The Mathematical Association of Victoria

Trial Examination 2018

FURTHER MATHEMATICS

Written Examination 1

STUDENT NAME: _____

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

MULTIPLE-CHOICE QUESTION BOOK

Structure of Book

Section	*Number of questions*	*Number of questions to be answered*	*Number of Modules*	*Number of modules to be answered*	*Number of marks*
A – Core	24	24			24
B - Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question book of 35 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet.
- Unless otherwise indicated, the diagrams in this book are NOT drawn to scale.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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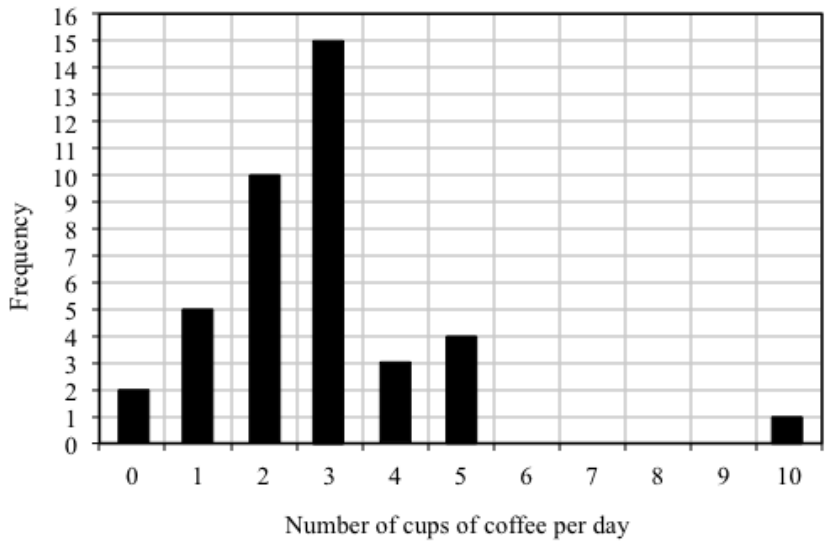
SECTION A – Core

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple – choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale

Data Analysis

The bar chart below displays the number of cups of coffee per day consumed by 40 people.



Question 1

The Q_3 value for the variable "number of cups of coffee per day" consumed by these 40 people is

A. 1
B. 2
C. 3
D. 4
E. 15

A table listing the ten major producers of Uranium in the world, and their production in thousands of tonnes in 2016, is shown below.

Country	Production
Australia	6.3
Canada	14.0
China	1.6
Kazakhstan	24.6
Namibia	3.7
Niger	3.5
Russia	3.0
Ukraine	1.0
United States	1.1
Uzbekistan	2.4

Question 2

The number of outliers in this data is:

A. 0
B. 1
C. 2
D. 3
E. 4

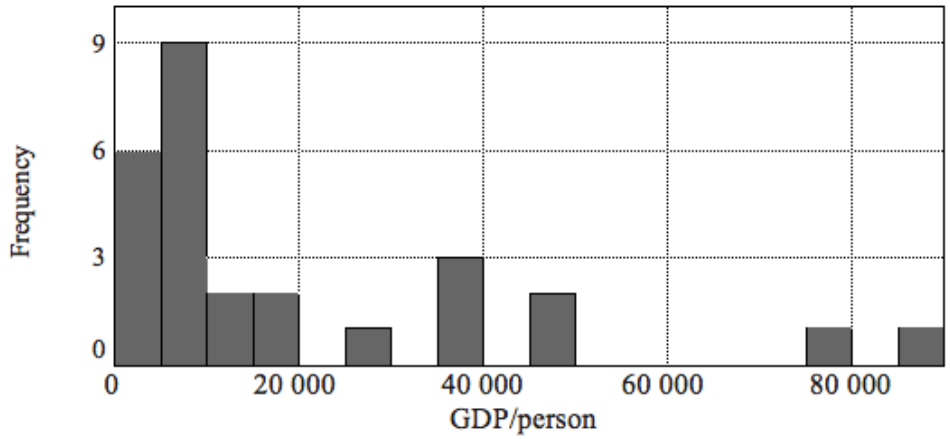
Question 3

The heights of boys in an Under-12 soccer competition are approximately normally distributed with a mean 153 cm and a standard deviation of 7 cm.
Fawad and Vinh both play in the soccer competition.
Fawad is 140 cm tall and Vinh is 165 cm tall.
Which of the following statements is **true**?

A. The standardised height values for Fawad and Vinh are identical.
B. If Vinh's height was increased by 1 cm he would have a standardised height value that is the same distance from the mean as Fawad's.
C. If Fawad's height was decreased by 1 cm he would have a standardised height value that is nearer to the mean than Vinh's.
D. Vinh's standardised height value of – 1.71 is further from the mean than Fawad's standardised height value of 1.86.
E. Fawad's standardised height value of 1.71 is further from the mean than Vinh's standardised height value of –1.86.

Question 4

The histogram below shows the distribution of gross domestic product (GDP) per person (in dollars per year) for a sample of 27 Asian countries in 2016.



Source: Wikipedia/World Bank downloaded 130118

Using this histogram, the percentage of these 27 countries with a GDP per person value of less than $20 000 per year is closest to

A. 15%
B. 19%
C. 55.6%
D. 70.4%
E. 75.0%

Question 5

Each Monday night, a group of people weigh themselves at a Weight Watchers® meeting. One Monday night, this group had a mean weight of 73 kg and a standard deviation of 6 kg. Six weeks later on a Monday night, each person had lost exactly 4 kg in weight.
The group statistics are now

A. a mean of 73 kg with a standard deviation of 6 kg
B. a mean of 73 kg with a standard deviation of 2 kg
C. a mean of 69 kg with a standard deviation of 6 kg
D. a mean of 69 kg with a standard deviation of 2 kg
E. a mean of 69 kg with a standard deviation of 0 kg

Question 6

Kate has scored the following marks in her last five tests:

79 84 75 82 76

What is the lowest mark Kate needs to score in her sixth test to have a **median** value of 80 for the six tests?

A. 78
B. 79
C. 80
D. 81
E. 82

Use the following information to answer Questions 7, 8 and 9
The following statitistics apply to the maximum temperature in degrees and the average gas usage in a home in MJ per day during September, recorded over a number of years. The distribution of maximum temperatures in September is normally distributed and the amount of gas use can be predicted from the maximum temperature.

	Mean	Standard deviation
Maximum temperature in degrees	17.2	3.6
Gas use in MJ per day	34.1	8.4
Pearson's Product Moment correlation coefficient	$r = -0.6$	

Question 7

Which of the following statements about the maximum temperatures in September is **not** true?

A. 84% of maximum temperatures in September would be above 13.6º
B. A maximum temperature of 24.4º would have a standardised score of 2
C. 68% of maximum temperatures would be between 13.6º and 20.8º
D. 13.5% of maximum temperatures would be between 24.4º and 28º
E. 0.3% of maximum temperatures would have been above 28º and below 6.4º

Question 8

The equation of least squares regression line for this relationship is closest to:

A. *gas use in MJ* = 58.2 – 1.4 × *maximum temperature*
B. *gas use in MJ* = 10.0 – 1.4 × *maximum temperature*
C. *gas use in MJ* = 58.2 + 1.4 × *maximum temperature*
D. *maximum temperature* = 58.2 – 1.4 × *gas use in MJ*
E. *maximum temperature* = 10.0 + 1.4 × *gas use in MJ*

Question 9

The value of the correlation coefficient (***r***) for this relationship indicates that

A. As the maximum temperature increases, the amount of gas usage will also increase
B. 60% of the variation in gas usage can be explained by the variation in the maximum temperature
C. As the temperature increases, people will need to use less gas
D. 36% of the variation in the maximum temperature can be explained by the variation in gas usage
E. An increase in the maximum temperature is moderately associated with a decrease in gas usage

Use the following information to answer Questions 10 and 11

With the increase in subscriber services for television, people have had the option of watching television shows in different ways. 410 people were surveyed and asked their age and whether they preferred to watch a TV series the traditional way, week by week, or whether they would rather binge watch a series, over a short period of time. The results are shown below:

		Age Group		
		Below 25 years	25 years to 45 years	Above 45 years
Preferred Method	Binge watching	134	65	24
	Traditional watching	26	72	89

Question 10

The percentage of people surveyed who were 25 years or over and who preferred to binge watch TV is closest to:

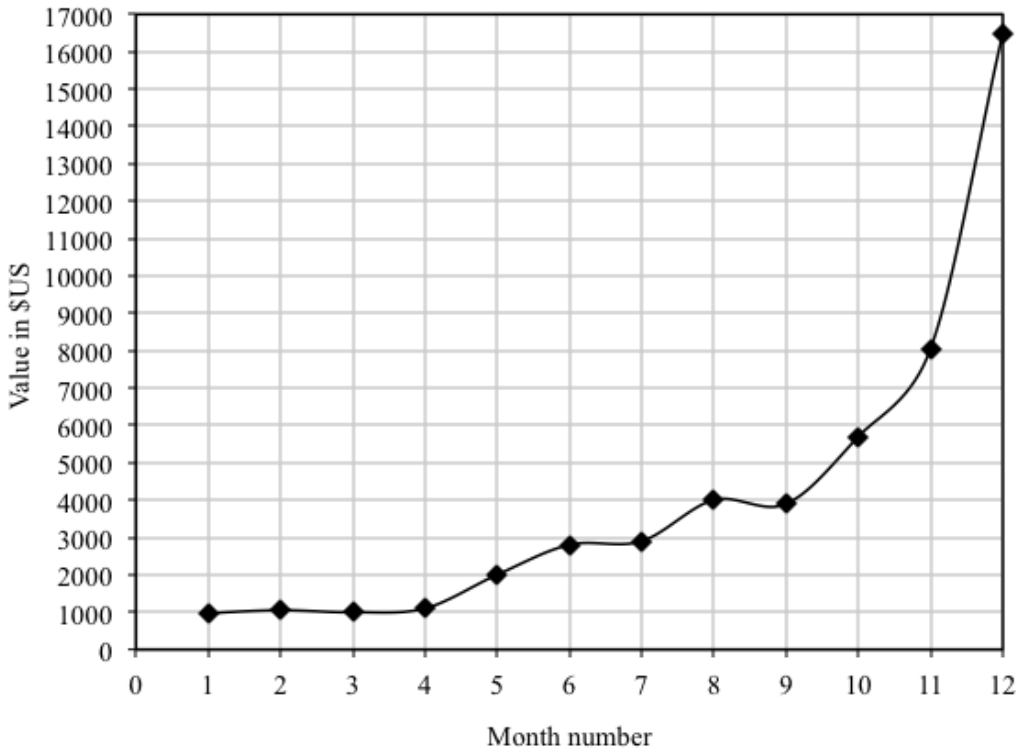
A. 16%
B. 22%
C. 36%
D. 47%
E. 61%

Question 11

The variables “age group in years” and “preferred method of watching TV” are

A. Both ordinal categorical variables
B. Both nominal categorical variables
C. An ordinal categorical variable and a nominal categorical variable respectively
D. A nominal categorical variable and an ordinal categorical variable respectively
E. An ordinal categorical variable and a numerical variable respectively

The value of Bitcoin has increased rapidly during 2017. The value in $US of each Bitcoin on the 20th of each month during 2017, correct to the nearest dollar, is shown on the graph below:



Question 12

This data is clearly non-linear.
The best transformation to linearise this data is

A. A linear relationship
B. A logarithmic transformation of the value axis
C. A reciprocal transformation of the month axis
D. A square transformation of the value axis
E. A logarithmic transformation of the month axis

Question 13

The least squares regression line for the relationship between the value of two different cryptocurrencies, Bitcoin and Ethereum in $US is shown below:

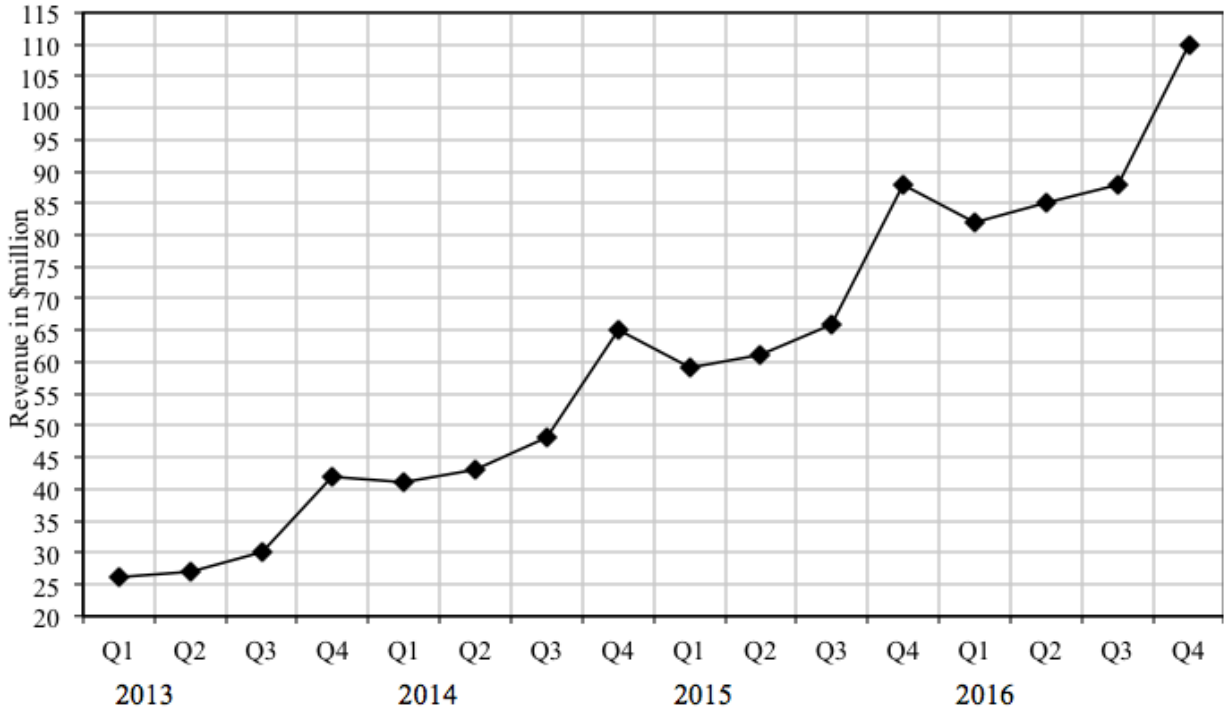
$$\textit{Value (Bitcoin)} = -285.43 + 18.72 \times \textit{Value (Ethereum)}$$

When Bitcoin was worth $2044, there was a residual of $26.87 from the least squares regression line prediction. The value of Ethereum at the time would have been closest to

A. $122
B. $123
C. $124
D. $125
E. $126

Question 14

A time series plot of Etsy's revenue in millions of dollars is shown below:



Which of the following best describes this time series?

A. Cyclic data with an increasing trend
B. An increasing trend with random fluctuation
C. Seasonality with no trend
D. Seasonality with an increasing trend
E. An increasing trend with structural changes

Question 15

Etsy's quarterly profits have been determined to be seasonal over the period 2013 through to 2016 inclusive. The seasonal indices for their revenue in millions of dollars are shown below:

Quarter 1	Quarter 2	Quarter 3	Quarter 4
0.86	0.89	0.97	1.28

The least squares equation that predicts the deseasonalised sales is

$$\textit{Deseasonalised sales in \$million} = 20.13 + 4.72 \times \textit{time}$$

where Quarter 1, 2013 is time = 1, Quarter 2, 2013 is time = 2, etc.
Etsy's **actual** sales in millions of dollars during the first quarter of 2017 is predicted to be closest to

A. \$86.3 million
B. \$100.4 million
C. \$81.4 million
D. \$70.0 million
E. \$82.2 million

Question 16

The seasonal index for the sales of cold drinks in a shop in July is 0.54.
To correct the July sales of cold drinks for seasonality, the actual sales should be

A. reduced by 46%
B. increased by 46%
C. increased by 54%
D. reduced by 85%
E. increased by 85%

Recursion and financial modelling

Question 17

A $7000 trailer depreciates by an equal amount of money each year until it reaches a book value of zero after 5 years. Which of the following recurrence relations could represent the book value of the trailer after n years?

A. $D_0 = 7000 \quad D_{n+1} = D_n + 1400$
B. $D_0 = 7000 \quad D_{n+1} = 0.8D_n - 1400$
C. $D_0 = 7000 \quad D_{n+1} = 1.2D_n$
D. $D_0 = 7000 \quad D_{n+1} = 0.8D_n$
E. $D_0 = 7000 \quad D_{n+1} = D_n - 1400$

Question 18

Consider the recurrence relation below:

$$A_0 = 15\,000 \qquad A_{n+1} = 1.003A_n - 45$$

This recurrence relation could be used to model

A. A reducing balance loan at 3.6% pa compounding monthly
B. An annuity investment at 3.6% pa compounding monthly
C. A reducing balance loan at 3% compounding annually
D. An interest only loan at 3.6% compounding monthly
E. An interest only loan at 3% compounding annually

Question 19

For insurance purposes, a new car was initially valued at $35 000.
Using the unit cost method, the value of the car was depreciated for each kilometre travelled.
At the start of 2016, the car was valued at $27 543.74.
At the start of 2017, the car was valued at $14 999.60
If the number of kilometres travelled in 2016 was 29687, the depreciation rate in ¢/km is closest to

A. 40
B. 41
C. 42
D. 43
E. 44

Craig takes out a loan of $2500 which is to be repaid in 12 months. Interest on this loan is charged at 10.5% per annum interest compounding monthly. An amortisation table for the duration of the 12 months loan, with some entries missing, is shown below:

Payment number (n)	Payment made ($)	Interest charged ($)	Reduction in loan balance ($)	Balance of Loan ($)
0				2500.00
1	220.00	21.88	198.12	2301.88
2	220.00	20.14	199.86	2102.02
3	220.00	18.39	201.61	1900.41
4	220.00	16.63	203.37	1697.04
5	220.00	14.85	205.15	1491.89
6	220.00	13.05	206.95	1284.94
7	220.00	11.24	208.76	1076.18
8	220.00	9.42	210.58	865.60
9	220.00	7.57	212.43	653.17
10	220.00	5.72	214.28	438.89
11	220.00	3.84	216.16	222.73
12				

Question 20

Craig's twelfth and final payment will be more than $220.00, due to the fact that he was happy to round his previous eleven repayments **down** to the nearest dollar.
How much will his final payment be to leave a balance of $0.00 after the twelfth payment?

A. $220.78
B. $221.95
C. $222.73
D. $224.68
E. $225.05

Question 21

George is about to invest his superannuation lump sum in an annuity. He has been offered an annuity paying interest at 4.24% pa compounding monthly which will give him $3500 per month for 20 years. He needs more than $3500 per month, but the amount he needs would result in a zero balance after exactly 13 years and 6 months at the same interest rate.
The **additional** amount he needs each month is closest to

A. $970.75
B. $1030.45
C. $1092.21
D. $1156.48
E. $4592.21

Question 22

Ian takes out a reducing balance loan at 6.1% pa compounding monthly and he makes equal monthly payments. Five years after he takes out the loan Ian owes $543 900, and nine years after he takes out the loan the balance owed is reduced to $412 700. The original amount that Ian borrowed correct to three significant figures was

A. $668 000
B. $669 000
C. $670 000
D. $708 000
E. $591 000

Question 23

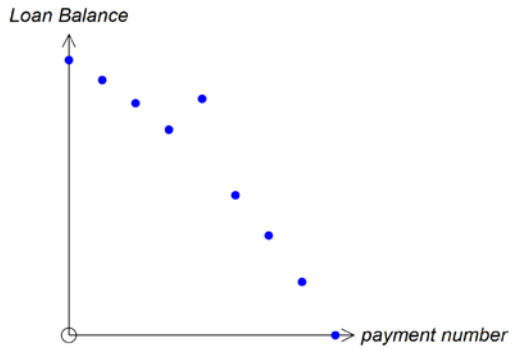
Travis starts with no money and over a 20 year period, he saves the same amount of money every month in an annuity investment.
After the 20 year period, Travis invests the balance into an annuity that will make him equal monthly payments until it reaches a zero balance after another 20 years.
The annuity investment and the annuity both compound monthly at 3.7% per annum.
Which of the following statements is **true**?

A. Travis' annuity will pay the same amount as his monthly payment into the annuity investment
B. Travis will receive 3.7% more each month in his annuity than his payment into the investment
C. Travis will receive 7.4% more each month in his annuity than his payment into the investment
D. Travis will receive 37% more each month in his annuity than his payment into the investment
E. Travis' annuity will pay more than double his payment into the investment every month

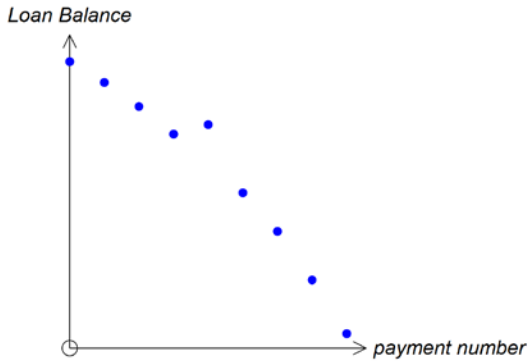
Question 24

Samuel takes out a reducing balance loan that he plans to pay off over 8 equal repayments. Samuel misses the 4th payment, but he makes a payment equivalent to double the original payment when the 5th repayment is due and then he continues to make the original repayments another three times. Which of the following graphs could show the balance of the loan over the 8 repayments?

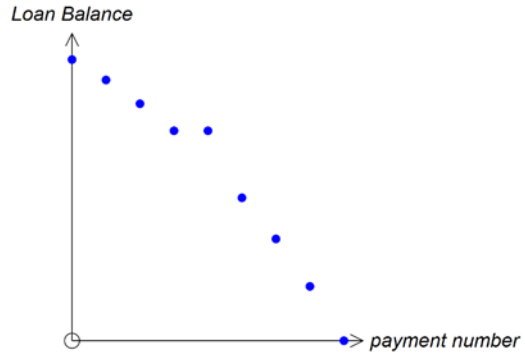
A.



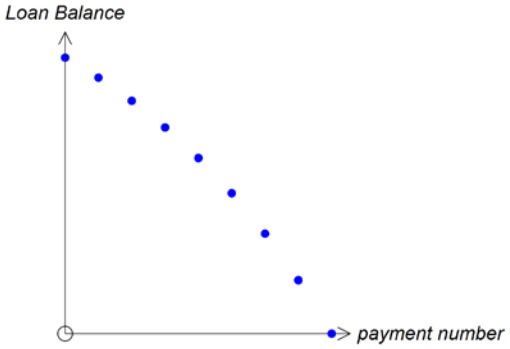
B.



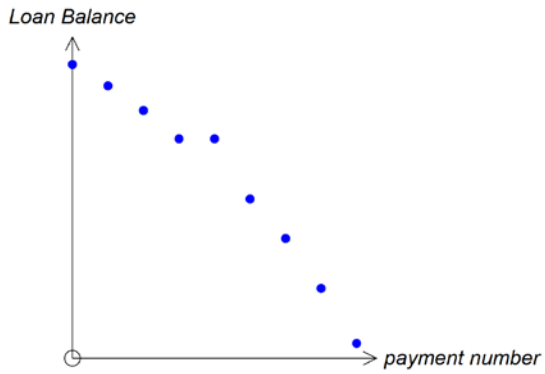
C.



D.



E.



END OF SECTION A

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END OF SECTION A

TURN OVER

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple – choice questions.
Show the modules you are answering by shading the matching boxes on your multiple - choice answer sheet **and** writing the name of the module in the box provided.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
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Contents **Page**

Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

$3\begin{bmatrix} 3 & 7 \\ 6 & 2 \end{bmatrix} + 2\begin{bmatrix} 3 & 7 \\ 6 & 2 \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 6 & 14 \\ 12 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 9 & 21 \\ 18 & 6 \end{bmatrix}$

C. $\begin{bmatrix} 15 & 35 \\ 30 & 10 \end{bmatrix}$

D. $\begin{bmatrix} 18 & 42 \\ 36 & 12 \end{bmatrix}$

E. $\begin{bmatrix} 414 & 462 \\ 396 & 348 \end{bmatrix}$

Question 2

Julia needs to buy some stationery for school.
Three pens and four exercise books will cost $20.65
Four pens and three exercise books will cost 20.30
Let p be the cost of a pen.
Let b be the cost of an exercise book.
Which of the following matrix equations could Julia solve to obtain a matrix that contains the price of a pen and the price of an exercise book ?

A. $\begin{bmatrix} p \\ b \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 20.65 \\ 20.30 \end{bmatrix}$

B. $\begin{bmatrix} p \\ b \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} 3 & -4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 20.30 \\ 20.65 \end{bmatrix}$

C. $\begin{bmatrix} p \\ b \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -3 & 4 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 20.65 \\ 20.30 \end{bmatrix}$

D. $\begin{bmatrix} p \\ b \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} 3 & -4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 20.65 \\ 20.30 \end{bmatrix}$

E. $\begin{bmatrix} p \\ b \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 20.65 \\ 20.30 \end{bmatrix}$

Question 3

Which matrix expression results in a matrix that contains the sum of the numbers 3, 8, 6, 4 and 9 ?

A. $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 & 8 & 6 & 4 & 9 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 8 & 6 & 4 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 8 \\ 6 \\ 4 \\ 9 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix}$

Question 4

The element in row i and column j of M is m_{ij}.
The elements in matrix M are determined using the rule $m_{ij} = 3i - 2j + 1$.
Matrix M could be

A. $\begin{bmatrix} 3 \end{bmatrix}$ **B.** $\begin{bmatrix} 2 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 0 \\ 5 & 3 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 0 & 8 \end{bmatrix}$

Question 5

If matrix $K = \begin{bmatrix} S \\ T \\ A \\ R \end{bmatrix}$ and matrix $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$,
then the word formed by the matrix product $P \times K$ is

A. $\begin{bmatrix} T \\ A \\ R \\ S \end{bmatrix}$ **B.** $\begin{bmatrix} R \\ A \\ T \\ S \end{bmatrix}$ **C.** $\begin{bmatrix} S \\ T \\ A \\ R \end{bmatrix}$

D. $\begin{bmatrix} T \\ S \\ A \\ R \end{bmatrix}$ **E.** $\begin{bmatrix} A \\ R \\ T \\ S \end{bmatrix}$

Question 6

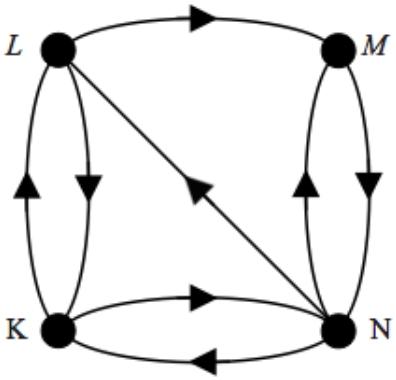
Kerri (K), Leanne (L), Maree (M) and Noni (N) are four key managers in a business who should be in communication with each other all the time. Unfortunately, personality clashes have meant that some of these managers will not directly communicate with some of their colleagues – either sending or receiving messages from their colleague.
The communication matrix below represents this current situation.
The elements in the matrix are such that :

"1" indicates that direct communication from one person to another is possible
"0" indicates that direct communication is not possible.

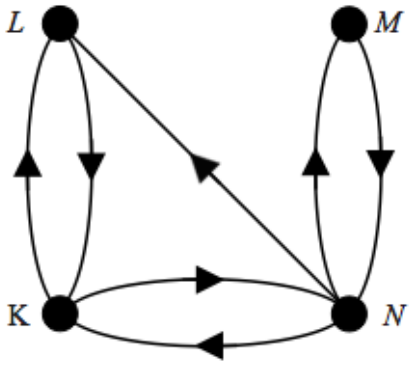
$$\begin{array}{cc} & \begin{array}{cccc} \multicolumn{4}{c}{to} \\ K & L & M & N \end{array} \\ \begin{array}{c} \\ from \\ \\ \\ \\ \end{array} & \begin{array}{c} K \\ L \\ M \\ N \end{array} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The diagram that correctly represents this matrix is :

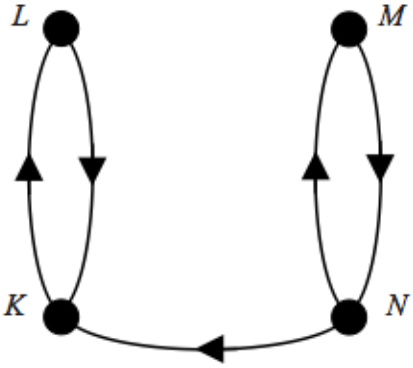
A.



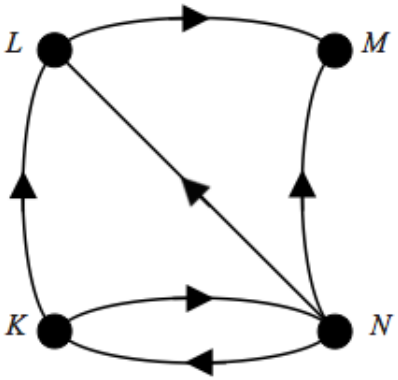
B.



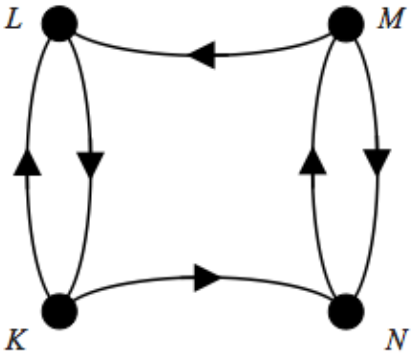
C.



D.



E.



Question 7

Consider the matrix recurrence relation below.

$$S_0 = \begin{bmatrix} 30 \\ 25 \\ 40 \end{bmatrix}, \qquad S_{n+1} = TS_n \qquad \text{where } T = \begin{bmatrix} 0.5 & 0.2 & Q \\ 0.3 & 0.6 & P \\ 0.2 & 0.2 & R \end{bmatrix}$$

Matrix T is a regular transition matrix.

Given the above and that $S_1 = \begin{bmatrix} 24 \\ 32 \\ 39 \end{bmatrix}$, which one of the following is **true** ?

A. $P = 0.1$
B. $Q = 0.2$
C. $Q = 0.4$
D. $R = 0.6$
E. $R = 0.7$

Question 8

Amy, Brian and Cary all service school kitchen equipment.
The matrix T shows the time that it takes (in minutes) for each of Amy (A), Brian (B) and Cary (C) to service a dishwasher (D), a hot water system (H) and a gas stove (G).

$$T = \begin{matrix} \\ D \\ H \\ G \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 13 & 14 & 15 \\ 15 & 17 & 12 \\ 21 & 18 & 20 \end{bmatrix} \end{matrix}$$

The matrix N below displays the number of dishwashers, hot water systems and gas stoves to be serviced in the Home Economics rooms of three schools, Upton (U), Verdant (V) and Wilton (W).

$$N = \begin{matrix} \\ U \\ V \\ W \end{matrix} \begin{matrix} \begin{matrix} D & H & G \end{matrix} \\ \begin{bmatrix} 2 & 2 & 12 \\ 1 & 1 & 8 \\ 2 & 2 & 10 \end{bmatrix} \end{matrix}$$

A matrix that displays the time that it would take each of Amy, Brian and Cary, working alone, to service the dishwashers, hot water systems and gas stoves in the Home Economics areas of each of the three schools is

A.
$$\begin{bmatrix} 15 & 16 & 27 \\ 16 & 18 & 20 \\ 23 & 20 & 30 \end{bmatrix}$$

B.
$$\begin{bmatrix} 11 & 12 & 3 \\ 14 & 16 & 4 \\ 19 & 16 & 10 \end{bmatrix}$$

C.
$$\begin{bmatrix} 70 & 70 & 418 \\ 71 & 71 & 436 \\ 100 & 100 & 596 \end{bmatrix}$$

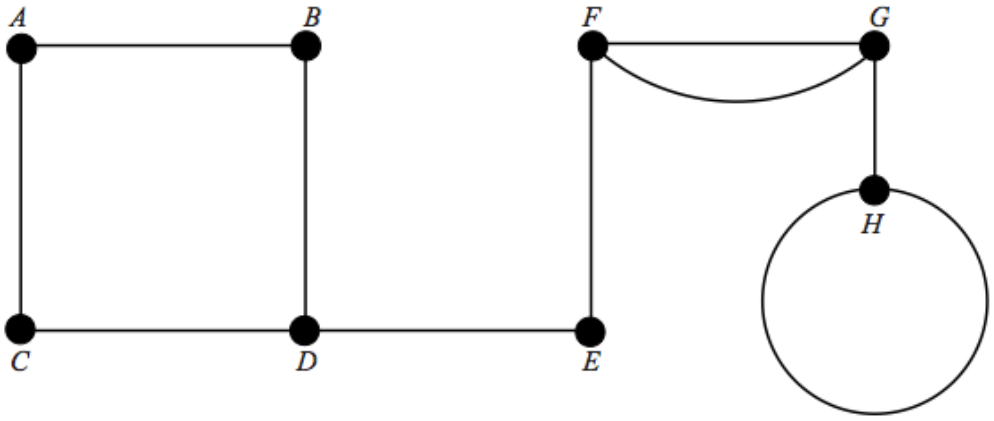
D.
$$\begin{bmatrix} 308 & 278 & 292 \\ 196 & 175 & 187 \\ 266 & 242 & 254 \end{bmatrix}$$

E.
$$\begin{bmatrix} 26 & 16 & 180 \\ 15 & 51 & 96 \\ 42 & 36 & 200 \end{bmatrix}$$

Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1
A network is shown below:



A number of statements, both correct and incorrect, have been made about this network:

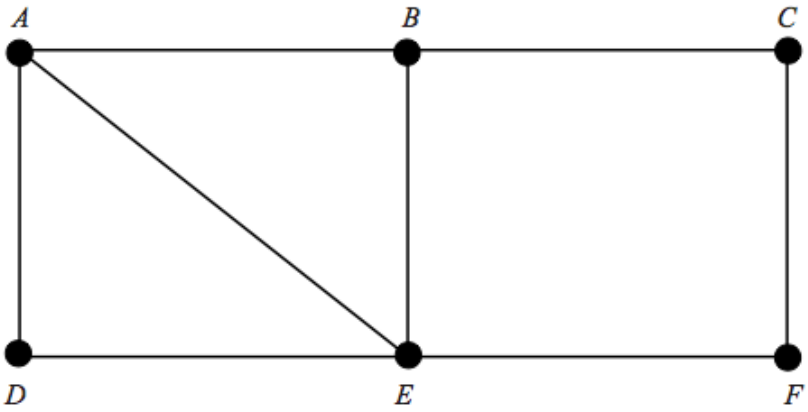
- There is a loop at vertex *H*.
- There are three faces in this network.
- The network is a simple network.
- *DE* is a bridge in the network.
- The degree of vertex *H* is three.

The number of correct statements is:

A. 1
B. 2
C. 3
D. 4
E. 5

Question 2

Aaron walks around the network shown below.

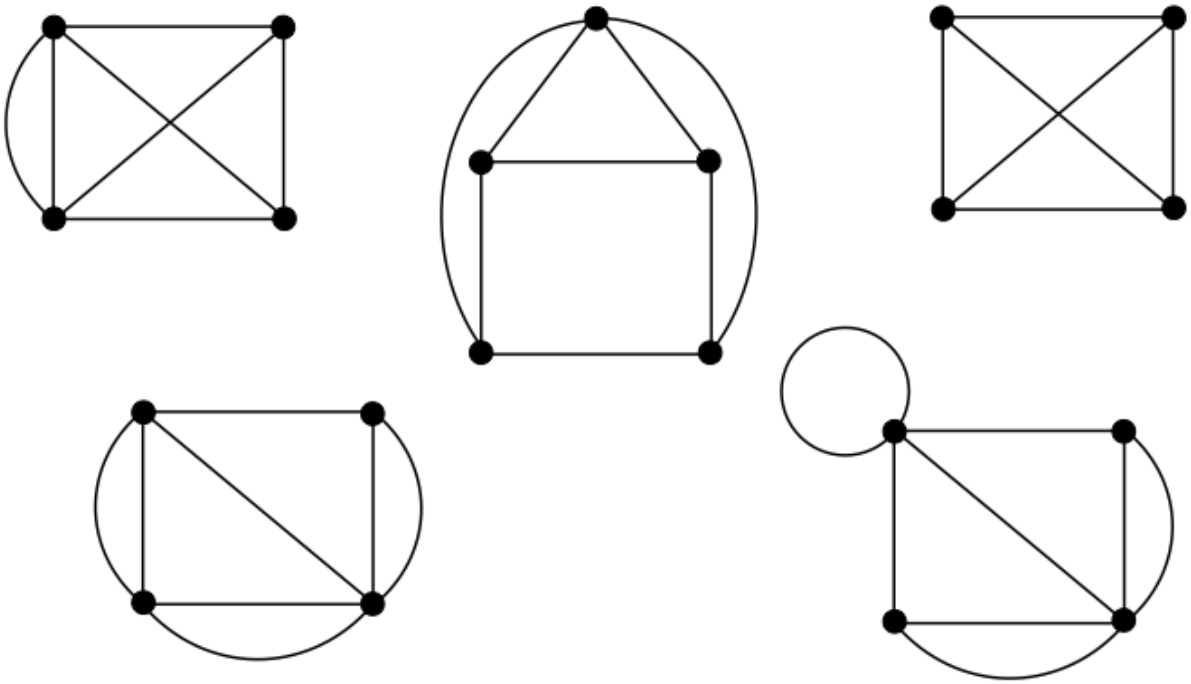


Which of the following statements is not true?

A. If Aaron walks along the route *ABCFEDA*, he has completed both a circuit and a cycle
B. If Aaron walks along the route *FEDAEDCF*, he has completed both a circuit and a cycle
C. If Aaron walks along the route *ABCFE*, he has completed both a trail and a path
D. If Aaron walks along the route *DAEBA*, he has completed a trail but not a path
E. If Aaron walks along the route *EADEBCF*, he has completed a trail but not a path

Question 3

How many of the graphs below consist of five faces?



A. None
B. One
C. Two
D. Three
E. Four

Question 4

The organiser of a fair has many attractions set up at a number of locations with paths laid out between the attractions. He wants to set up lights at each of the attractions using the shortest amount of electrical cable possible.
In order to determine the length of wire required he should

A. Use Dijkstra's algorithm
B. Use Prim's algorithm
C. Use the Hungarian algorithm
D. Use an activity network
E. Use a Hamiltonian cycle

Question 5

A connected, planar network has the same number of vertices as edges.

Which of the following statements is true?

A. If one edge were removed, the network could be a tree.
B. Any Hamiltonian cycle would not be an Eulerian circuit.
C. Any Hamiltonian path would also be an Eulerian trail.
D. Any spanning tree would also be a Hamiltonian path.
E. The network must have an Eulerian circuit.

Question 6

The following matrix is to be reduced so that a minimum allocation can be made using the Hungarian algorithm.

$$\begin{bmatrix} 2 & 7 & 8 & 6 \\ 5 & 7 & 9 & 6 \\ 3 & 4 & 8 & 3 \\ 3 & 5 & 7 & 7 \end{bmatrix}$$

The reduced matrix that would be ready for allocation is

A.
$$\begin{bmatrix} 0 & 5 & 6 & 4 \\ 0 & 2 & 4 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 2 & 4 & 4 \end{bmatrix}$$

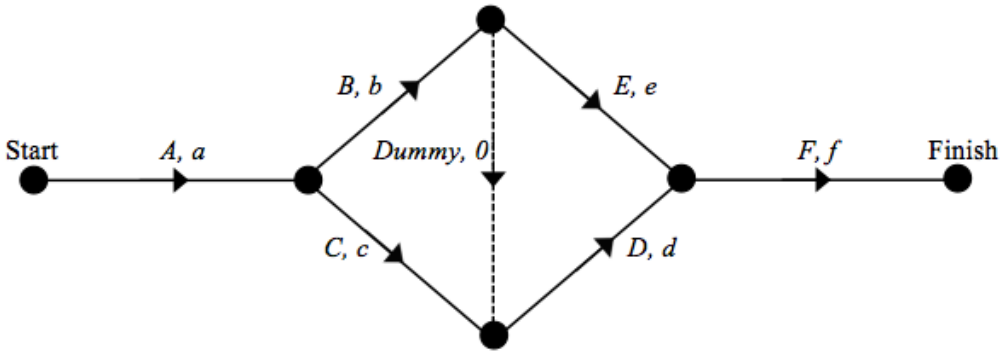
B.
$$\begin{bmatrix} 0 & 4 & 2 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

C.
$$\begin{bmatrix} 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

D.
$$\begin{bmatrix} 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

E.
$$\begin{bmatrix} 1 & 3 & 2 & 3 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 3 \end{bmatrix}$$

Question 7



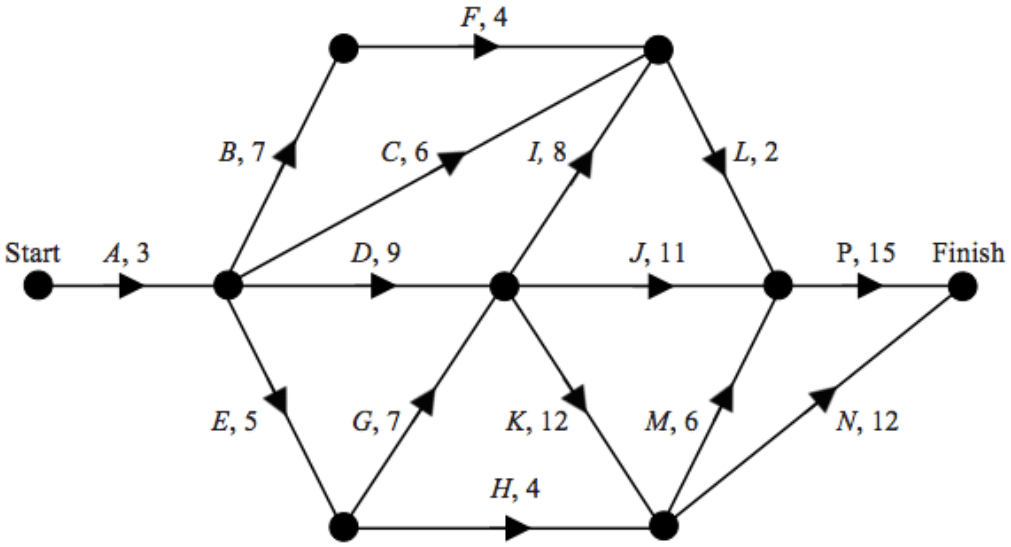
The activity network above shows the activities *A*, *B*, *C*, *D*, *E* and *F*, along with their durations in hours of *a*, *b*, *c*, *d*, *e* and *f* respectively.

Which of the following statements must be **true** based on this information?

A. If $d > e$, then the critical path must be *ACDF*
B. There are two paths through this network
C. The critical path must have a duration of $a + b + c + d + e + f$
D. If $b = c$ and $d = e$ there will be three critical paths through the network
E. If $c > b$ then the critical path must be *ABEF*

Question 8

An activity network is shown below, with durations of each activity in days:



The float time of activity *D* is:

A. 3
B. 5
C. 7
D. 9
E. 10

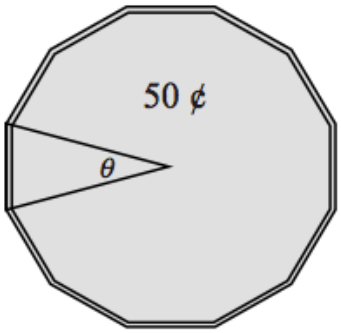
SECTION B – Module 2 continued
TURN OVER

Geometry and measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

A 50 cent coin has 12 sides of equal length.
The angle (θ) subtended at the centre of the coin by each side is identical.



The value of θ is :

A. 15º
B. 20º
C. 30º
D. 45º
E. 60º

Question 2

All parts of the nation of China are in the same time zone (by Government decree).
Shanghai (31º N, 122º E) and Chengdu (30º N, 103º E) are two significant cities in China.
On one day in January, the sun set at 5.18 pm in Shanghai.
Assuming that 15º of longitude equates to one-hour time difference, the time that the sun was expected to set in Chengdu is

A. 4.02 pm
B. 4.59 pm
C. 5.18 pm
D. 5.37 pm
E. 6.34 pm

Question 3

Palmerston North (New Zealand) is situated at 40º S, 176º E.
Which one of the following locations would be on the same great circle of longitude ?

A. Great Sitkin Island (52º N, 176º W)
B. Valdivia (Chile) (40º S, 73º W)
C. Montpellier (France) (43º N, 4º E)
D. Madrid (Spain) (40º N, 4º W)
E. Providenija (Siberia) (64º N, 172º W)

Question 4

Jerry and Kaz are racing their land yachts across a large desert space, having started from the same place.
Jerry travels 20 km on bearing 337º T and stops.
Kaz travels 21 km on bearing 067º T and also stops.
The straight line distance, in kilometres correct to one decimal place, between Jerry and Kaz now is closest to

A. 6.4
B. 26.3
C. 31.4
D. 29.0
E. 42.0

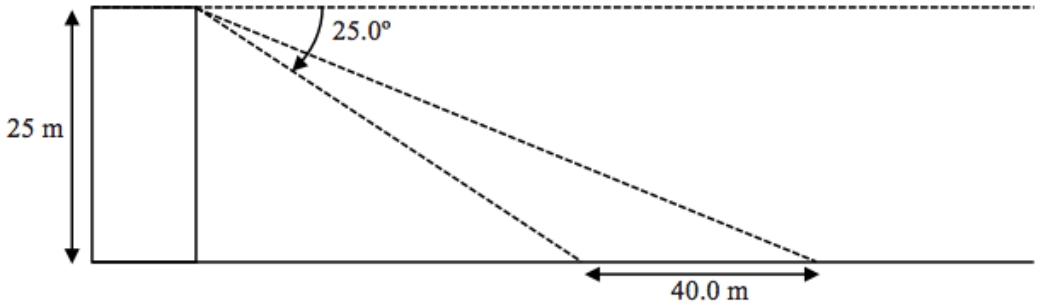
Question 5

Two fishing boats travel in a straight line from the same harbour to their favourite fishing grounds.
Boat A travels 14.8 km on bearing 125º T.
Boat B travels 16.3 km on a bearing that is greater than the bearing for Boat A.
When anchored at their fishing grounds, the direct distance between Boat A and Boat B is 13.2 km.
The bearing that Boat B travelled on, in degrees correct to the nearest degree is

A. 50º T
B. 75º T
C. 130º T
D. 150º T
E. 175º T

Question 6

From the top of a 25 m building, Josie sees two flagsticks planted on the level ground in front of her building. The two flagsticks are in a direct line from the building and the ground distance between them is 40.0 m. The angle of depression to the closest flagstick is 25.0º.

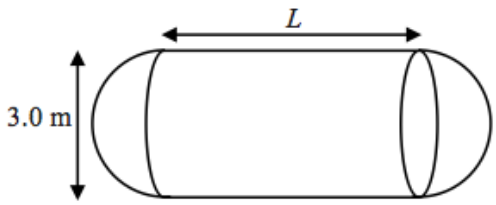


The angle of depression to the furthest flagstick, in degrees correct to one decimal place, is

A. 13.9
B. 14.7
C. 15.0
D. 15.4
E. 16.3

Question 7

A railway tanker wagon has a tank consisting of a cylinder with two hemispherical ends, as shown below.



The diameter of the cylinder is 3.0 m.
What will the length of the cylindrical part of the tanker (shown by L above), in metres correct to one decimal place, if the tanker hold 50 000 L of fluid ?
Note that 1 m^3 can hold 1000L.

A. 4.5
B. 4.7
C. 4.9
D. 5.1
E. 5.3

Question 8

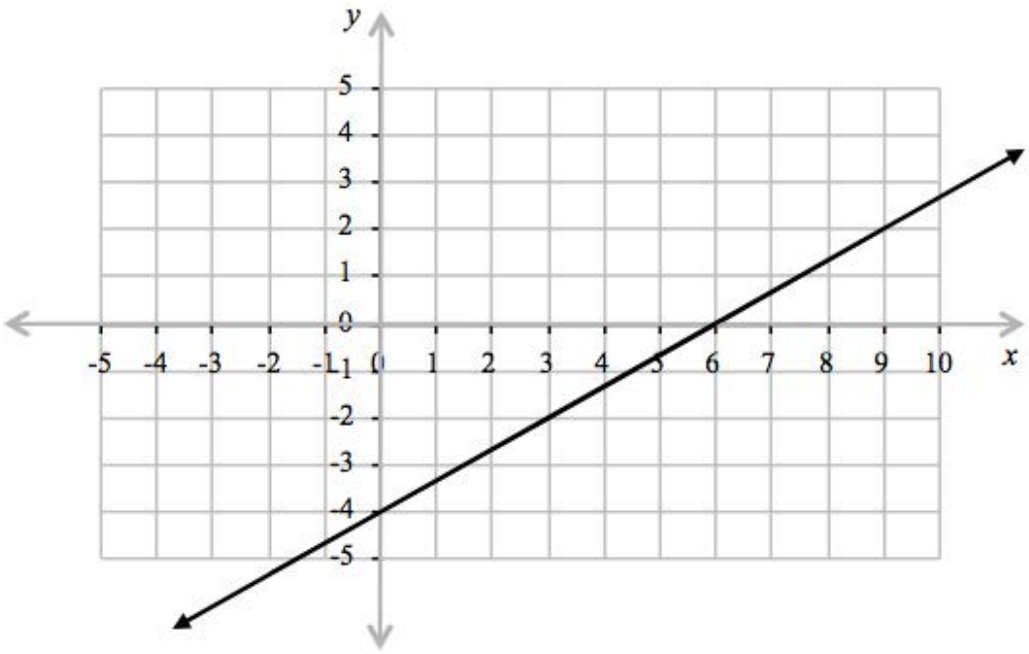
A particular country decided to redesign their $1 and $2 coins.
The new $1 coin has a diameter of 20.0 mm and a thickness of 2.0 mm.
The new $2 has both a diameter and thickness which is 50% greater than the $1 coin.
Both coins are made of exactly the same nickel-based alloy, so the weight is related to the volume.
Hank has five $2 coins and 4 $1 coins in his pocket.
The total weight of these nine coins is closest to

A. 9 times the weight of one $1 coin
B. 12 times the weight of one $1 coin
C. 14 times the weight of one $1 coin
D. 15 times the weight of one $1 coin
E. 21 times the weight of one $1 coin

Module 4 – Graphs & Relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

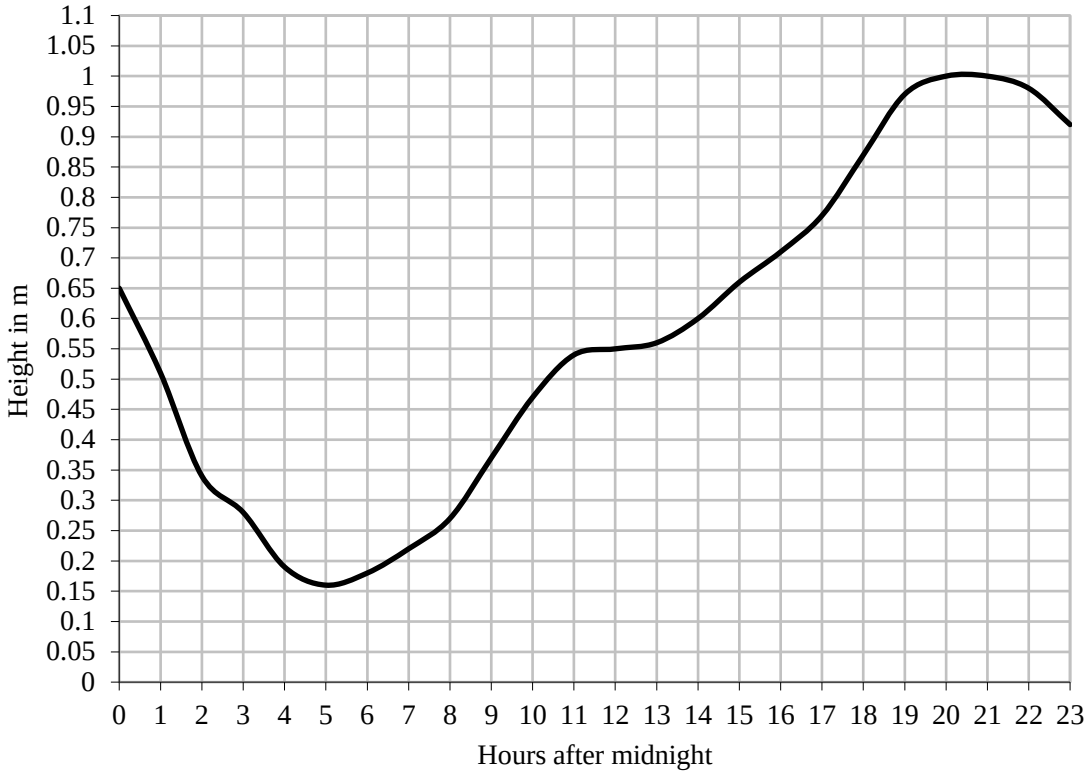


The equation of the line shown above is

A. $2x + 3y = 12$
B. $2x - 3y = 12$
C. $3x + 2y = 12$
D. $3x - 2y = 12$
E. $2x - 3y = -12$

Question 2

The graph below shows the height in metres of the tide during the 1st of January, 2018 at Bunbury, WA.



The average rate of change in the height in metres per hour between 5 am and midday is closest to

A. 0.15 m/hr
B. 0.55 m/hr
C. 0.02 m/hr
D. 0.06 m/hr
E. 0.08 m/hr

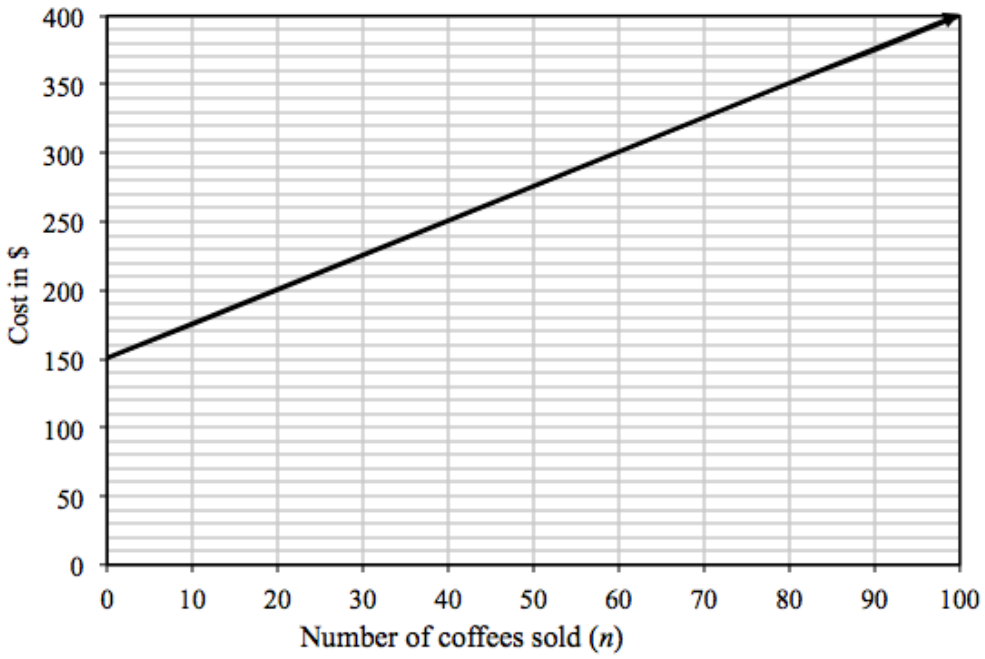
Question 3

Christie runs a cattery and kennel. When the cattery and kennel is full, Christie keeps 35 animals and she makes $330 a day. Christie charges $8 per day for cats and $10 per day for dogs. If the number of cats is x and the number of dogs is y, a set of simultaneous equations that could determine how many of each animal Christie is keeping would be

A. $y = 35 + x$ and $8x + 10y = 330$
B. $y = 35 - x$ and $10x + 18y = 330$
C. $y = 35 - x$ and $8x + 10y = 330$
D. $y = 330 - x$ and $8x + 10y = 35$
E. $y = 35 + x$ and $8x - 10y = 330$

Question 4

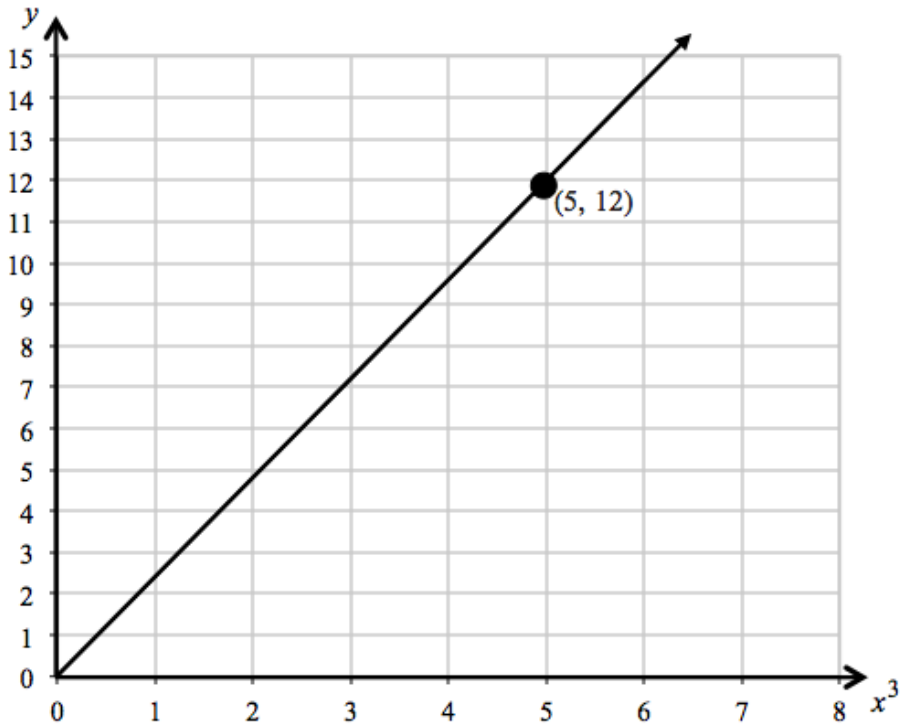
Bella runs a mobile coffee business. The cost to her business based on the number of coffees that she sells (n) is shown in the graph below:



Bella makes $30 profit if she sells 80 coffees.
Bella sells each cup of coffee for

A. $2.85
B. $4.35
C. $4.50
D. $4.75
E. $4.85

Question 5



The equation of the relationship shown above is

A. $y = 2.4x^3$

B. $y = \frac{5x^3}{12}$

C. $y = 2.4x$

D. $y = \frac{5}{12x^3}$

E. $y = \frac{12x^3}{125}$

Question 6

A particular cryptocurrency's price follows a relationship of the form $P = km^2$ where P is the price and m is the number of months after purchase.
Five months after purchase, the value of the cryptocurrency was $150.
What was the value of the cryptocurrency nine months after purchase?

A. $225
B. $270
C. $384
D. $486
E. $1176

Question 7

A gym employs trainers and assistants. Every shift there must be at least six staff on duty, with at least three trainers working per shift. The number of trainers must be more than or equal to half the number of assistants. Every trainer can work with four people at a time and every assistant can work with two people at a time. The gym guarantees that they will have staff to work with at least 20 people at a time.
If x is the number of assistants and y is the number of trainers, the constraint inequations for this situation would be:

A. $x \geq 0, \quad y \geq 3, \quad x + y \geq 6, \quad y \geq 2x, \quad 2x + 4y \geq 20$

B. $x \geq 0, \quad y \geq 3, \quad x + y \geq 6, \quad y \geq \frac{x}{2}, \quad 2x + 4y \geq 20$

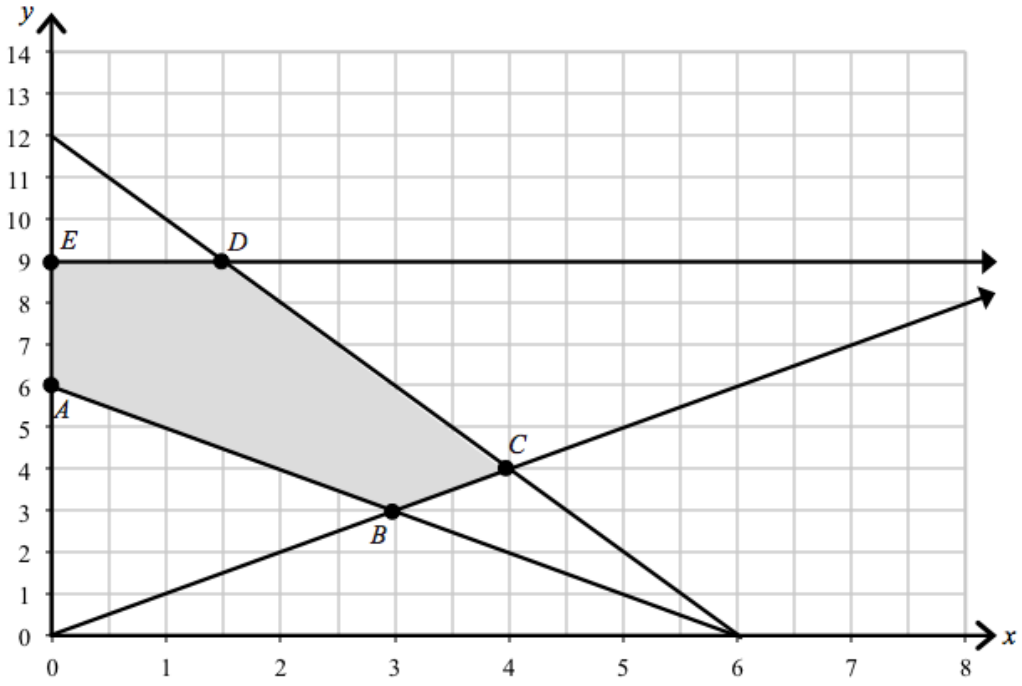
C. $x \geq 0, \quad y \geq 3, \quad x + y \geq 6, \quad y \leq \frac{x}{2}, \quad 2x + 4y \geq 20$

D. $x \geq 0, \quad y \geq 3, \quad x + y \geq 6, \quad y \leq 2x, \quad 4x + 2y \geq 20$

E. $x \geq 0, \quad y \geq 3, \quad x + y \geq 6, \quad y \geq \frac{x}{2}, \quad 4x + 2y \geq 20$

Question 8

A feasible region is shown below:



An objective function for this linear programming problem is $F = mx + ny$ where m and n are both positive integers.
Which of the following statements is **not** true?

A. If $m = n$ and F is to be minimised the line segment from A to B would be the minimum
B. If $m = 2n$ and F is to be maximised the line segment from C to D would be the maximum
C. If $m > 2n$ and F is to be maximised then C would always be the maximum
D. If $m < n$ and F is to be minimised then E could be a minimum
E. If $n < m < 2n$ and F is to be maximised then D would always be the maximum

END OF MULTIPLE - CHOICE QUESTION BOOKLET