

YEAR 12 Trial Exam Paper

2018

FURTHER MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- worked solutions
- mark allocations
- tips on how to approach the exam.

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SECTION A – CORE

Data analysis

Question 1a.

Worked solution

The frequency for students who selected geometry is 3.

$$\frac{3}{25} \times 100 = 12\%$$

Mark allocation: 1 mark

- 1 mark for a correct answer of 12%

Note: The ‘%’ symbol is not required to receive the mark.

Question 1b.

Worked solution

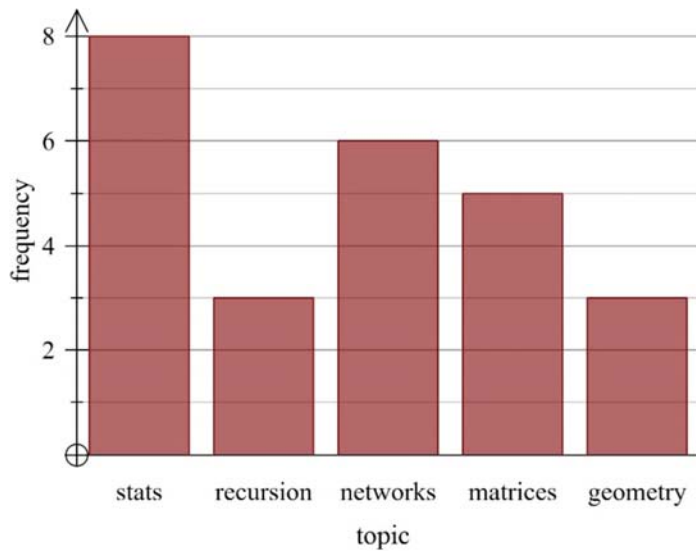
The total number of students in the class is 25.

We must subtract the frequencies that we have been given to find the remainder. This is the number of students who selected matrices.

$$25 - 8 - 3 - 6 - 3 = 5$$

Five students selected matrices.

Draw in the bar for this, as shown below.



Mark allocation: 1 mark

- 1 mark for correct bar



Tip

- When graphing a categorical variable, such as ‘topic’ on a histogram, the bars must not touch.

Question 2a.**Worked solution**

The answer can be found with or without your CAS calculator.

Without the CAS calculator:

Write the data values in a list in ascending order.

1 1 1 2 2 3 3 3 4 4 4 4 **5** 5 5 6 6 7 7 7 8 8 9 9 9

Find the middle number of this data list.

In this case, the middle number is 5. Therefore, the median effort level given is 5.

With the CAS calculator (example from TiNspire):

Enter data into a spreadsheet.

	A effort	B	C	D
1	1			
2	1			
3	1			
4	2			
5	2			

Use the statistics function to find the summary statistics.

Sx := $\sum_{i=1}^n x_i$	27.9079
" $\sigma_x := \sigma_{nX}$ "	2.54433
"n"	25.
"MinX"	1.
"Q ₁ X"	3.
"MedianX"	5.
"Q ₃ X"	7.
"MaxX"	9.
"SSX := $\sum(x - \bar{x})^2$ "	161.84

Mark allocation: 1 mark

- 1 mark for the correct answer of 5

Question 2b.**Worked solution**

There are 9 numbers between 3 and 7 (4, 4, 4, 4, 5, 5, 5, 6, 6).

$$\frac{9}{25} \times 100 = 36\%$$

Mark allocation: 1 mark

- 1 mark for 36%

Question 2c.**Worked solution**

The variable is numerical discrete.

Although not always the case for discrete data, the values given in this survey are whole numbers and are therefore discrete.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3a.**Worked solution**

The boxplot for female *end of year result* is positively skewed.

Mark allocation: 1 mark

- 1 mark for stating positively skewed

Note: Answers that will not be accepted include positive, positive trend, increasing trend and approximately symmetric.

Question 3b.**Worked solution**

Minimum = 3

$Q_3 = 41$

These answers should be read directly from the boxplot.

Mark allocation: 2 marks

- 1 mark for minimum of 3
- 1 mark for Q_3 of 41

**Tip**

- Remember that although the value of 3 is an outlier, it is still a part of the data set. It is, therefore, the minimum.

Question 3c.**Worked solution**

$$\begin{aligned} \text{Lower fence} &= Q_1 - 1.5 \times \text{IQR} \\ &= 26 - 1.5 \times (41 - 26) \\ &= 26 - 1.5 \times 15 \\ &= 3.5 \end{aligned}$$

$3 < 3.5$; therefore, it is an outlier.

Mark allocation: 2 marks

- 1 mark for correct calculation of the lower fence
- 1 mark for statement that 3 is below the lower fence and is therefore an outlier

Question 3d.**Worked solution**

We can see that the median *end of year result* differs for males and females. The median for females was 26, compared with the median of 32 for males. This suggests that *end of year result* is associated with *gender* because the median is higher for males than for females.

Mark allocation: 2 marks

- 1 mark for a statement that implies a change or difference in median
- 1 mark for numerical values for median quoted accurately for both male and female data

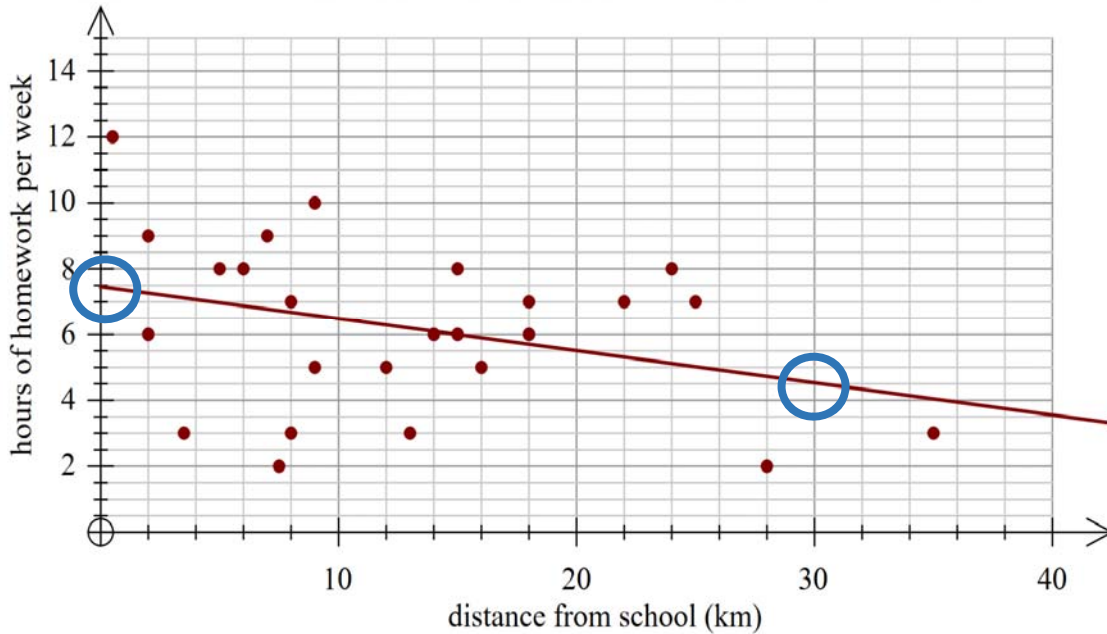
**Tip**

- *Ensure that you use comparative language for this type of question. Use words such as 'differ', 'increase', 'decrease' and 'change'.*

Question 4a.**Worked solution**

Below is the scatterplot with line sketched in.

Two points have been circled, as a guide to ensure accuracy of the line.

**Mark allocation: 2 marks**

- 1 mark for line passing through correct y-intercept
- 1 mark for line passing through another accurate point, for example, the point at (30, 4.5) is circled above

**Tip**

- *When plotting a straight line, it can be helpful to identify two points on the line, and plot these before joining these points. This can be done by substituting values of x into the equation.*

Question 4b.**Worked solution**

Moderate, negative, linear

Mark allocation: 1 mark

- 1 mark for all of the above correct
- Note:** Weak, negative, linear is also acceptable.

Question 4c.**Worked solution**

On average, as the *distance from school* increases by one kilometre, the *hours of homework per week* completed by a student will decrease by 0.0977 hours.

Mark allocation: 1 mark

- 1 mark for correct interpretation of slope, including units for variables and a statement that as x increases, y decreases

Question 4d.**Worked solution**

The coefficient of determination is the r^2 value. We can find the square root of this to find the Pearson's correlation coefficient, r .

$$r = \sqrt{0.1117} = 0.3342 \text{ or } -0.3342$$

This relationship is negative and we must therefore select the negative value of r .

Pearson's correlation coefficient, r , is equal to -0.3342 .

Mark allocation: 1 mark

- 1 mark for correct answer, including the negative

**Tip**

- *Be careful when calculating the r -value from r^2 . Generally, calculators will always give the positive answer. You should always check the slope of the graph or equation to ensure that the r -value is correctly stated as negative or positive.*

Question 4e.**Worked solution**

residual = actual – predicted

residual = $3 - (7.46 - 0.0977 \times 8)$

residual = -3.68

Mark allocation: 1 mark

- 1 mark for correct answer

Note: The answer must be a negative.

Question 4f.i.**Worked solution**

hours of homework per week = $7.46 - 0.0977 \times \text{distance from school}$

hours of homework per week = $7.46 - 0.0977 \times 11$

hours of homework per week = 6.3853

hours of homework per week = 6.4

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4f.ii.**Worked solution**

This prediction is interpolation, within the given data range.

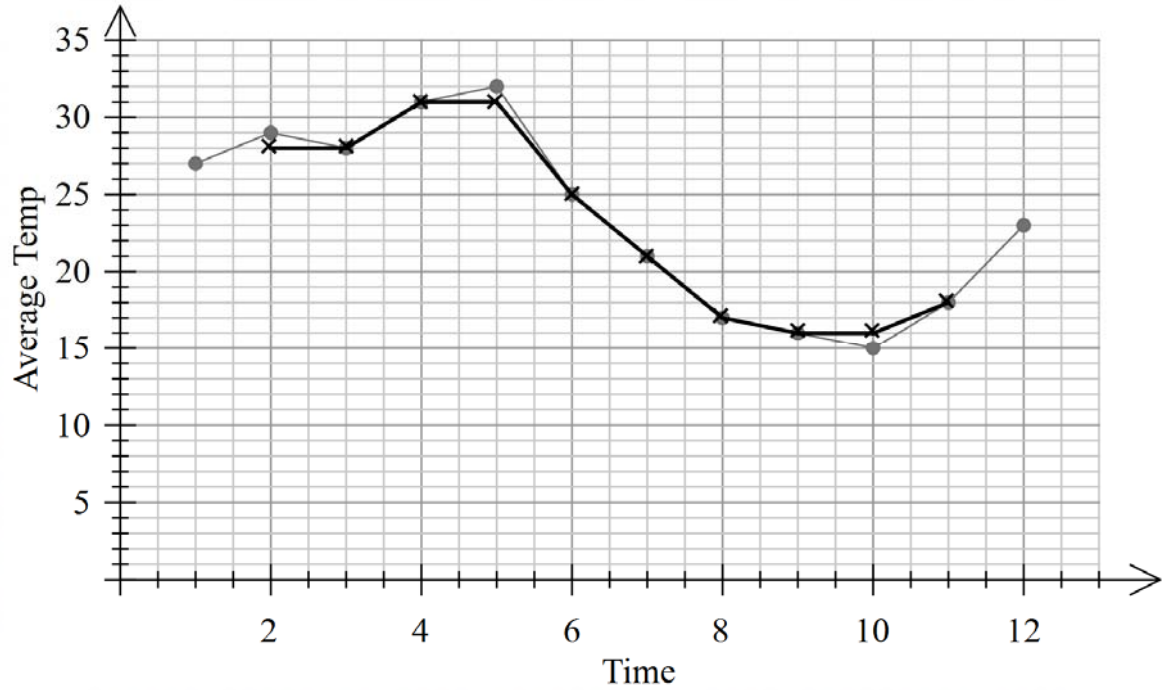
Mark allocation: 1 mark

- 1 mark for statement that the data is within the given range

Question 5a.**Worked solution**

Three-median smoothing requires finding the median of three values, written in order.

In this case: 15 **18** 23



Mark allocation: 1 mark

- 1 mark for correct plot of point (11, 18) and line joining point

Question 5b.**Worked solution**

$$\text{deseasonalised} = \frac{\text{actual}}{\text{seasonal index}}$$

$$\text{deseasonalised} = \frac{16}{0.68}$$

$$\text{deseasonalised} = 23.5$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5c.**Worked solution**

$$\text{deseasonalised temperature} = 23.71 + 0.457 \times \text{month}$$

$$\text{deseasonalised temperature} = 23.71 + 0.457 \times 13$$

$$\text{deseasonalised temperature} = 29.651$$

$$\text{predicted temperature} = 29.651 \times 1.13$$

$$\text{predicted temperature} = 33.5^\circ$$

$$\text{predicted temperature} = 34^\circ$$

Mark allocation: 2 marks

- 2 marks for correct answer of 34

Note: If incorrect, 1 mark can be given for a correct substitution of 13 into the equation OR for a multiplication of the answer by 1.13.

**Tip**

- *Always remember that deseasonalised data has been altered to remove the influence of the seasons to identify a trend or make predictions. The actual value will always require the seasonal index, or influence of the season, to be taken into consideration.*

Recursion and financial modelling

Question 6a.

Worked solution

For this question, you must show calculation to show that the value is \$24 960.

Substituting 20 000 into the rule:

$$V_n = 24\,990 - 0.0015n$$

$$V_n = 24\,990 - 0.0015 \times 20\,000$$

$$V_n = 24\,960$$

Mark allocation: 2 marks

- 1 mark for a substitution into the rule
- 1 mark for a result of \$24 960



Tips

- A 'show that' question will always require working to be shown.
- As a general rule, if the question is worth 2 marks, you are required to show two pieces of working. You will not receive marks for only an answer.

Question 6b.

Worked solution

The unit cost depreciation is 0.0015 according to the rule.

This is written in dollars, and so this must be multiplied by 100 to convert to cents.

$$0.0015 \times 100 = 0.15 \text{ cents}$$

Mark allocation: 1 mark

- 1 mark for correct answer, written in cents

Question 6c.

Worked solution

Find the total depreciation over 3 years:

$$24\,990 - 23\,191 = 1799$$

Divide this amount by 3 to find the depreciation in one year:

$$\frac{1799}{3} = 599.67$$

Calculate this amount as a percentage of the original amount:

$$\frac{599.67}{24\,990} \times 100 = 2.39964\%$$

$$\therefore \approx 2.4\%$$

Mark allocation: 1 mark

- 1 mark for correct answer

Note: An unrounded answer will *not* be accepted.

Question 6d.**Worked solution**

The general relation for reducing balance depreciation is:

$$V_0 = \text{initial value, } V_{n+1} = R \times V_n, \text{ where } R = 1 - \frac{r}{100}$$

In this question we have been given the following:

$$V_0 = 24\,990$$

$$r = 5.8\% \text{ p.a.}$$

This means that we can work out R .

$$R = 1 - \frac{r}{100}$$

$$R = 1 - \frac{5.8}{100}$$

$$R = 0.942$$

Therefore, our recurrence relation is:

$$A_0 = 24\,990, A_{n+1} = 0.942 \times A_n$$

Mark allocation: 2 marks

- 1 mark for accurate form of relation, including A_0
- 1 mark for correct value of R included in relation

Question 7a.**Worked solution**

We need to calculate 6 iterations of our recurrence relation to find the value of Jonathan's loan.

Working using the CAS calculator is shown below.

15000	15000
$15000 \cdot 1.0022 - 200$	14833.
$14833. \cdot 1.0022 - 200$	14665.6
$14665.6326 \cdot 1.0022 - 200$	14497.9
$14497.89699172 \cdot 1.0022 - 200$	14329.8
$14329.792365102 \cdot 1.0022 - 200$	14161.3
$14161.317908305 \cdot 1.0022 - 200$	13992.5

The value of the loan after 6 months is \$13 992.47.

Note that the calculator will often round this value and we need to check what the unrounded value is. This can be avoided by ensuring that your float setting on the calculator does not restrict the answer.

Mark allocation: 1 mark

- 1 mark for correct answer of \$13 992.47, dollar sign not required

Question 7b.**Worked solution**

We will need to use the Finance solver in the CAS calculator to find Jonathan's monthly payment.

Variable	Value
N:	84
I(%):	4.5
PV:	15000
Pmt:	-208.50241948681
FV:	0.
PpY:	12

Finance Solver info stored into tvn.n, tvn.i, tvn.pv, tvn.pmt, ...

Jonathan's monthly repayment will be \$208.50.

Mark allocation: 1 mark

- 1 mark for correct answer

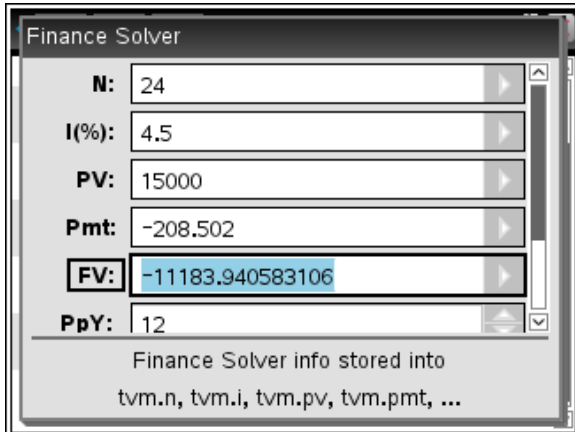
Note: A dollar sign is not required; negative answers are not accepted; unrounded answers are accepted.

**Tip**

- Remember, when using the Finance solver on your calculator:
 - money coming to you is a positive value
 - money going to the bank is a negative value.

Question 7c.**Worked solution**

First, we must find the value of Jonathan's loan after 2 years:



Jonathan still owes \$11 183.94 on his loan.

We must now find how much Jonathan has paid off his loan in the first 2 years:

$$15\,000 - 11\,183.94 = 3816.06$$

Next, we calculate how much money Jonathan has put into his loan:

$$208.50 \times 24 = 5004$$

The interest that Jonathan has paid is the difference between the amount of money he has contributed, and the amount that he has paid off the principal of his loan.

$$5004 - 3816.06 = 1187.94$$

Jonathan has paid \$1187.94 interest in the first 2 years.

Mark allocation: 2 marks

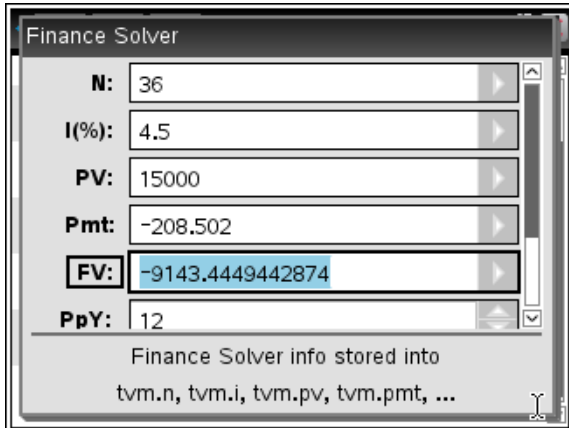
- 2 marks for correct answer

Note: 2 marks can be awarded if the answer to Question 7b. was incorrect but has been used correctly in all parts of this question. This is a consequential mark, so working must be shown.

Note: If the answer is incorrect, 1 mark may be given for a subtraction of any value from 5004 and 1 mark can be given for a correct calculation of the value after 2 years.

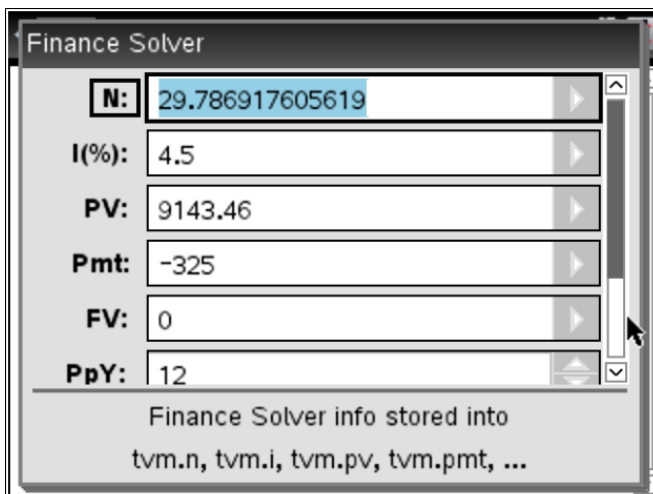
Question 7d.**Worked solution**

We must first find the value of Jonathan's loan after 3 years at the original conditions.



Jonathan still owes \$9143.44 after 3 years.

This now becomes our new principal. We also alter our pmt amount and FV becomes 0, as below:



We can see that it will take Jonathan a further 30 months (rounding up) to pay off his loan.

With the original arrangement, Jonathan would have taken another 4 years, or 48 months, to pay off his loan.

With the new payment amount, he will save 18 months on his loan.

Mark allocation: 2 marks

- 2 marks for an answer of 18, with no working required

Note: If incorrect, 1 mark can be given for a correct FV value after 3 years, if shown and clearly defined.

Module 1 – Matrices

Question 1a.

Worked solution

254 memberships were taken out in February.

We add the numbers across row 2: $107 + 95 + 52 = 254$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 1b.

Worked solution

The number of occasional memberships taken at the gym in February was 52.

Mark allocation: 1 mark

- 1 mark for correctly identifying the element

Question 1c.

Worked solution

Multiply row 2 of matrix G by the column matrix.

$$107a + 95b + 52c = 19\ 474$$

Mark allocation: 1 mark

- 1 mark for correct equation, as above

Question 1d.

Worked solution

$$\begin{bmatrix} 125 & 76 & 35 \\ 107 & 95 & 52 \\ 95 & 120 & 43 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 19\ 045 \\ 19\ 474 \\ 19\ 831 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 125 & 76 & 35 \\ 107 & 95 & 52 \\ 95 & 120 & 43 \end{bmatrix}^{-1} \times \begin{bmatrix} 19\ 045 \\ 19\ 474 \\ 19\ 831 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 95 \\ 75 \\ 42 \end{bmatrix}$$

An occasional membership is \$42 per month.

Mark allocation: 1 mark

- 1 mark for correct answer of 42

Question 2a.**Worked solution**

72% of members who chose the aerobics class in the first week will choose it again the next week. This means that $100\% - 72\% = 28\%$ will not choose the aerobics class the following week.

Mark allocation: 1 mark

- 1 mark for correct answer of 28

Note: A percentage sign is not required.

Question 2b.**Worked solution**

23% of members choosing *C* will change to $W = 0.23 \times 51 = 11.73 \approx 12$

9% of members choosing *A* will change to $W = 0.09 \times 73 = 6.57 \approx 7$

65% of members choosing *W* will continue with $W = 0.65 \times 101 = 65.65 \approx 66$

In total, 85 members will choose the weights class in the second week.

Mark allocation: 1 mark

- 1 mark for correct answer of 85

Question 2c.**Worked solution**

55% of members will continue with the cycle class from one week to next.

From week one to week two: $0.55 \times 51 = 28.05 \approx 28$

From week two to week three: $0.55 \times 28 = 15.4 \approx 15$

Thus, 15 of the original members who started the cycle class will remain in the cycle class during the third week

Mark allocation: 1 mark

- 1 mark for correct answer of 15

Question 3a.**Worked solution**

We must consider the people who change from this year to next, for each fitness centre.

$$\begin{aligned} & \text{change from } F \quad + \quad \text{change from } H \quad + \quad \text{change from } S \\ & (0.18 \times 120 + 0.32 \times 120) + (0.34 \times 120 + 0.22 \times 120) + (0.12 \times 120 + 0.11 \times 120) \\ & = 154.8 \approx 155 \end{aligned}$$

Mark allocation: 1 mark

- 1 mark for correct answer of 155

Question 3b.**Worked solution**

$$M_3 = \begin{bmatrix} 91 \\ 62 \\ 146 \\ 60 \end{bmatrix} \begin{matrix} F \\ H \\ S \\ L \end{matrix}$$

Mark allocation: 1 mark

- 1 mark for correct matrix M_3 , with all elements correct

**Tips**

- You need to check the initial state matrix very carefully.
 - As you have been given M_1 , you only require 2 transitions to get M_3 .
You can use the equation $M_n = T^{n-1} \times M_1$
 - If you had been given M_0 , you would need to do 3 transitions. You can use the equation $M_n = T^n \times M_0$

Question 3c.**Worked solution**

The year 2022 is year 5. Therefore, we are finding M_5 .

$$\begin{bmatrix} 0.44 & 0.34 & 0.12 & 0 \\ 0.18 & 0.35 & 0.11 & 0 \\ 0.32 & 0.22 & 0.66 & 0 \\ 0.06 & 0.09 & 0.11 & 1 \end{bmatrix}^4 \cdot \begin{bmatrix} 120 \\ 120 \\ 120 \\ 0 \end{bmatrix} = \begin{bmatrix} 69.809 \\ 48.4909 \\ 129.16 \\ 112.54 \end{bmatrix}$$

129 people will choose Smash It in 2022.

Mark allocation: 1 mark

- 1 mark for correct answer of 129

Question 3d.**Worked solution**

The year 2025 is year 8. Therefore, we are finding M_8 .

$$\begin{bmatrix} 0.44 & 0.34 & 0.12 & 0 \\ 0.18 & 0.35 & 0.11 & 0 \\ 0.32 & 0.22 & 0.66 & 0 \\ 0.06 & 0.09 & 0.11 & 1 \end{bmatrix}^7 \cdot \begin{bmatrix} 120 \\ 120 \\ 120 \\ 0 \end{bmatrix} = \begin{bmatrix} 51.3211 \\ 35.9101 \\ 97.9473 \\ 174.822 \end{bmatrix}$$

There will be 36 members at the Healthy Life (H) fitness centre in 2025.

$$\frac{36}{120} \times 100 = 30\%$$

The manager is not correct, the membership has not fallen below 25% of its original number.

Mark allocation: 2 marks

- 1 mark for a statement that the manager is incorrect
- 1 mark for a calculation showing that the number is 30% of the original

Module 2 – Networks and decision mathematics

Question 1a.

Worked solution

C – B – A – D – H – G – F – E – C Total distance = 5250 metres

C – G – H – D – A – B – F – E – C Total distance = 6925 metres

Other possible answers will exist. To correct this, check that:

- all vertices have been included exactly once in your circuit, aside from C at the start and end
- you have added the distances on all roads used in the circuit.

Mark allocation: 2 marks

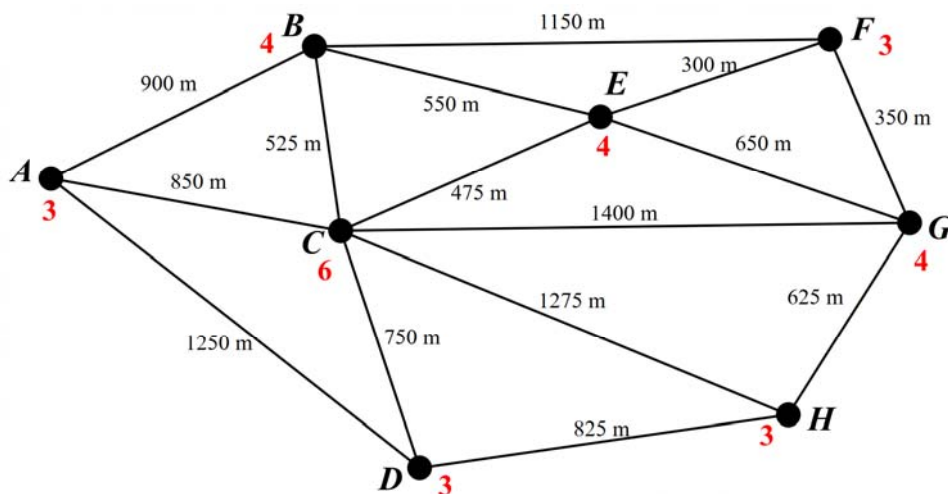
- 1 mark for a correct circuit
- 1 mark for corresponding total

Question 1b.

Worked solution

An Euler trail is not possible because there are more than 2 vertices with an odd degree.

See the diagram below for degree of each vertex.



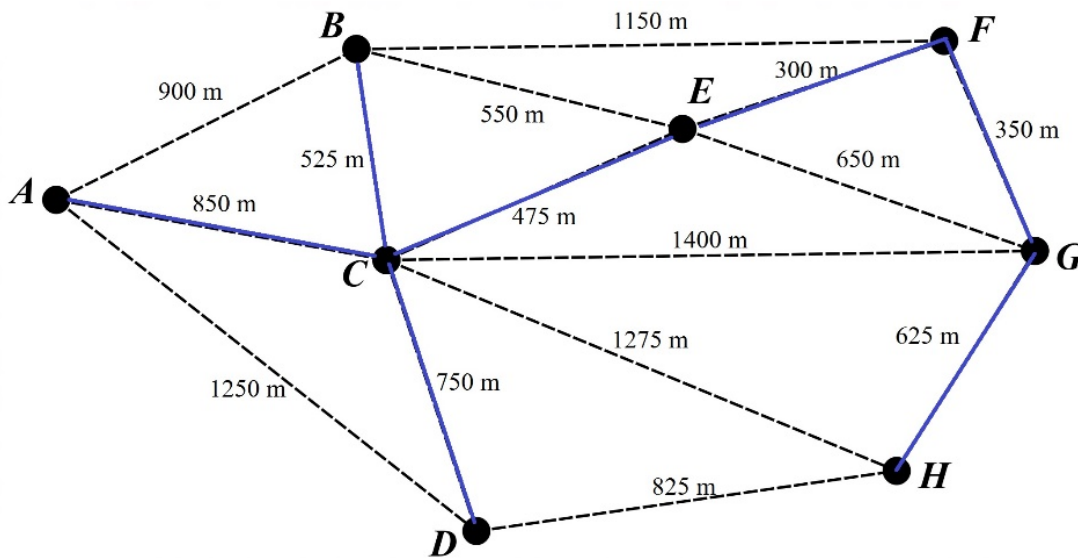
Mark allocation: 1 mark

- 1 mark for similar statement to above



Tip

- Remember that an Euler trail is only possible if there are two odd vertices in a network, where all other vertices have an even degree. Similarly, an Eulerian circuit is possible if all vertices have an even degree.

Question 1c.i.**Worked solution****Mark allocation: 1 mark**

- 1 mark for correct minimum spanning tree

Question 1c.ii.**Worked solution**

$$525 + 850 + 475 + 750 + 350 + 625 + 300 = 3875 \text{ metres}$$

Mark allocation: 1 mark

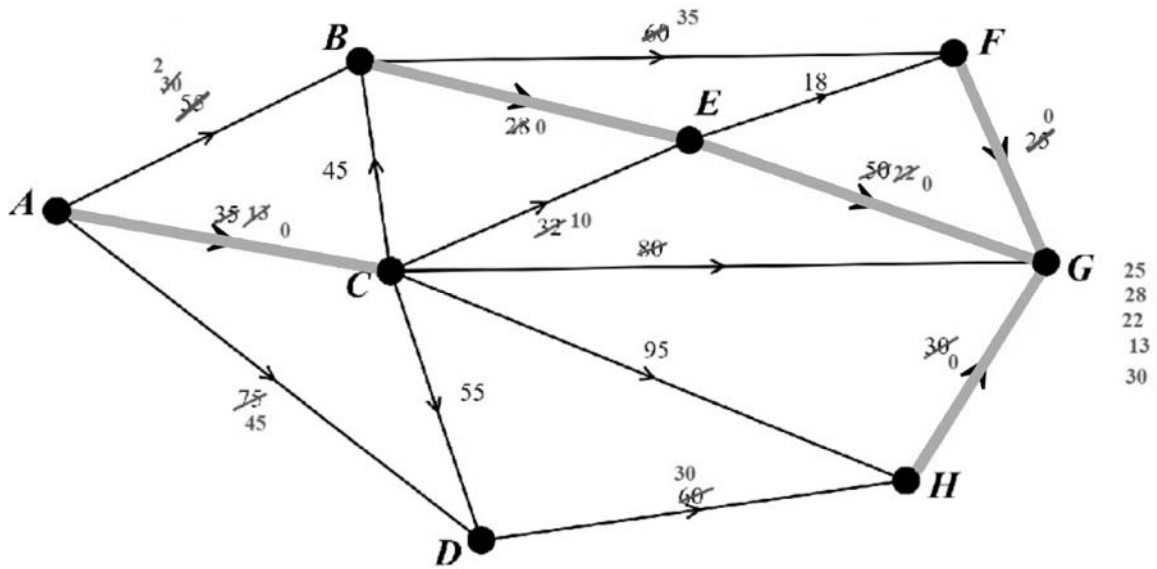
- 1 mark for correct length

Note: This is an answer that results from your answer to Question 1c.i. If your answer to this question is incorrect, you will need to check that your minimum spanning tree is correct.

Question 1d.**Worked solution**

118 children.

Below is an example of using the plotting method. Other methods for reaching the final answer exist.



Mark allocation: 1 mark

- 1 mark for correct answer of 118

Question 2a.**Worked solution**

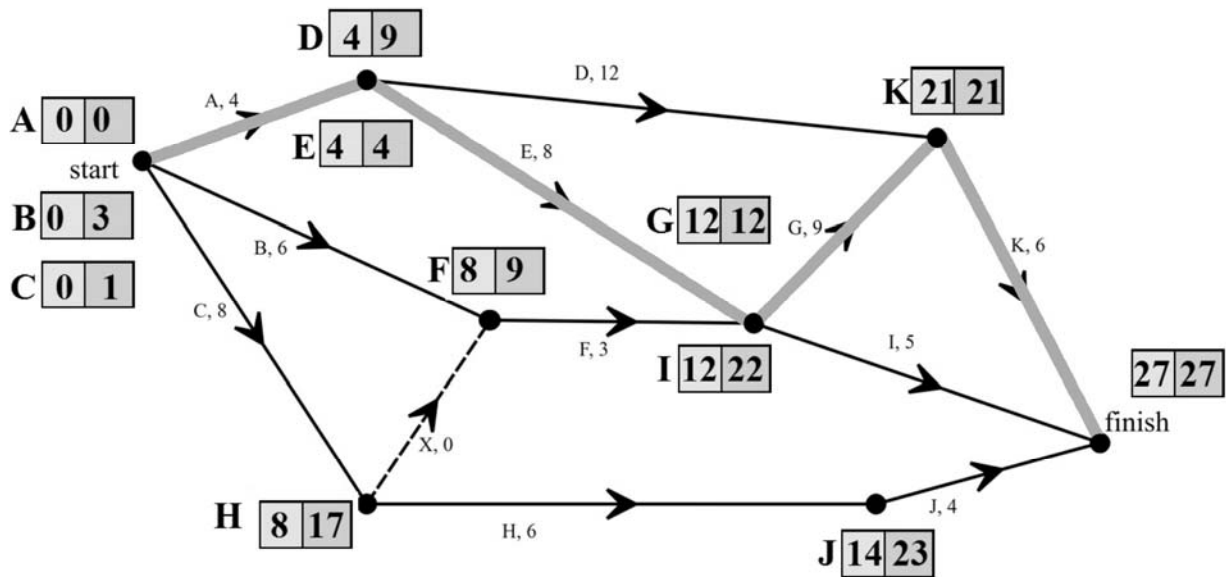
A, B, C, E and F are all predecessors of activity G.

Mark allocation: 1 mark

- 1 mark for correct list of activities

Question 2b.**Worked solution**

Minimum completion time is 27 days. Critical path is A – E – G – K



Mark allocation: 2 marks

- 1 mark for correct critical path
- 1 mark for correct minimum completion time

Question 2c.**Worked solution**

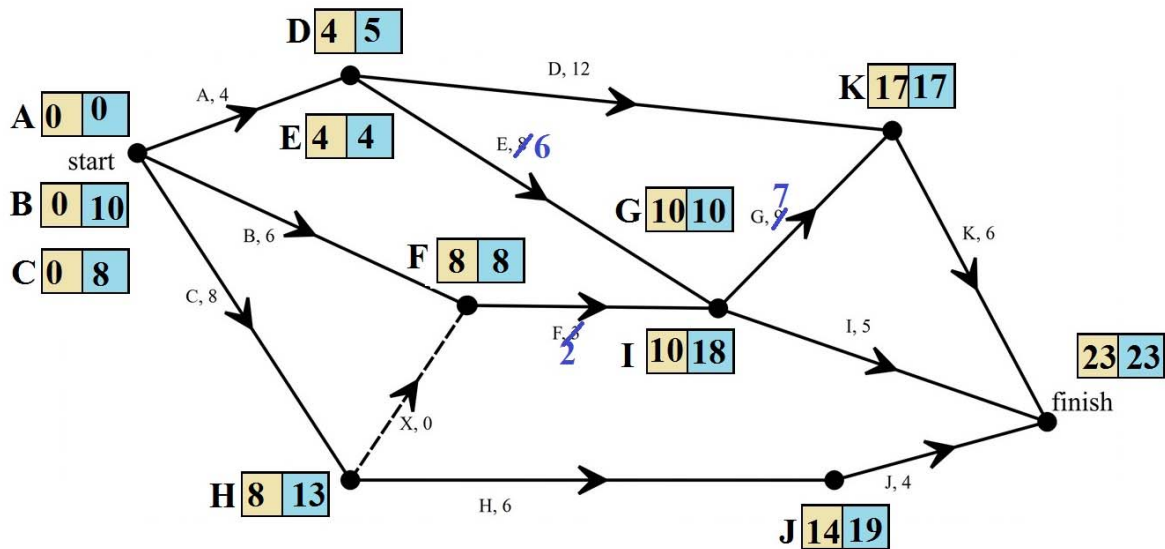
Float time = LST – EST = 17 – 8 = 9

Mark allocation: 1 mark

- 1 mark for correct answer of 9

Question 2d.i.**Worked solution**

We start by crashing the activities that are on the critical path, *E* and *G*.



We also need to crash activity *F* by 1 day to ensure that this does not create a new critical path.

The minimum completion time is now 23 days.

Mark allocation: 1 mark

- 1 mark for correct answer of 23

Question 2d.ii.**Worked solution**

The cost is calculated by:

Cutting *E* by 2 days = \$4000

Cutting *G* by 2 days = \$4000

Cutting *F* by 1 day = \$2000

Total cost is $4000 + 4000 + 2000 = \$10\,000$

Mark allocation: 1 mark

- 1 mark for correct answer of \$10 000

**Tip**

- *When crashing a project, start by considering activities that are on the critical path. As this is the largest path through the network, this is the way to reduce the overall completion time. Once this has been done, look at activities that are not on the critical path and see if these will influence completion time.*

Module 3 – Geometry and measurement

Question 1a.

Worked solution

We are finding the perimeter here.

P = circumference of circle + perimeter of rectangle

$$P = 2\pi r + 2l + 2w$$

$$P = 2 \times \pi \times (3 - 1.5) + 2 \times 7.5 + 2 \times 3$$

$$P = 30.4248$$

$$P \approx 30 \text{ m}$$

Mark allocation: 1 mark

- 1 mark for correct answer of 30

Note: Units are not required.

Question 1b.

Worked solution

A = area of equipment + area of seating

A = area of trapezium + area of trapezium

$$A = \frac{1}{2}(a + b)h + \frac{1}{2}(a + b)h$$

$$A = \frac{1}{2} \times (4.5 + 7.5) \times 2.8 + \frac{1}{2} \times (3 + 6.4) \times 4.5$$

$$A = 16.8 + 21.15$$

$$A = 37.95$$

$$A \approx 38 \text{ m}^2$$

Mark allocation: 1 mark

- 1 mark for correct answer of 38

Question 1c.

Worked solution

We are trying to find the volume of the sandpit.

$$V = \pi r^2 h$$

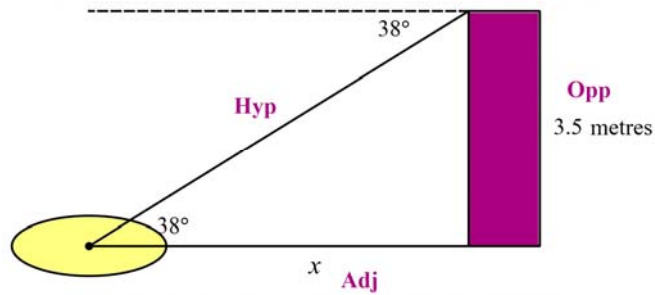
$$V = \pi \times 1.5^2 \times 1.3$$

$$V = 9.189$$

$$V \approx 9 \text{ m}^3$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 1d.**Worked solution**

To find length x , we need to use SOH CAH TOA.

$$\tan \theta = \frac{O}{A}$$

$$\tan 38 = \frac{3.5}{x}$$

$$x = \frac{3.5}{\tan 38}$$

$$x = 4.4798$$

$$x \approx 4.5 \text{ m}$$

Mark allocation: 1 mark

- 1 mark for correct answer 4.5

**Tip**

- *When a diagram is not provided in the question, drawing one can always be helpful to be clear on which formula is appropriate to use.*

Question 2a.**Worked solution**

The angles in a triangle will equal 180° .

$$\text{Angle } a = 180 - 36 - 86 = 58^\circ$$

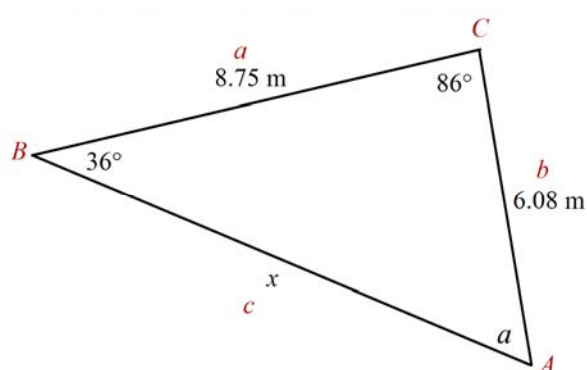
Mark allocation: 1 mark

- 1 mark for correct answer of 58

Question 2b.**Worked solution**

We can use the sine rule to find the unknown length.

Start by labelling the sides and angles.



$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 86} = \frac{6.08}{\sin 36}$$

$$x = \frac{6.08}{\sin 36} \times \sin 86$$

$$x = 10.3187$$

$$x \approx 10 \text{ m}$$

Mark allocation: 1 mark

- 1 mark for correct answer of 10 m

Note: Units are not required.

Question 2c.**Worked solution**

$$A = \frac{1}{2}ab\sin C$$

$$A = \frac{1}{2} \times 8.75 \times 6.08 \times \sin 86$$

$$A = 26.5352$$

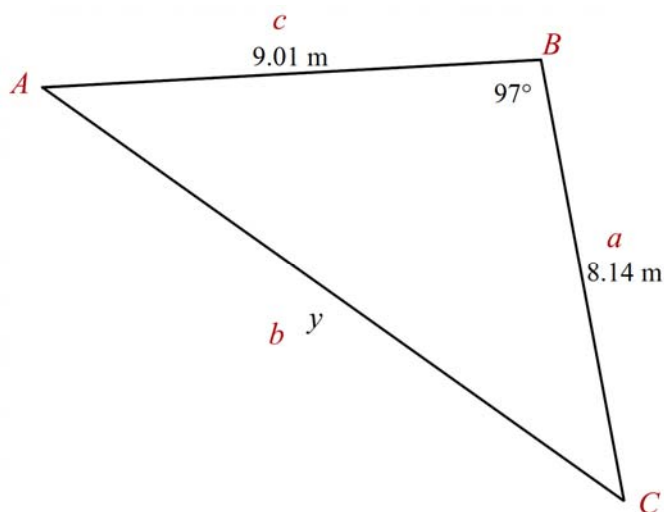
$$A \approx 27 \text{ m}^2$$

Mark allocation: 1 mark

- 1 mark for correct answer of 27

Question 2d.**Worked solution**

Start by labelling the sides and angles of the triangle to identify what we don't know.



For this, we are trying to find a side length, and so we will use

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = 8.14^2 + 9.01^2 - 2 \times 8.14 \times 9.01 \times \cos 97$$

$$b = \sqrt{8.14^2 + 9.01^2 - 2 \times 8.14 \times 9.01 \times \cos 97}$$

$$b = 12.8575$$

$$b \approx 12.9 \text{ m, as required}$$

Mark allocation: 2 marks

- 1 mark for substitution into the correct form of the cosine rule
- 1 mark for a calculation that results in 12.9

Question 3a.**Worked solution**

There is a 3-hour time difference, as Mildura is further east than Beijing; we must subtract this time difference.

Therefore, it will be 6:30 am on Tuesday in Beijing.

Mark allocation: 1 mark

- 1 mark for correct day and time

Question 3b.**Worked solution**

$$\frac{40}{180} \times \pi \times 6400 = 4468.04$$

$$\approx 4468 \text{ km}$$

Mark allocation: 1 mark

- 1 mark for correct answer

Note: If the answer is incorrect, a working mark may be given for:

- correct formula used, even if incorrect values used
- correct answer, but not rounded.

Module 4 – Graphs and relations

Question 1a.

Worked solution

100 km

Mark allocation: 1 mark

- 1 mark for correct answer

Question 1b.

Worked solution

The family was stationary for 2 hours when they were 100 km from home.

Mark allocation: 1 mark

- 1 mark for description above, or similar

Question 1c.

Worked solution

Identify two points: (5, 100) and (6, 50)

Use the formula, $y - y_1 = m(x - x_1)$ to find the equation of the line.

$$y - 100 = \left(\frac{50 - 100}{6 - 5} \right) \times (x - 5)$$

$$y - 100 = \frac{-50}{1}(x - 5)$$

$$y - 100 = -50(x - 5)$$

$$y - 100 = -50x + 250$$

$$y = -50x + 350$$

Mark allocation: 1 mark

- 1 mark for correct equation

Question 1d.**Worked solution**

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 100}{9 - 8} = \frac{-30}{1} = -30$$

Mark allocation: 1 mark

- 1 mark for clear indication of gradient = $\frac{\text{rise}}{\text{run}}$ and a calculation that results in -30

**Tip**

- A 'show that' question requires you to clearly state the formula or process that you are using. You must also show any substitution into this formula, as well as the calculation that results in the answer required.

Question 2a.**Worked solution**

The mother can drive less than or equal to one third of the time the father can drive.

Mark allocation: 1 mark

- 1 mark for above statement, or similar

Question 2b.**Worked solution**

Substitute $y=8$ into Constraint 1.

$$x \leq \frac{y}{3}$$

$$x \leq \frac{8}{3}$$

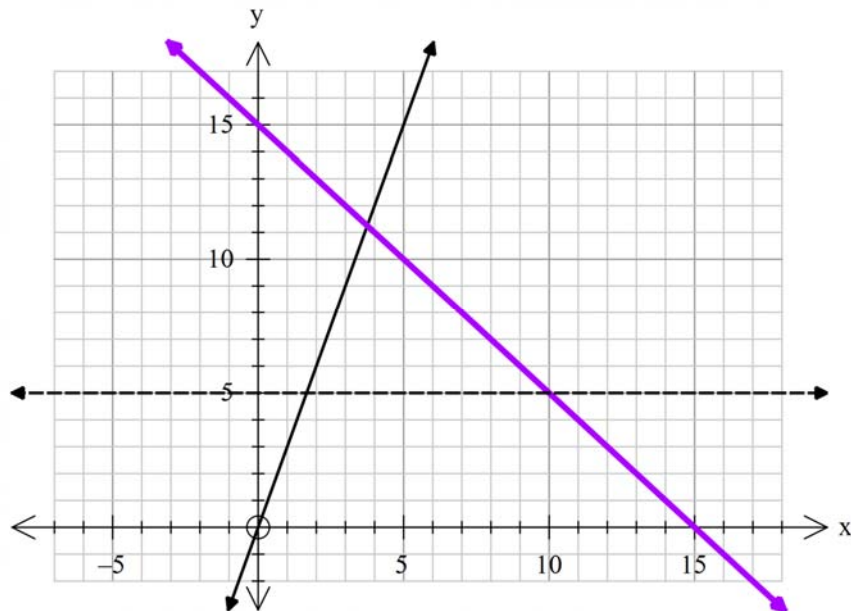
$$x \leq 2.666667$$

$x \leq 2$ hours and 40 minutes

As $\frac{2}{3} = 0.66667$, we can see that this is two-thirds of an hour. Therefore, 40 minutes.

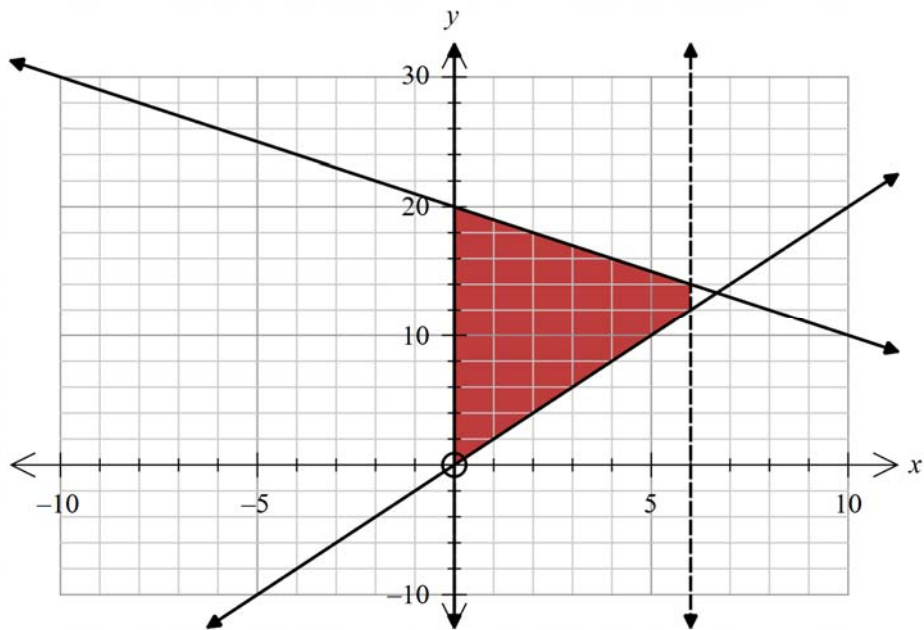
Mark allocation: 1 mark

- 1 mark for correct answer

Question 2c.**Worked solution****Mark allocation: 2 marks**

- 2 marks for accurate graph

Note: If incorrect, 1 mark can be awarded for correct y-intercept or for accurate intersection with Constraint 1.

Question 2d.**Worked solution****Mark allocation: 2 marks**

- 2 marks for correct shaded area.

Note: If incorrect, 1 mark can be awarded for an attempt to shade any region that is in a region where both x and y are greater than 0.

Question 2e.**Worked solution**

The maximum point can be found at $(6, 14)$.

This means that the son does 6 hours of driving, and the mother and father do 14 hours between them.

Therefore, the maximum amount of driving in one day is 20 hours.

This can also occur at $(0, 20)$, however this would mean that the son would do no driving.

Mark allocation: 2 marks

- 2 marks for an answer of 20

Note: If incorrect, 1 mark can be awarded for an identification that the maximum occurs at $(6, 14)$ or $(0, 20)$.

END OF WORKED SOLUTIONS