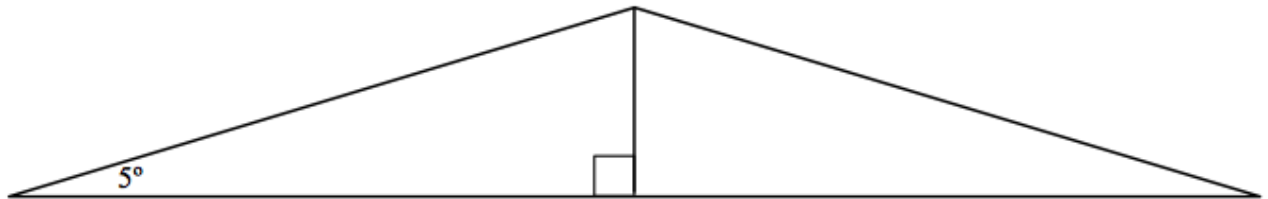
The triangle formed by the sides of the glass panel and the centre of the lobby forms an isosceles triangle when viewed from overhead. This is shown in the diagram below.



- (c) Use this information to determine the diameter of the lobby, in metres correct to one decimal place. 2 marks

**Question 2** (2 marks)

The circular roof was originally designed to be flat, but the local planning authority insisted on an angle of  $5^\circ$  between the roof line and the horizontal to improve water drainage. The diagram below shows a cross-section of the roof with the base being a diameter.

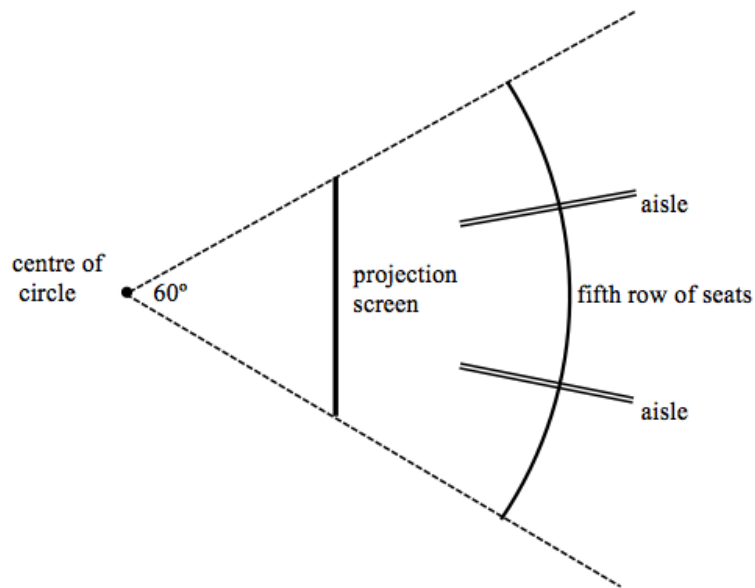


- (a) Show that, if the roof has a diameter of 48 m, the height of the centre, in metres correct to two decimal places, is 2.10 m. 1 mark
- (b) The builder of the roof estimates that if he adds 1% to the area of a circle with a diameter of 48 m, he should have enough roof material to complete the job. The architect who designed the roof claims that  $1820 \text{ m}^2$  of roofing will be required. How many  $\text{m}^2$  excess to the architect's calculation does the builder order? Give your answer correct to the nearest whole number. 1 mark

**SECTION B – Module 3 – continued**  
**TURN OVER**

**Question 3** (3 marks)

The actual lecture theatre has been designed in the shape of a sector of a circle, with an angle of  $60^\circ$ . The centre of the circle is behind the projection screen as shown in the figure below.



- (a) If the projection screen is 5 m wide, how far from the centre of the circle is the centre of the screen, in metres correct to two decimal places? 1 mark
- (b) The fifth row is 13.2 m from the centre of the circle  
 Each of the two aisles in the fifth row is 1.2 m wide, and they have been positioned to allow the maximum number of seats possible in that row.  
 If each seat is 600 mm wide, how many seats could be put into the fifth row? 2 marks

**SECTION B – Module 3 – continued**  
**TURN OVER**

**Question 4** (3 marks)

The underground lecture theatre is located in Perth, Western Australia ( $32^\circ$  S,  $115^\circ$  E) and will be the site of an international conference. Participants are flying directly into Perth from around Australia and overseas.

A large contingent is flying in from Ganzhou, China ( $25^\circ$  N,  $115^\circ$  E).  
Assume that the radius of Earth is 6400 km.

- (a) If their plane flies the great circle route along the  $115^\circ$  E meridian of longitude, what is the distance, in kilometres correct to three significant figures, between these two cities? 1 mark

A senior German academic will be flying to Perth from Hamburg in Germany ( $53^\circ$  N,  $10^\circ$  E) to present at the conference.

Assume that each  $15^\circ$  of longitude corresponds to 1 hour time difference.

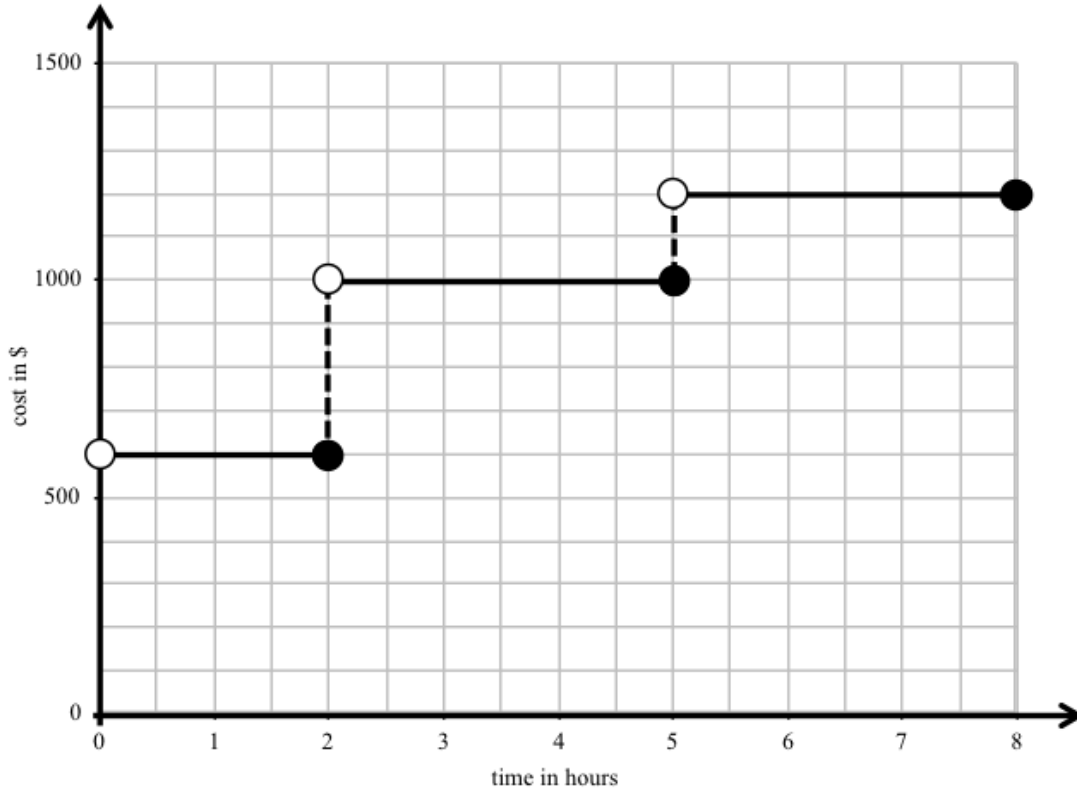
- (b) The academic leaves Hamburg at 10.45 pm local time on a Tuesday evening and, with all connecting flights, is expected to take 24 hours 25 minutes to land at Perth.  
At what local time on what day in Perth will the academic arrive? 2 marks

**END OF MODULE 3 – SECTION B – continued**

### Module 4 – Graphs and relations

#### Question 1 (2 marks)

Spitfire World provide helicopter flights up to a maximum of eight hours along the coast. The cost of each flight is determined by the amount of time taken in hours using the graph shown below:



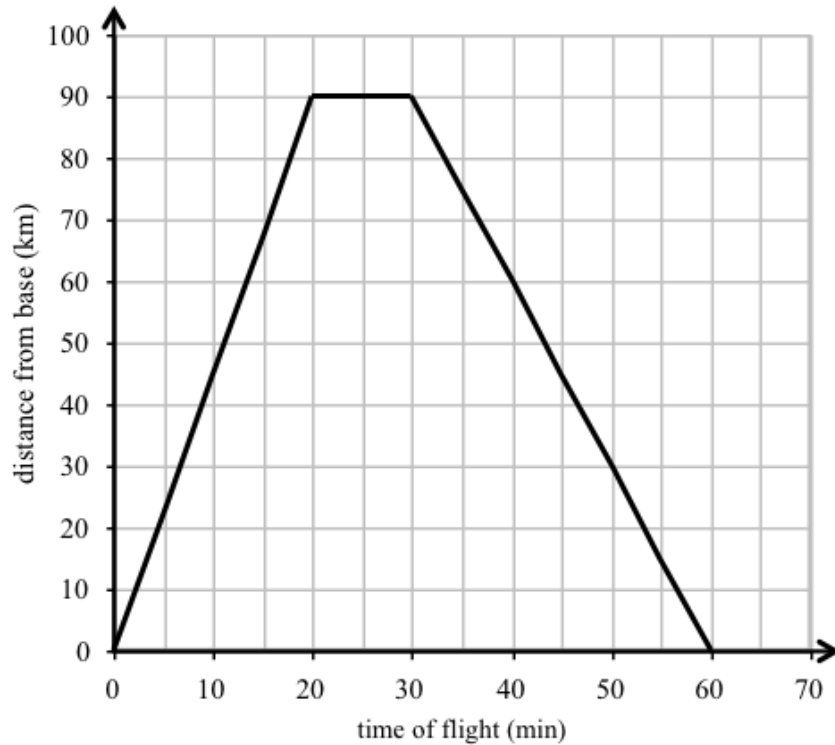
- (a) What is the cheapest average hourly rate for a helicopter flight through Spitfire world? 1 mark
- (b) During one particular week Spitfire world collected \$12 000 for helicopter rides. During this week there were four flights of up to two hours, six flights of more than two hours and up to five hours and a number of flights of 6 hours. How many six hour flights were there? 1 mark

**SECTION B – Module 4 – continued**  
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**Question 2** (2 marks)

Vera hires a helicopter from Spitfire World for a 60-minute period. The helicopter flies in a straight line directly to a viewing platform where it lands before taking off again and flying in a straight line back to the base.

The graph below shows the distance that the helicopter is from the Spitfire World base  $t$  minutes after the start of the hire period.



- (a) How far has the helicopter flown 40 minutes after the start of the 60-minute hire period? 1 mark
- (b) Show that the helicopter flies at 3 km/min on Vera's return journey. 1 mark



**Question 3** (4 marks)

Todd also hires a helicopter from Spitfire World. His flight goes in a straight line from the base to the same viewing platform and immediately leaves again, returning in a straight line. His flight leaves at a different time and flies at a different speed to Vera's flight.

The equation below gives the distance,  $D$  in kilometres, Todd's flight is from the base  $t$  minutes after the start of Vera's flight:

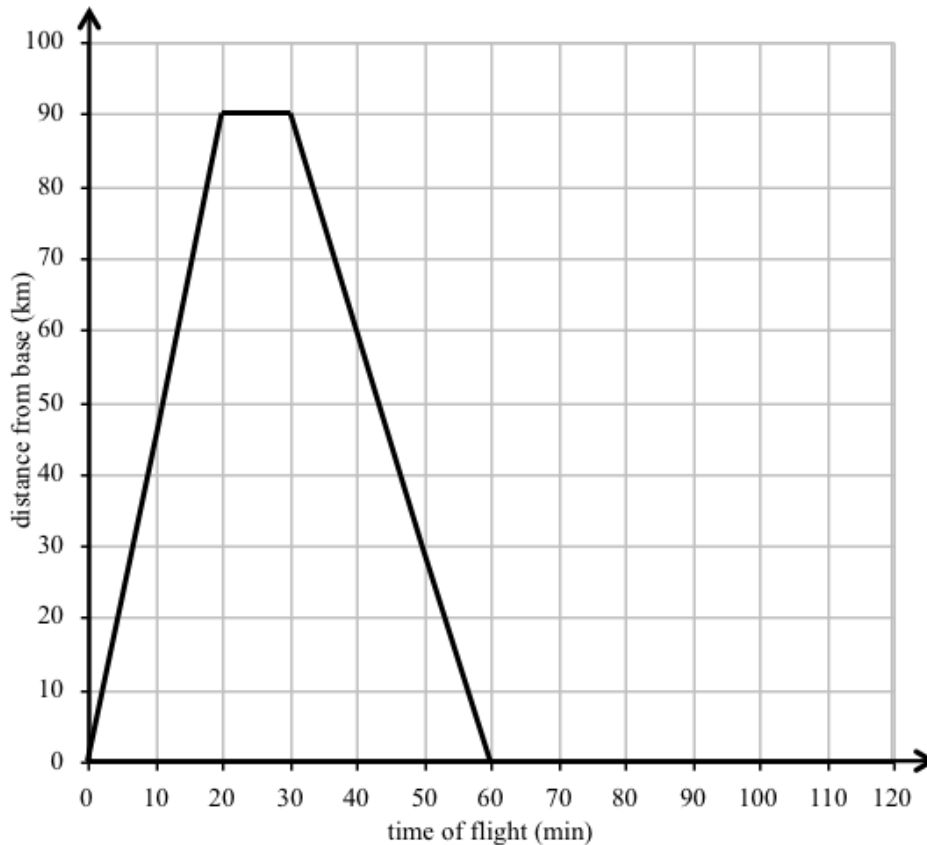
$$D = \begin{cases} 1.5t - 15, & e < t \leq f \\ -2t + 230, & f < t \leq g \end{cases}$$

where  $e$  is the time that Todd's flight leaves the base

$f$  is the time when Todd's flight reaches the viewing platform and

$g$  is the time that Todd's flight returns to the base.

- (a) Add the graph that represents Todd's flight to the graph below that follows the path of Vera's flight. 2 marks



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- (b) The helicopter pilots can communicate directly without going via the base when they are 15 kilometres or less apart. For how many minutes can the two flights communicate directly? Give your answer in minutes correct to two decimal places. 2 marks

**SECTION B – Module 4 – continued**

**Question 4** (4 marks)

Spitfire World employ both flight staff and administration staff.

Let  $x$  be the number of flight staff and  $y$  be the number of administration staff.

There are a number of constraints on the operation of Spitfire World.

On any one day there must be at least 5 staff in total working to run the operation with a minimum of 4 administration staff. The flight staff are paid \$80 per hour and the administration staff are paid \$45 per hour with a maximum hourly wage bill of \$780.

These constraints can be expressed as the following inequalities :

$$\text{Constraint 1 : } x \geq 0$$

$$\text{Constraint 2 : } y \geq 5$$

$$\text{Constraint 3 : } x + y \geq 5$$

$$\text{Constraint 4 : } 80x + 45y \leq 780$$

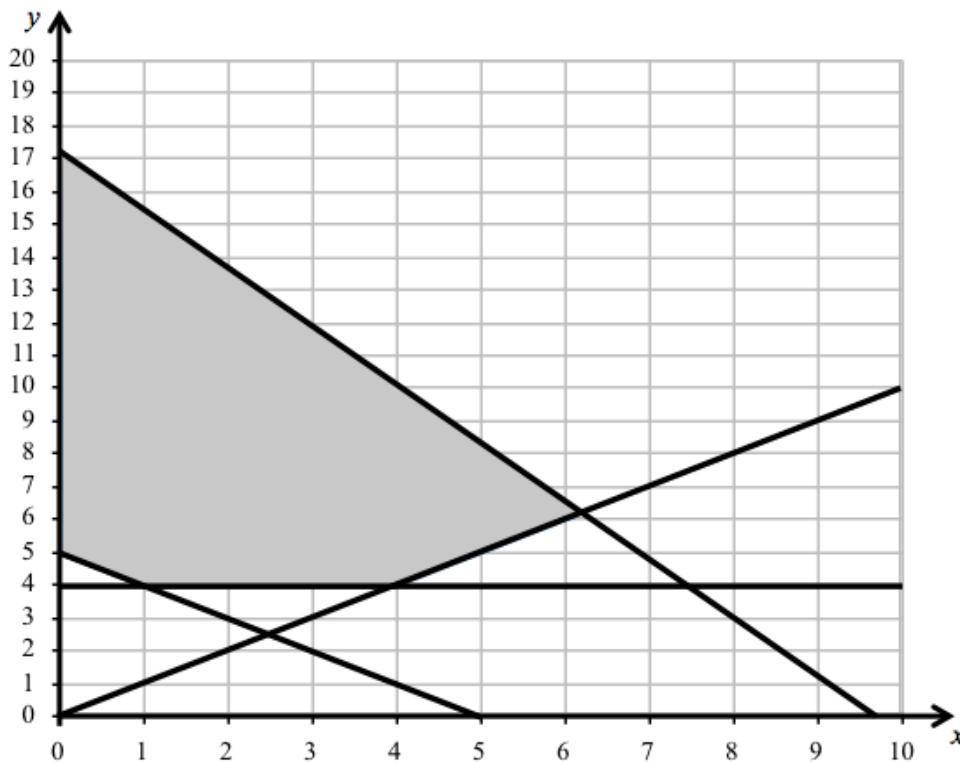
Another constraint inequality applies to the operation of Spitfire World :

$$\text{Constraint 5 : } y \geq x$$

(a) Explain the meaning of Constraint 5 in real terms in this situation.

1 mark

The feasible region for this problem is shaded in the graph shown below.



(b) What is the maximum number of flight staff that could be employed on any one day?

1 mark

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- (c) On weekends staff are paid a bonus per day. The flight staff receive a \$72 bonus each day and the administration staff are paid a bonus of \$40 each day. What is the minimum and maximum bonus that could be paid on any weekend day by Spitfire World given Constraints 1 to 5 inclusive? 2 marks

**END OF QUESTION AND ANSWER BOOK**