The Mathematical Association of Victoria

Trial Examination 2018

FURTHER MATHEMATICS

Trial Written Examination 2 - SOLUTIONS

SECTION A

CORE – Data Analysis

Question 1 (9 marks)

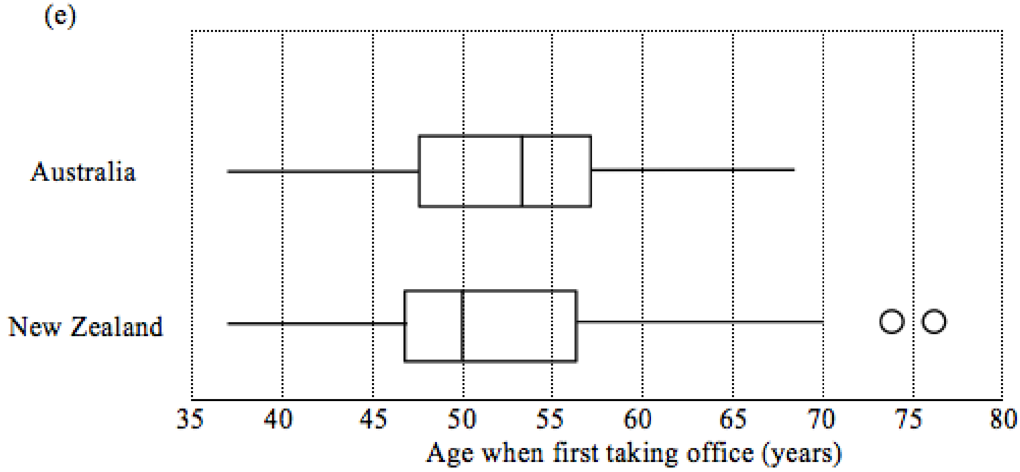
1. **(a)** modal age = 53 years **A1**

(b) median age = 53 years **A1**

(c) IQR = 57.0 – 47.5 = 9.5 **M1**
Lower whisker limit = 47.5 – 1.5 × 9.5 = 33.25
As 37 > 33.25, it CANNOT be an outlier **A1**

(d) Positively skewed with two (2) outliers (74, 76) at the upper (higher) end **A1**

(e) A1



(f) There is a positive skew, and there are two (2) high value outliers to the New Zealand data **M1**
As the value of the mean will be affected by the outliers, it is more appropriate to use the median to indicate the centre of the distribution **A1**

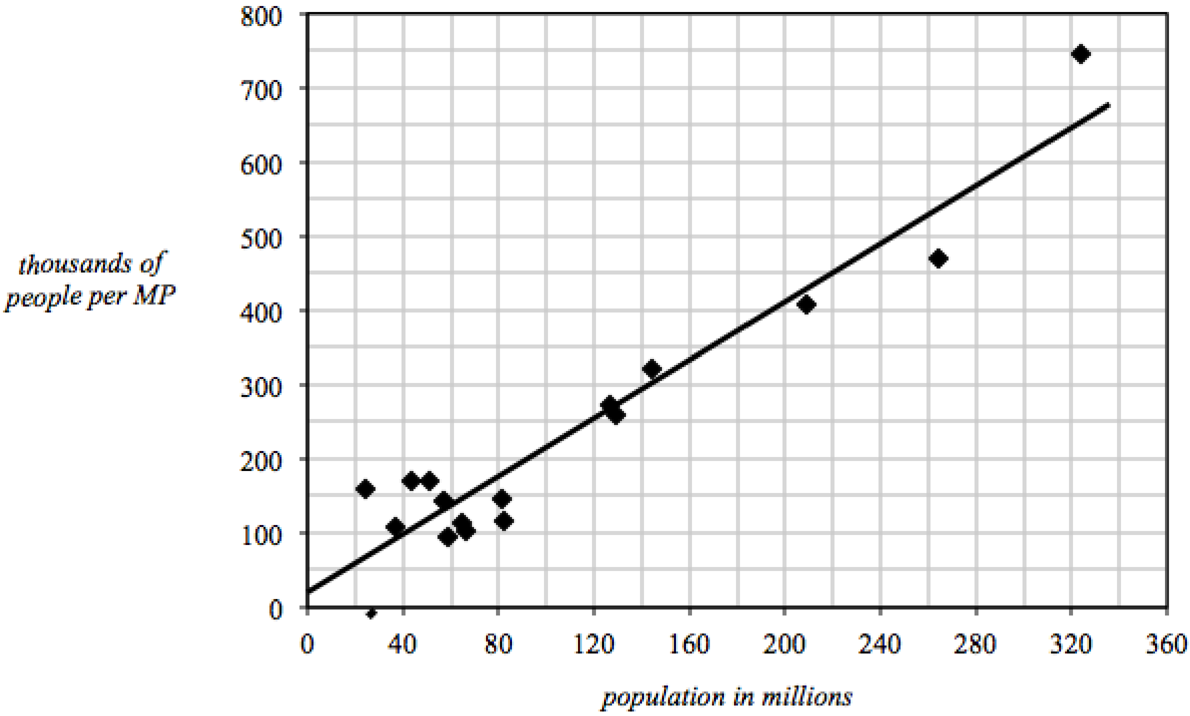
(g) z–score = $\frac{25-48.7}{7.7}$ = – 3.0779… ≈ – 3.08 **A1**

2. **(a)** $10^1 = 10$ **A1**

(b) Positively skewed **A1**

(c) The median for 150 electorates is between 75 and 76 in rank order
The frequency sum of the first two columns is $40 + 52 = 92$
Hence the median lies between 10^2 and 10^3, i.e. between 100 and 1000 **A1**

3. **(a)**



Key points : (0, 22.8) (40, 100) (220, 452) (320, 647) (360, 724.8) **A1**

(b) On average, the number of people per MP increases by 1950 for each 1 million increase in the population. **A1**

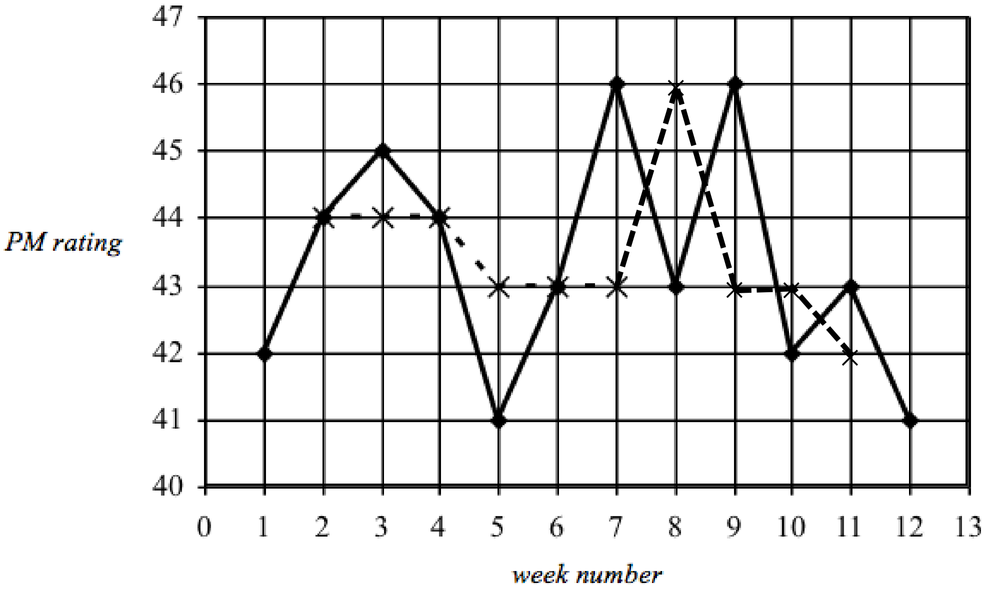
(c) $(0.925)^2 = 0.8556\ldots \approx 85.6\%$ **A1**

(d) For a population of 24 000 000, the number of people per member
$= 22.8 + 1.95 \times 24 = 69.6 = 69600$ **M1**
Hence number of members $= 24\,000\,000 \div 69\,600 = 344.8 \approx 345$ **A1**

4. **(a)** *Thousands of people per MP* = **1.8** + **55** $\times \log_{10}$ (*population in millions*) **A2**

(b) *Thousands of People per MP* $= 1.8 + 55 \times \log_{10}(9.7) = 56.072\ldots$
Number of People per MP $\approx 56\,000$ **A1**

5. **(a)** New points at (8, 46), (9, 43), (10, 43) and (11, 42). Any two **A1**
Other two **A1**



(b) Four-mean (6, 7, 8, 9) = $\frac{(40+43+41+42)}{4}$ = 41.5

Four-mean (7, 8, 9, 10) = $\frac{(43+41+42+44)}{4}$ = 42.5 Either correct **A1**

$\frac{(41.5+42.5)}{2}$ = 42.0 MUST SHOW ALL CALCULATIONS **A1**

CORE – Recursion and financial modeling

6. **(a)** A multiple of 1.26 is a multiple of $\left(1+\frac{26}{100}\right)$ which is a 26% increase. **A1**

(b) $R_0 = 450$

$R_1 = 1.26 \times R_0 = 1.26 \times 450 = \$567$ **A1**

$R_2 = 1.26 \times R_1 = 1.26 \times 576 = \$714.42$ **A1**

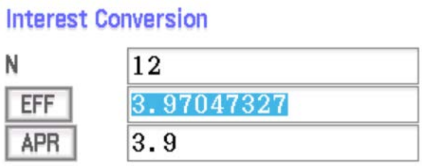
(c) $R_9 = 1.26^9 \times R_0 = \$3602.03$ correct to nearest cent **A1**

(d) $D_0 = 15\,000$, $D_{n+1} = 0.68\ D_n$ **A1**

7. **(a)** 3.9% per annum is $\frac{3.9}{12}$ = 0.325 % per month

$K_0 = 3\,000, \quad K_{n+1} = 1.00325 \times K_n + 150$ **A1**

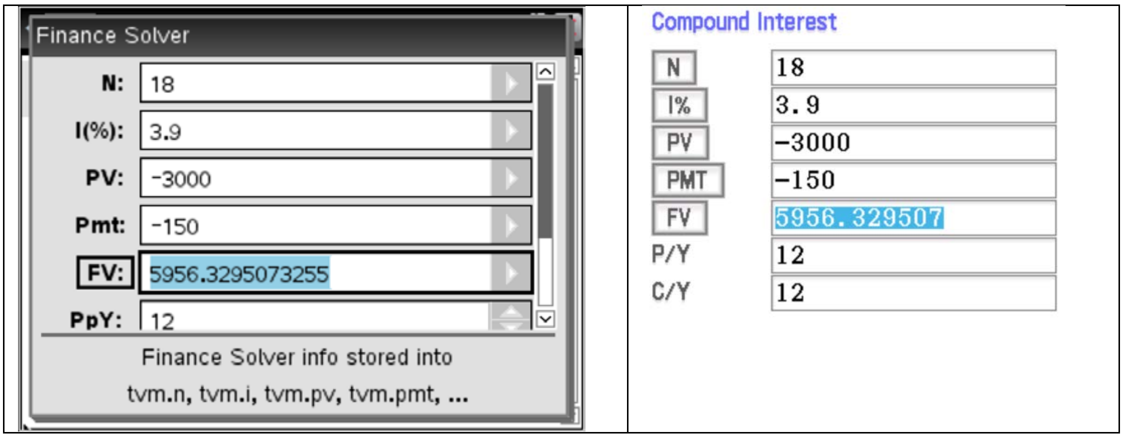
(b) The effective rate of interest can be calculated using the calculator finance menu: 3.97% **A1**



OR $r_{\text{effective}} = \left(\left(1+\frac{r}{100n}\right)^n - 1\right) \times 100\%$

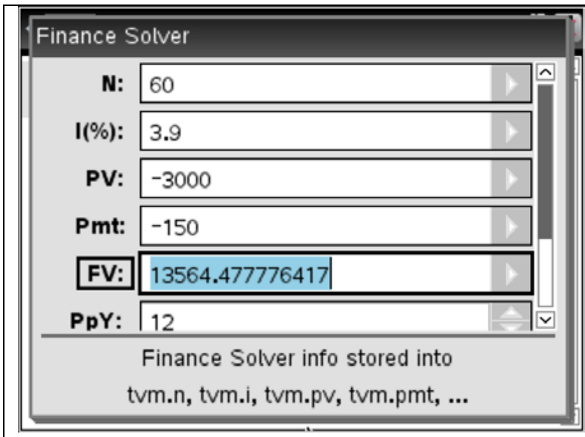
$= \left(\left(1+\frac{3.9}{100 \times 12}\right)^{12} - 1\right) \times 100\% = 3.9704\ldots\% = 3.97\%$

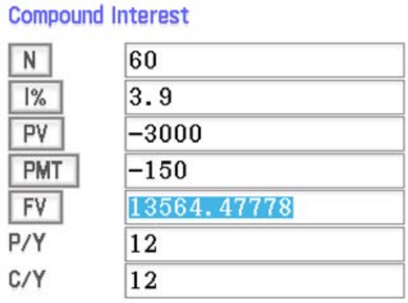
(c) The balance after 18 months can be determined to be \$5956.33 using Finance:



The amount of interest is \$5956.33 – (\$3000 + \$150 × 18) = \$256.33 **A1**

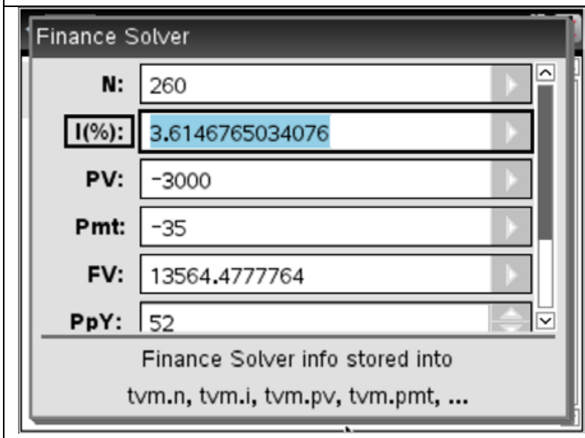
(d) The interest rate in Penny's account is 3.61% correct to two decimal places. The working is shown below.
The balance in Kevin's account after 5 years is determined to be $13564.48. **M1**

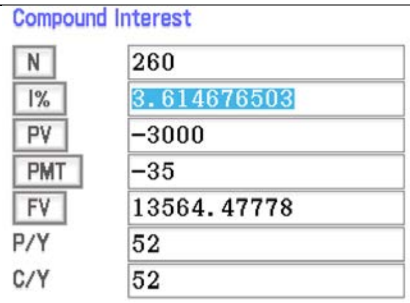




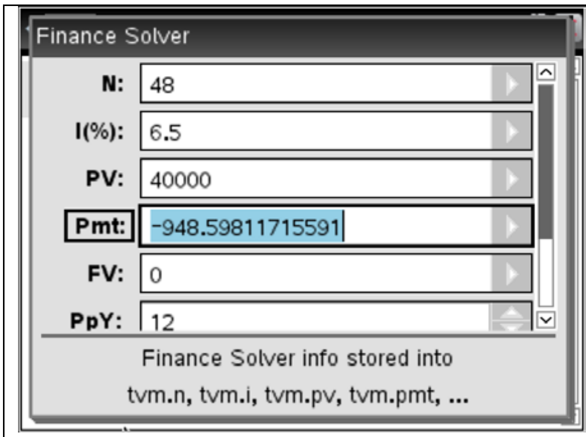
Penny's account after 5 years has a balance of $13564.48. The rate is solved to be 3.61%.

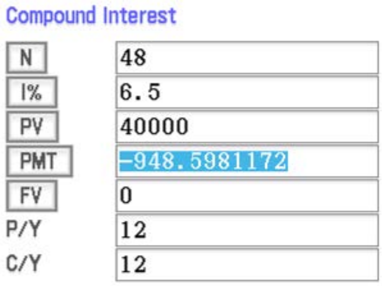
A1



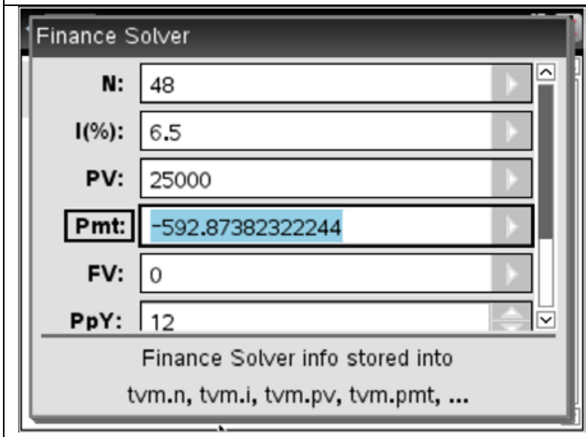


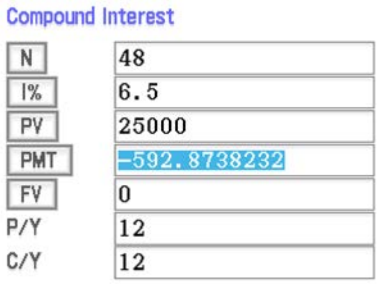
8. Kevin will pay $948.60 \times 48 - (592.87 \times 48 + 15\,000) = \$2075.04 \approx \$2075$ more.
The working in the Finance menu is shown below.
The payments if Kevin borrows the entire \$40000 is \$948.60 per month,
so he pays $948.60 \times 48 = \$45\,532.80$ in total. **M1**





If Kevin borrows \$25000, the payment is \$592.87 per month
so he pays $592.87 \times 48 + 15\,000 = \$43\,457.76$ in total.





$\$45\,532.80 - \$43\,457.76 = \$2075.04 \approx \$2075$ more **A1**

Module 1 – Matrices

1. **(a)** 3×1 **A1**

(b) $G = \begin{matrix} B & D & T \\ \end{matrix}$ $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$ **A1**

(c) $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \times \begin{bmatrix} 12 \\ 9 \\ 11 \end{bmatrix} = \$95.00$ **A1**

(d) Need sum of the multiples of any two of 12, 9 and 11 which add to 40.
$9 + 11 = 20$, so $2 \times 9 + 2 \times 11 = 40$

$\begin{bmatrix} 0 & 2 & 2 \end{bmatrix} \times \begin{bmatrix} 12 \\ 9 \\ 11 \end{bmatrix} = \$40.00$ **A1**

(e) $\begin{bmatrix} 3 & 0 & 2.5 \\ 0 & 2.5 & 3 \\ 2 & 3.5 & 0 \end{bmatrix} \begin{bmatrix} b \\ d \\ t \end{bmatrix} = \begin{bmatrix} 48.25 \\ 44.25 \\ 44.25 \end{bmatrix}$ **A1**

(f) $\begin{bmatrix} b \\ d \\ t \end{bmatrix} = \begin{bmatrix} 3 & 0 & 2.5 \\ 0 & 2.5 & 3 \\ 2 & 3.5 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 48.25 \\ 44.25 \\ 44.25 \end{bmatrix} = \begin{bmatrix} 9.00 \\ 7.50 \\ 8.50 \end{bmatrix}$ **A1**

Hence the drumsticks cost \$7.50 per kilogram.

2. **(a)** $B \rightarrow D$ 5% $D \rightarrow T$ 15% $T \rightarrow B$ 20% **A1**

(b) $\begin{bmatrix} 0.75 & 0.15 & 0.20 \\ 0.05 & 0.70 & 0.05 \\ 0.20 & 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 275 \\ 265 \\ 230 \end{bmatrix} = \begin{bmatrix} 292.00 \\ 210.75 \\ 267.25 \end{bmatrix}$

hence 292 kg breast fillets, 211 kg drumsticks and 267 kg thigh fillets. **A1**
(Correct answers MUST be written correct to nearest kilogram)

3. $$S_1 = \begin{bmatrix} 0.80 & 0.10 & 0.15 \\ 0.05 & 0.75 & 0.05 \\ 0.15 & 0.15 & 0.80 \end{bmatrix} \times \begin{bmatrix} 375 \\ 345 \\ 370 \end{bmatrix} + \begin{bmatrix} 75 \\ 65 \\ 80 \end{bmatrix} = \begin{bmatrix} 465.0 \\ 361.0 \\ 484.0 \end{bmatrix}$$ **M1**

$$S_2 = \begin{bmatrix} 0.80 & 0.10 & 0.15 \\ 0.05 & 0.75 & 0.05 \\ 0.15 & 0.15 & 0.80 \end{bmatrix} \times \begin{bmatrix} 465 \\ 361 \\ 484 \end{bmatrix} + \begin{bmatrix} 75 \\ 65 \\ 80 \end{bmatrix} = \begin{bmatrix} 555.7 \\ 383.2 \\ 591.1 \end{bmatrix}$$ **A1**

Hence 591 kg thigh fillets were sold in the second week.

4. Kitty has 1 two-step dominance over Nona, meaning Kitty defeated someone who defeated Nona. In round 1 Leo defeated Nona, so Kitty must have defeated Leo.

Nona has 2 two-step dominances over Leo, meaning that Nona has defeated two teams that defeated Leo, and since Nona lost to Leo in round 1, Nona must have defeated Kitty and Max, who each in turn have defeated Leo. This supports the values in Leo's row, gaining 1 two-step dominance over Kitty and Max, since Leo defeated Nona who defeated Kitty and Max.

M1

The results determined to date are :

Kitty defeated Leo	Nona defeated Kitty
	Nona defeated Max

The last game to determine is therefore the one between Leo and Max.
Given Max has 1 two-step dominance over Nona, and Nona only lost to Leo, Max must have defeated Leo.

The four results are :

Kitty defeated Leo	Nona defeated Kitty	Max defeated Leo
	Nona defeated Max	

A1

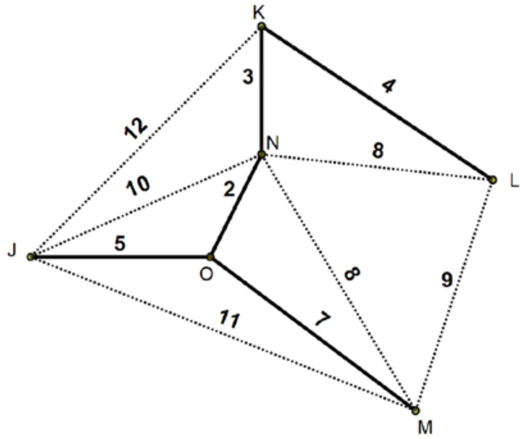
Module 2 – Networks and decision mathematics

1. **(a)** The shortest path is *JONKL* with a length of 14 km. **A1**

(b) For this network v = 6, f = 7, e = 11, so
6 + 7 = 11 + 2
13 = 13 The network is planar. **A1**

(c) Hamiltonian cycle **A1**

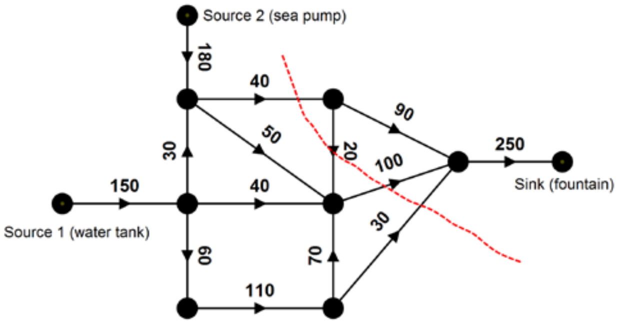
(d) A minimum spanning tree using Prim's Algorithm is used to produce the tree below:



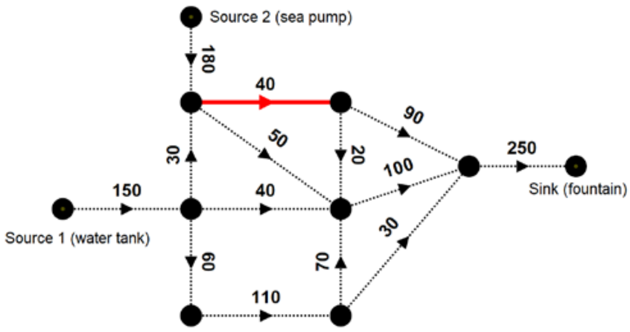
A1

(e) Alice should complete an Eulerian circuit, but she cannot because *K*, *L*, *N* and *O* each have odd degree. The minimum distance must therefore repeat two extra roads connecting these four vertices. The two edges that do this with the **minimum** extra distance are *NO* with a length of 2 km and *KL* with a distance of 4 km. *KN* is not appropriate to use because there is no edge between *L* and *O*. The total distance is 85 km. **A1**

2. **(a)** The required cut is shown below. **A1**



(b) The pipe that must be replaced is shown below. The replaced pipe must be on the minimum cut and by choosing the pipe of capacity 40, the next minimum cut is also increased. **A1**



(c) 220 litres **A1**

220 litres is the maximum flow, because the minimum cut that does NOT include the edge selected is 220 litres through 90 + 100 +30 = 220 litres

3. **(a)** 26 days **A1**

The following table of earliest and latest starting times and float times is used for this network:

Activity	EST	LST	Float
A	0	7	7
B	0	4	4
C	0	0	0
D	7	11	4
E	8	12	4
F	7	7	0
G	7	18	11
H	7	12	5
I	7	11	4
J	12	16	4
K	14	18	4
L	17	17	0

From this table it can be seen that the critical path is *CFL* with a length of 26 days.

(b) The float time for activity *D* is 4 days. **A1**

(c) 22 days **A1**

There are a number of paths through this network:

Path	Length
CFL	26 days
CIJK	22 days
CHJK	21 days
CG	15 days
CDEL	22 days
BHJK	17 days
BG	11 days
BFL	22 days
BDEL	18 days
AEL	19 days

If every path with F was reduced by 6 days, then the new longest path is 22 days using either *CIJK* or *CDEL*.

(d) $1200 **A1**

Any path longer than 21 days must be reduced. Three of the paths greater than 21 days, *CFL*, *BFL* and *CDEL* all have activity *L* in common and so reducing *L* by 5 days reduces these paths to 21 or below at a cost of $1000. However *CIJK* is also 22 days and also needs to be reduced by 1 day. Only *I* or *K* can be reduced, so either of these activities can be reduced by 1 at a cost of $200. This is a total cost of $1200.

Module 3 – Geometry and measurement

1. **(a)** Area = $2.93 \times 5.00 = 14.65$ m^2 **A1**

(b) Volume = area × thickness = $14.65 \times 0.0095 = 0.139175$ m^3
Weight = $0.139175 \times 3000 = 417.5$ kg **A1**

(c) EITHER
Angle in triangle at centre of lobby = $360 \div 45 = 8°$
Angle at base = $(180° - 8°) \div 2 = 86°$ **M1**
Let r be the radius of the building
From Sine Rule,
$\frac{r}{\sin 86°} = \frac{2.93}{\sin 8°}$, giving r = 21.001… ≈ 21.0 m
Diameter = radius × 2 = $21.0 \times 2 = 42.0$ m **A1**

OR
Angle in triangle at centre of lobby = $360 \div 45 = 8°$
Let r be the radius of the building
From Cosine Rule,
$r^2 + r^2 - 2 \times r \times r \times \cos 8° = 2.93^2$ **M1**
Solving for r gives $r = 21.001\ldots$
Diameter = radius × 2 = $21.0 \times 2 = 42.0$ m **A1**

2. **(a)** Base right triangle = $48 \div 2 = 24$ m
Let h be height
From SOHCAHTOA
$\tan 5° = \frac{h}{24}$, giving h = 2.0997 ≈ 2.10 m MUST SHOW CALCULATION **A1**

(b) Area circle = $\pi \times 24 \times 24 = 1809.557\ldots$
To add 1%, multiply by 1.01
Area bought = $1.01 \times 1809.557 = 1827.652\ldots$
Excess area = $1827.652 - 1820 = 7.652\ldots \approx 8$ m^2 **A1**

3. **(a)** Let d be the distance from the screen centre to the centre of the circle.
The triangle formed by the screen and the sector lines is equilateral
EITHER
From Pythagoras,
$d^2 = 5^2 - 2.5^2$, giving $d = 4.3301 \approx 4.33$ m **A1**

OR
From SOHCAHTOA
$\tan 30° = \frac{2.5}{d}$, giving $d = 4.3301 \approx 4.33$ m **A1**

OR
From SOHCAHTOA
$\cos 30° = \frac{d}{5.0}$, giving $d = 4.3301 \approx 4.33$ m **A1**

3. **(b)** Length fifth row = $\frac{13.2 \times 2 \times \pi \times 60^\circ}{360^\circ}$ = 13.8230…

Length allowing for two aisles = 13.8230 – 2 × 1.2 = 11.4230… **M1**

Number of seats = 11.4230 ÷ 0.600 = 19.038…

There will be 19 seats in the row. **A1**

4. **(a)** Angle between Ganzhou and Perth = 25º + 32º = 57º

Great circle distance = $\frac{6400 \times 2 \times \pi \times 57^\circ}{360^\circ}$ = 6366.9611… ≈ 6370 km **A1**

(b) Longitude difference = 115º – 10º = 105º

Time difference = 105º ÷ 15º = 7 hours **M1**

EITHER

10.45 pm + 24 hours 25 minutes = 11.10 pm Wednesday Hamburg time

Perth is East and therefore ahead in time of Hamburg

Perth time = 11.10 pm + 7 hours = 6.10 am Thursday **A1**

OR

Departure time (Perth time) = 10.45 pm + 7 hours = 5.45 am Wednesday

Arrival time = 5.45 am + 24 hours 25 minutes = 6.10 am Thursday **A1**

Module 4 – Graphs and relations

1. **(a)** \$150 per hour. **A1**

The cheapest rate is for an 8 hour flight. This is the cheapest rate because the gradient of the line from the origin would be least to this point. The rate is therefore $\frac{1200}{8}$ = \$150 per hour.

(b) Three six hour flights. **A1**

The following calculation is required:
$4 \times 600 + 6 \times 1000 + 1200x = 12\,000$
$x = (12\,000 - 4 \times 600 - 6 \times 1000) \div 1200 = 3$

2. **(a)** A total of 90 + 30 = 120 km **A1**

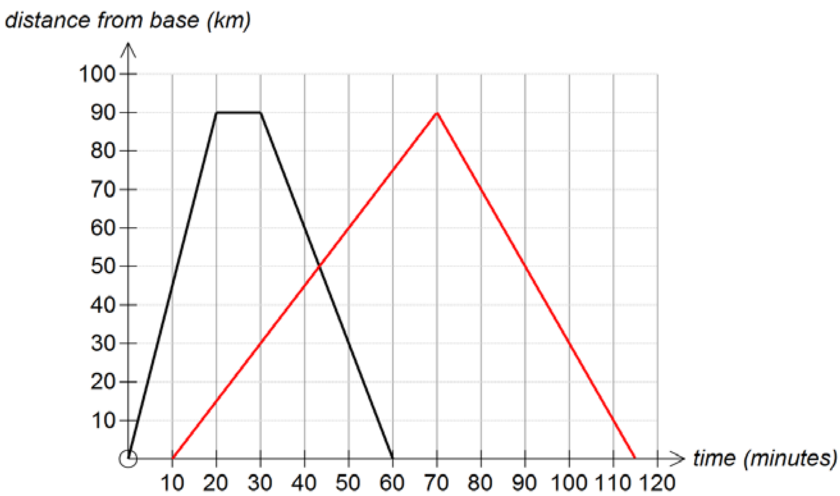
(b) speed = gradient = $\frac{90-0}{30-60} = -3$ Speed is 3 km/min **M1**

3. **(a)** Line 1 from (10, 0) to (70, 90) **A1**

These points are obtained from the line $D = 1.5t - 15$.
When the helicopter leaves $D = 0$, so $0 = 1.5t - 15$, $t = 10$.
The platform is 90 km away so $90 = 1.5t - 15$, $t = 70$.

Line 2 from end of Line 1 (70, 90) to (115, 0) **A1**

These points are obtained from the line $D = 230 - 2t$.
When the helicopter is at the platform $D = 90$, $90 = 230 - 2t$, $t = 70$.
When the helicopter is back at the base $D = 0$, $0 = 230 - 2t$, $t = 115$.



(b) The times when the helicopters are 15 minutes apart can be determined by looking at the graph and seeing that the two lines $D = 1.5t - 15$ and $D = 180 - 3t$ cross and the helicopters are within a short distance of each other. The two equations to be solved are:
$180 - 3t - (1.5t - 15) = 15$ so $t = 40$
$1.5t - 15 - (180 - 3t) = 15$ so $t = 46.6666\ldots$
Setting up either equation showing difference between these two lines **M1**
Distance = 46.666… − 40 = 6.67 minutes

4. **(a)** The number of administration staff must be the same as or more than the number of flight staff. **A1**

(b) 6 **A1**

x is the number of flight staff and the feasible region extends just past 6 in the x direction. As the number of staff must be whole, the answer is 6.

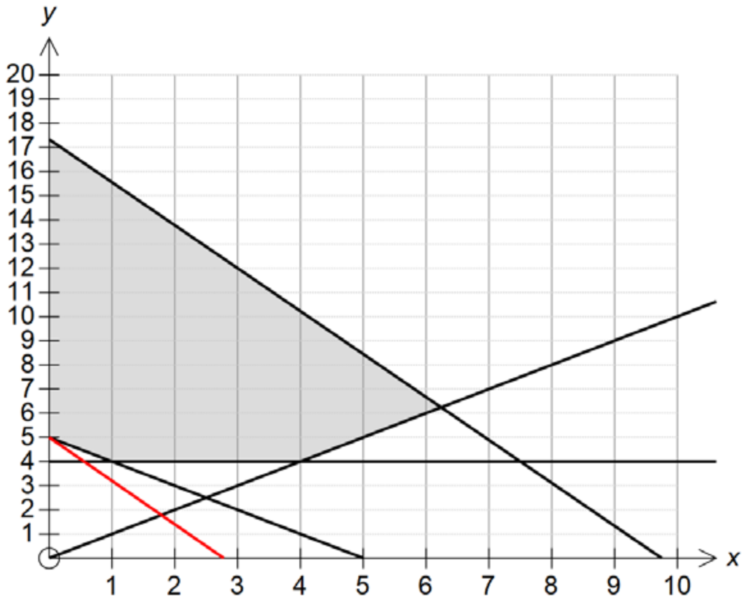
(c) The maximum bonus is \$696.

This occurs at the point (3,12) as $3 \times 72 + 12 \times 40 = \$696$ **A1**

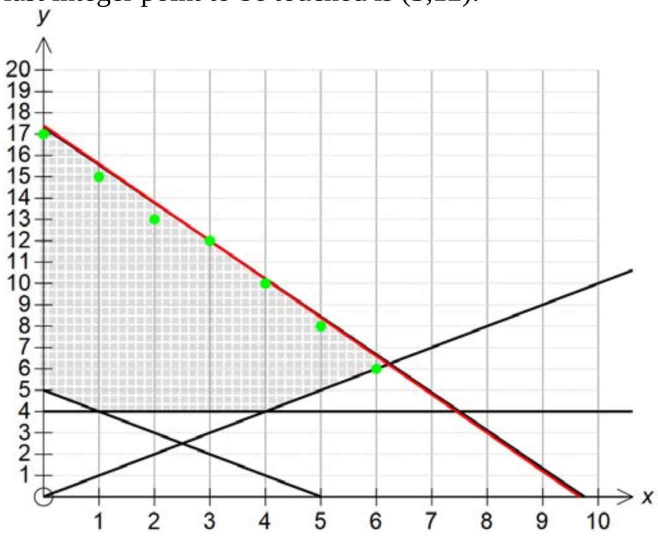
The minimum bonus is \$200. This occurs at the point (0,5) as $40 \times 5 = \$200$. **A1**

The objective function for the bonus is $B = 72x + 40y$. This line can be added to the feasible region and slid up to maximize and down to minimize.

The minimum is shown with the objective function added to the graph below. The last point in the region touched by the objective function is (0, 5):



When the maximum is explored the fact that this is an integer situation must be considered. The integer points along the upper boundary of the region have been marked and as the line is slid up the last integer point to be touched is (3,12):



END OF SOLUTIONS