

YEAR 12 Trial Exam Paper

2017

FURTHER MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions
- mark allocations
- tips on how to approach the exam

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SECTION A – Core

Data analysis

Question 1a.

Worked solution

The data is discrete numerical.

In this case, the data has been counted as whole numbers, and is therefore discrete.

Mark allocation: 1 mark

- 1 mark for ‘discrete numerical’; ‘numerical’ alone not accepted

Question 1b.

Worked solution

$$\frac{17}{44} \times 100 = 38.6363\dots = 38.6\%$$

Mark allocation: 1 mark

- 1 mark for correct answer, rounded accurately to one decimal place

Question 1c.

Worked solution

The median is 17.5.

The stem plot can be used to find the median value, by counting to the middle.

There are 44 data values, and therefore the median lies between 17 (the 22nd data value) and 18 (the 23rd data values).

Alternatively, the data can be entered into your CAS and a statistical summary obtained.

Statistic	Value
"X"	17.1136
"ΣX"	753.
"ΣX ² "	16743.
"sX := s _{n-1} X"	9.47019
"σX := σ _n X"	9.36195
"n"	44.
"MinX"	3.
"Q ₁ X"	9.
"MedianX"	17.5
"Q ₃ X"	25.
"MaxX"	45.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 1d.**Worked solution**

Step 1: Find the interquartile range

$$IQR = Q_3 - Q_1 = 25 - 9 = 16$$

Step 2: Use the IQR and Q_3 to find the upper fence

$$\text{upper fence} = Q_3 + 1.5 \times IQR = 25 + 1.5 \times 16 = 49$$

Step 3: Clearly answer the question

$$45 < 49$$

\therefore it is not an outlier.

The statistician is **not correct**.

Mark allocation: 2 marks

- 1 mark for correct calculation showing that upper fence is 49
- 1 mark for a clear statement that the value of 45 is not an outlier

Question 2a.**Worked solution**

The boxplot is approximately symmetric, with a median at 35 and an IQR of 20.

Mark allocation: 1 mark

- 1 mark for identifying the boxplot as approximately symmetric and stating the numerical values for the median and IQR
- Note: if the median is given in conjunction with the range (48), it can be accepted.

Question 2b.**Worked solution**

The median would be the most suitable measure of centre. This is the case when data is either skewed or has an outlier.

Mark allocation: 2 marks

- 1 mark for stating median
- 1 mark for explaining that the data is skewed

Question 2c.**Worked solution**

Yes, there is a correlation between *age group* and *hours worked*.

As the age of the person surveyed increases, from 18–24 to 25–30, we can see that the median number of hours worked also increases, from 17.5 hours for 18–24 to 35 hours for 25–30.

OR

As the age of the person surveyed increases, from 18–24 to 25–30, we can see that spread of hours worked, as shown by the IQR, increases from 16 to 20 hours.

Mark allocation: 2 marks

- 1 mark for clearly stating that a correlation (relationship) exists
- 1 mark for using either median or IQR to support explanation

Question 3a.**Worked solution**

Moderate, positive, linear relationship

Mark allocation: 1 mark

- 1 mark for correct answer – all three aspects correct

Question 3b.**Worked solution**

$$r = 0.718274\dots = 0.72$$

Use your CAS calculator to obtain a statistical summary for the linear regression between *hourly pay rate* and *hours worked*.

Pearson's correlation coefficient, r , will be a part of this summary.

Mark allocation: 1 mark

- 1 mark for stating the value of r , correct rounding included

Question 3c.**Worked solution**

The equation obtained when linear regression is performed between *hourly pay rate* and *hours worked* is:

$$\text{hours worked} = -4.857157 + 1.004465 \times \text{hourly pay rate}$$

When rounding to two significant figures:

The intercept:

$$-4.857157 \approx -4.9$$

The slope:

$$1.004465 \approx 1.0$$

Therefore, the equation, rounded to two significant figures, is:

$$\text{hours worked} = -4.9 + 1.0 \times \text{hourly pay rate}$$

Mark allocation: 2 marks

- 2 marks for two variables, correct to two significant figures
- If this is not the case:
 - 1 mark can be awarded if **one coefficient** is correct, and rounded to two significant figures

OR

 - 1 mark can awarded if **both coefficients are correct**, but are not rounded correctly

Question 3d.**Worked solution**

On average, as the *hourly pay rate* increases by one dollar, the *number of hours worked* will increase by one hour.

Mark allocation: 1 mark

- 1 mark for correct interpretation as above (or equivalent statement)

**Tips**

- *If the slope is positive, you must make sure that you have stated that the response variable will **increase** by the slope.*
- *If the slope is negative, you must make sure that you have stated that the response variable will **decrease** by the slope.*

Question 3e.**Worked solution**

The coefficient of determination tells us that, on average, 51.59% of the variation in *hours worked* can be explained by the variation in *hourly pay rate*.

Mark allocation: 1 mark

- 1 mark for correct interpretation

Question 3f.i.**Worked solution**

Yes, the residual plot shows no clear pattern, and therefore confirms an assumption of linearity.

Mark allocation: 2 marks

- 1 mark for a clear indication that the assumption is correct
- 1 mark for an explanation specific to the residual plot; for example, random scattering or no pattern

Question 3f.ii.**Worked solution**

$$\text{residual} = \text{actual} - \text{predicted}$$

$$\text{residual} = 17 - (-4.9 + 1 \times 25)$$

$$\text{residual} = 17 - 20.1$$

$$\text{residual} = -3.1$$

Mark allocation: 1 mark

- 1 mark for correct answer of -3.1

Question 4a.**Worked solution**

A decreasing trend only

Mark allocation: 1 mark

- 1 mark for correct answer
- Note: a mark will not be awarded if additional information is provided, such as reference to seasonality or variation, with one exception – a mention of random variation is acceptable.

Question 4b.**Worked solution**

$$\text{sleep} = 299.6 - 12.7 \times \text{month}$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4c.**Worked solution**

This would be extrapolating, and would therefore not be a suitable prediction. Also, we must take into account that this data is for this student's final year of high school only. This pattern would not continue after exams have finished.

Mark allocation: 1 mark

- 1 mark for 'extrapolating' or an explanation as above

Question 4d.**Worked solution**

Step 1: Find the average for the 10-month period

$$\frac{282 + 252 + 265 + 257 + 262 + 224 + 218 + 198 + 172 + 165}{10} = 229.5$$

Step 2: Find the seasonal index, by dividing the actual figure for July by the average

$$\frac{218}{229.5} = 0.949891 = 0.95$$

The seasonal index for July is 0.95, as required.

Mark allocation: 2 marks

- 1 mark for showing calculation of average
- 1 mark for use of formula: $\text{seasonal index} = \frac{\text{actual}}{\text{average}}$

Recursion and financial modelling

Question 5a.

Worked solution

The initial amount comes from V_0 in the recurrence relation.

This is therefore \$6000.

Mark allocation: 1 mark

- 1 mark for correct answer; dollar sign not required

Question 5b.

Worked solution

The annual interest is \$216.

As a percentage of the initial amount this is:

$$\frac{216}{6000} \times 100 = 3.6\% \text{ p.a.}$$

Mark allocation: 2 marks

- 2 marks for correct answer of 3.6%
- Note: 1 working mark can be given for stating annual interest or attempting to calculate a percentage.

Question 5c.i.

Worked solution

The first box is V_0 and is therefore 6000.

The second box is the annual interest, and is therefore 216.

Mark allocation: 1 mark

- 1 mark for both values correct

Question 5c.ii.**Worked solution**

$$V_5 = 6000 + 216 \times 5$$

$$V_5 = \$7080$$

Mark allocation: 1 mark

- 1 mark for correct answer; dollar sign not required

Question 5d.**Worked solution**

$$V_0 = 6000$$

$$V_1 = 1.054 \times V_0 = 1.054 \times 6000 = 6324$$

$$V_2 = 1.054 \times V_1 = 1.054 \times 6234 = 6665.496 = 6665.50$$

Mark allocation: 1 mark

- 1 mark for showing two calculations, the second of which results in \$6665.50

Question 6a.**Worked solution**

Annual depreciation is:

$$\frac{37000 - 29600}{5} = 1480$$

Recurrence relation is: $C_0 = 37000$, $C_{n+1} = C_n - 1480$

Mark allocation: 2 marks

- 2 marks for correct recurrence relation, including clear indication of $C_0 = 37000$
- Note: if relation is not correct, 1 mark may be given for an accurate calculation of the annual depreciation of \$1480.

Question 6b.**Worked solution**

The general recurrence relation for unit cost depreciation is $V_n = V_0 - nD$.

We can use this general relation to find D , the cost per kilometre.

$$V_1 = V_0 - nD$$

$$(37000 - 1480) = 37000 - 4000 \times D$$

$$35520 = 37000 - 4000 \times D$$

Using solve on our CAS, we find that $D = 0.37$.

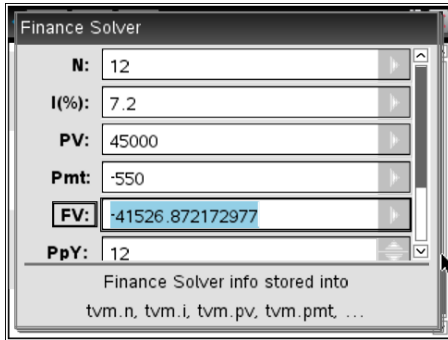
Therefore, the unit cost depreciation is 37 cents per kilometre.

Mark allocation: 1 mark

- 1 mark for correct answer of \$0.37 or 37 cents; in this case, a unit is required

Question 7a.**Worked solution**

Use the finance solver on your CAS calculator.



Katerina will still owe \$41 526.87 after 12 months.

Mark allocation: 1 mark

- 1 mark for correct answer

**Tip**

- *Note that there are no specific instructions here on rounding. When we are working with money, unless otherwise specified, give your answer to the nearest cent.*

Question 7b.**Worked solution**

First, we need to find the value of Katerina's loan after 4 years, with her monthly repayment of \$550.

The Finance Solver dialog box shows the following input values:

N:	48
I(%):	7.2
PV:	45000
Pmt:	-550
FV:	-29478.199049955
PpY:	12

Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

Katerina still owes \$29 478.20 after 4 years.

We must now calculate her payment, in order to repay this fully in a further 2 years.

The Finance Solver dialog box shows the following input values:

N:	24
I(%):	7.2
PV:	29478.2
Pmt:	-1322.4893538129
FV:	0
PpY:	12

Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

Katerina will need to make repayments of \$1 322.49 per month, in order to repay her loan in a further 2 years.

Mark allocation: 2 marks

- 2 marks for correct answer of \$1 322.49
- Note: if this is not achieved, 1 mark may be awarded for calculating the value of the loan after 4 years.

SECTION B – Modules

Module 1 – Matrices

Question 1a.

Worked solution

Matrix P has 4 rows and 1 column and therefore the order of P is 4×1 .

Mark allocation: 1 mark

- 1 mark for correct answer of 4×1

Question 1b.i.

Worked solution

$$[4.00 \quad 5.60 \quad 7.20 \quad 9.00] \times \begin{bmatrix} 250 \\ 135 \\ 180 \\ 65 \end{bmatrix} = [3637]$$

Mark allocation: 1 mark

- 1 mark for correct matrix

Question 1b.ii.

Worked solution

Matrix M represents the total amount that the postal company charged for parcels for the month of January in 2016.

Mark allocation: 1 mark

- 1 mark for correct statement as above or equivalent



Tip

- Note that this matrix is not showing the amount charged **for each type of parcel**; this is a combined total for **all parcels**. You should be clear on this in your explanation.

Question 1b.iii.

Worked solution

The matrix product $P \times C$ exists because the number of columns in matrix P is equal to the number of rows in matrix C .

Mark allocation: 1 mark

- 1 mark for correct statement as above

Question 2a.**Worked solution**

$$S_1 = TS_0 = \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix} = \begin{bmatrix} 152.5 \\ 132 \\ 133.75 \\ 181.75 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix} = \begin{bmatrix} 153 \\ 132 \\ 134 \\ 182 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

Therefore, $f = 132$

Mark allocation: 1 mark

- 1 mark for correct answer of 132

Question 2b.**Worked solution**

The answer for Cally, which is represented by g in the matrix S_1 , is found by multiplying row 3 in matrix T , with column 1 of matrix S_0 .

Therefore, we can find this answer through the following calculation.

$$0.13 \times 250 + 0.11 \times 125 + 0.62 \times 125 + 0.10 \times 100 = 133.75 = 134$$

Mark allocation: 1 mark

- 1 mark for correct calculation, in full, as above
- Note: a mark will not be awarded if $S_1 = TS_0$ is given as an answer.

Question 2c.i.**Worked solution**

We read this from the transition matrix, T .

0.68 customers who chose Dovan this month will choose it next month. Therefore, 68%.

Mark allocation: 1 mark

- 1 mark for 68% as an answer.
- Note: the proportion, 0.68, is not accepted.

Question 2c.ii.**Worked solution**

0.68 of customers will choose Dovan again in February; therefore, 0.32 will choose a different distribution centre.

In January 2016, 100 customers chose Dovan.

$$0.32 \times 100 = 32 \text{ customers choosing another distribution centre.}$$

Mark allocation: 1 mark

- 1 mark for 32

Question 2d.**Worked solution**

To find the long term, we must find the steady state matrix.

We find this by calculating two consecutive matrices with identical elements.

$$S_{50} = \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix}^{50} \times \begin{bmatrix} 250 \\ 125 \\ 125 \\ 100 \end{bmatrix} = \begin{bmatrix} 110.183 \\ 115.693 \\ 134.297 \\ 239.827 \end{bmatrix}$$

$$S_{51} = \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix}^{51} \times \begin{bmatrix} 250 \\ 125 \\ 125 \\ 100 \end{bmatrix} = \begin{bmatrix} 110.183 \\ 115.693 \\ 134.297 \\ 239.827 \end{bmatrix}$$

Therefore, in the long term, 110 customers will choose Abbey as their preferred distribution centre.

Mark allocation: 2 marks

- 1 mark for 110 customers
- 1 mark for showing two consecutive matrices, as a proof of steady state

Question 2e.**Worked solution**

$$F_{2017} = T \times J_{2017} + B$$

$$F_{2017} = \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix} \times \begin{bmatrix} 350 \\ 225 \\ 270 \\ 240 \end{bmatrix} + \begin{bmatrix} 45 \\ 65 \\ 70 \\ 40 \end{bmatrix}$$

$$F_{2017} = \begin{bmatrix} 287.1 \\ 297.1 \\ 331.65 \\ 389.15 \end{bmatrix}$$

$$M_{2017} = T \times F_{2017} + B$$

$$M_{2017} = \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix} \times \begin{bmatrix} 287.1 \\ 297.1 \\ 331.65 \\ 389.15 \end{bmatrix} + \begin{bmatrix} 45 \\ 65 \\ 70 \\ 40 \end{bmatrix}$$

$$M_{2017} = \begin{bmatrix} 294.752 \\ 345.042 \\ 384.542 \\ 500.665 \end{bmatrix}$$

In March 2017, 345 customers choose Burton as their distribution centre.

Mark allocation: 2 marks

- 2 marks for an answer of 345
- Note: if this is not achieved, 1 mark can be allocated for a correct calculation of the matrix for February 2017.

Module 2 – Networks and decision mathematics

Question 1a.

Worked solution

Possible paths (by inspection) from A to E :

$$\begin{aligned} A - F - E & \quad 1\,400 + 750 = 2\,150 \\ A - C - F - E & \quad 950 + 900 + 750 = 2\,600 \\ A - D - E & \quad 1\,300 + 900 = 2\,200 \\ A - B - D - E & \quad 1\,100 + 850 + 900 = 2\,850 \end{aligned}$$

Therefore, the shortest path is $A - F - E$

Mark allocation: 1 mark

- 1 mark for correct path; can also be written as AFE



Tip

- *Read the question carefully. Note that the length is not what is being requested here.*

Question 1b.

Worked solution

$$B - A - C - F - E - D - B$$

OR

$$B - D - E - F - C - A - B$$

Mark allocation: 1 mark

- 1 mark for either of the Hamiltonian circuits listed above



Tip

- *Remember that a Hamiltonian circuit should have no repeated vertices, aside from the start/end point.*

Question 1c.**Worked solution**

In order for an Eulerian circuit to exist, all vertices must be of an even degree. Vertices F and D have a degree of 3, and are therefore odd.

Mark allocation: 1 mark

- 1 mark for correct statement as above, or similar

Question 1d.**Worked solution**

Start at D and finish at F .

OR

Start at F and finish at D .

Mark allocation: 1 mark

- 1 mark for one of the correct combinations above

**Tip**

- Remember that an Eulerian trail will start and finish at the odd vertices.

Question 2a.

Worked solution

The earliest start time for activity *F* is 4 days.

The predecessor for *F* is Activity *B*, which takes 4 days.

Mark allocation: 1 mark

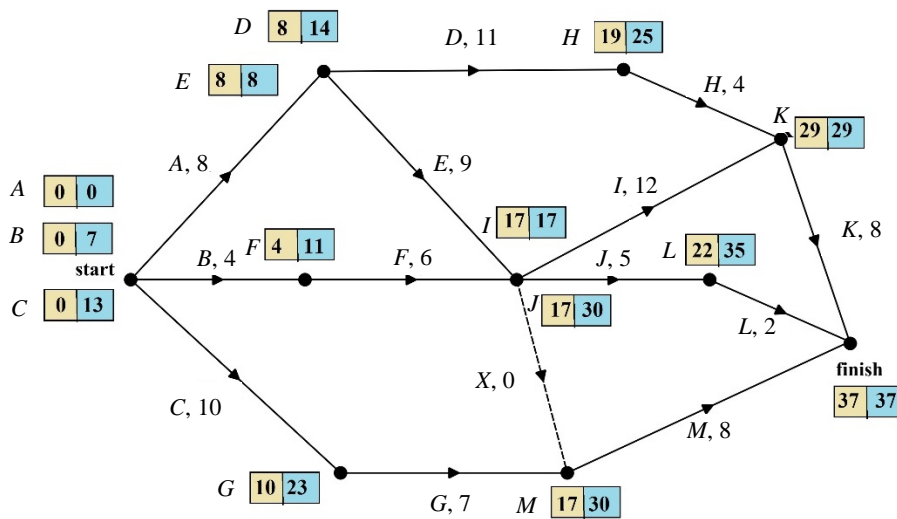
- 1 mark for an answer of 4

Question 2b.

Worked solution

A – E – I – K

Completion time is 37 days.



Mark allocation: 1 mark

- 1 mark for **both** the critical path and completion time.

Question 2c.**Worked solution**

<i>B</i>	EST = 0	LST = 7	Float = 7
<i>F</i>	EST = 4	LST = 11	Float = 7

Mark allocation: 1 mark

- 1 mark for stating both *B* and *F*

Question 2d.**Worked solution**

The dummy activity is required as activities *E*, *F* and *G* are all predecessors for activity *M*. However, only activity *E* and *F* are required for activity *J*.

Mark allocation: 1 mark

- 1 mark for statement above, or equivalent

Question 2e.**Worked solution**

The minimum cost is \$8000. And the project can now be completed in 35 days. Only activities *E* and *I* will affect the minimum completion time for the project, as these form the critical path. If we cut *C*, *F* or *M*, the minimum time will not go below 35 days, and therefore there is no need to pay for this.

Mark allocation: 1 mark

- 1 mark for \$8000

**Tip**

- *When ‘crashing’ a project, start by considering activities that are on the critical path. As this is the largest path through the network, this is the way to reduce the overall completion time. Once this has been done, look at activities not on the critical path, and see if these will influence completion time.*

Question 3a.**Worked solution**

Step 1: Row reduction

	A	W	P	S
Al	8	18	21	0
Br	10	13	14	0
Ch	9	20	21	0
Da	9	13	15	0

Step 2: Column reduction

	A	W	P	S
Al	0	5	7	0
Br	2	0	0	0
Ch	1	7	7	0
Da	1	0	1	0

Step 3: Cover the zeros with the minimum number of lines

	A	W	P	S
Al	0	5	7	0
Br	2	0	0	0
Ch	1	7	7	0
Da	1	0	1	0

As this requires 4 lines to cover all zeros, we can perform our allocation.

Step 4: Use the zeros to allocate a job to each person

	A	W	P	S
Al	0	5	7	0
Br	2	0	0	0
Ch	1	7	7	0
Da	1	0	1	0

Mark allocation: 1 mark

- 1 mark for table **after** both row and column reduction is completed.



Tip

- Remember that the minimum number of lines should equal the number of tasks/jobs being allocated.

Question 3b.

Worked solution

Using the allocation in part a.

We have:

Alistair completing Addressing

Brianna completing Packaging

Christine completing Sorting

Danielle completing Weighing

The total time for this allocation is $22 + 32 + 16 + 30 = 100$ minutes

Mark allocation: 2 marks

- 1 mark for correct allocation
- 1 mark for 100 minutes

Module 3 – Geometry and measurement

Question 1a.

Worked solution

The formula for the surface area of a sphere is: $s = 4\pi r^2$

In this question we have $r = 8.6$

$$s = 4\pi r^2$$

$$s = 4 \times \pi \times (8.6)^2$$

$$s = 929.4087$$

$$\therefore s = 929 \text{ mm}^2$$

Mark allocation: 1 mark

- 1 mark for correct answer, rounded to a whole number



Tip

- *Be sure to read the instructions carefully; geometry and measurement is a module where the units used and rounding required change from question to question.*

Question 1b.

Worked solution

The formula for the volume of a sphere is: $s = \frac{4}{3}\pi r^3$

In this question we have $r = 8.6$

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3} \times \pi \times (8.6)^3$$

$$V = 2664.3051$$

$$\therefore V = 2664 \text{ mm}^3$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 2a.**Worked solution**

The formula for the area of a sector is: $A = \frac{\pi r^2 \theta}{360}$

We have values of $A = 9642.4$ and $\theta = 93$

We can therefore solve for r , the radius, which in this case is d on our diagram.

$$9642.4 = \frac{\pi \times r^2 \times 93}{360}$$

Using your CAS calculator to solve, we find that $r = 109$ m

Mark allocation: 1 mark

- 1 mark for correct answer

Question 2b.**Worked solution**

From part a., the radius of the larger circle is 109 m. We can therefore find the area of the *major arc*, for the larger circle, using an angle of $360 - 93 = 267$.

$$A = \frac{\pi r^2 \theta}{360}$$

$$A = \frac{\pi \times 109^2 \times 267}{360}$$

$$A = 27682.9029$$

The radius of the smaller circle is 45 m. Again, find the area of the *major arc*, using an angle of 87.

$$A = \frac{\pi r^2 \theta}{360}$$

$$A = \frac{\pi \times 45^2 \times 267}{360}$$

$$A = 4718.2795$$

The paved area is therefore the difference between the larger area and the smaller:

$$27\,682.9029 - 4\,718.2795 = 22\,964.6234$$

Therefore, the paved area is equal to 22 965 m².

Mark allocation: 2 marks

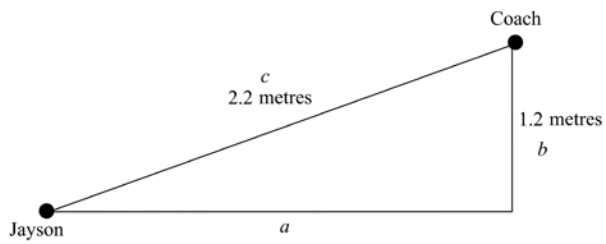
- 2 marks awarded for correct answer
- Note: if an incorrect answer was obtained in part a., 2 marks may be awarded if the correct procedure has been followed, using the incorrect answer from part a. The smaller circle area **must** be correct, however.

OR

- If the correct answer is not achieved, 1 mark can be given for correctly calculating the larger or smaller area, or for subtracting the smaller area from the larger, even if one is incorrect.

Question 3a.**Worked solution**

Use Pythagoras' theorem to find the length of the distance between Jayson and the bottom of the coach's ladder.



$$c^2 = a^2 + b^2$$

$$2.2^2 = a^2 + 1.2^2$$

$$\therefore a^2 = 2.2^2 - 1.2^2$$

$$a^2 = 3.4$$

$$a = \sqrt{3.4}$$

$$a = 1.84391$$

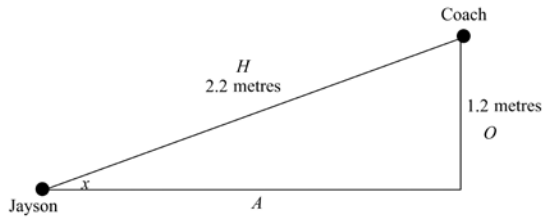
$$a = 1.8 \text{ m}$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3b.**Worked solution**

We must first find the internal angle in the triangle, marked as x in the diagram below.



We can use SOH CAH TOA.

Label the sides Opposite, Adjacent and Hypotenuse, as shown above.

We have information for O and H, and are trying to find an angle, so we will use sin.

$$\sin \theta = \frac{O}{H}$$

$$\sin \theta = \frac{1.2}{2.2}$$

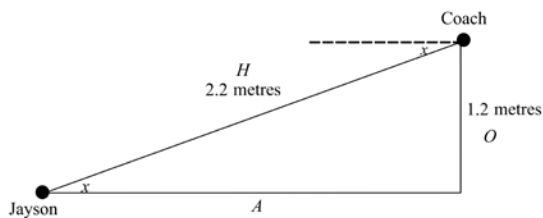
$$\therefore \theta = \sin^{-1}\left(\frac{1.2}{2.2}\right)$$

$$\theta = 33^\circ$$

Now we can find the angle of depression.

Using our knowledge of vertically opposite angles, we know that angle of depression is also 33° .

The diagram below shows the vertically opposite and equivalent angles both marked as an x .

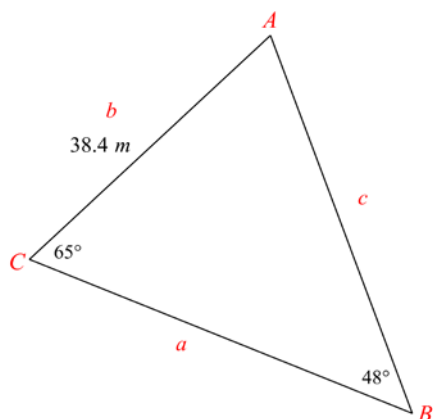
**Mark allocation: 1 mark**

- 1 mark for angle of depression equals 33°

Question 4a.**Worked solution**

We will use the sine rule to find the length of the side AB .

Start by labelling the diagram.



We have side b , angle B , angle C . We are trying to find side c .

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{38.4}{\sin 48} = \frac{c}{\sin 65}$$

$$\therefore c = \sin 65 \times \frac{38.4}{\sin 48}$$

$$c = 46.831$$

$$\therefore c = 47 \text{ m}$$

Mark allocation: 1 mark

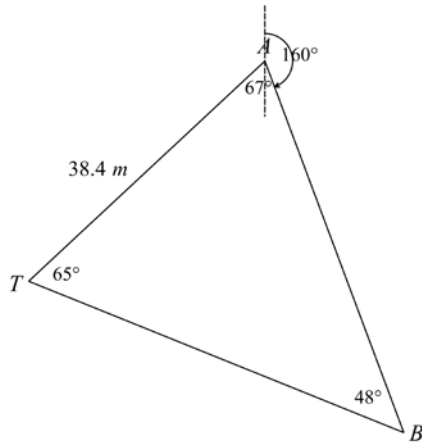
- 1 mark for correct answer, rounded to a whole number

Question 4b.**Worked solution**

Use the two known angles in the triangle to find the angle $\angle TAB$.

$$180^\circ - 65^\circ - 48^\circ = 67^\circ$$

We know that the bearing from A to T is 227° . Use this to find the bearing from A to B .



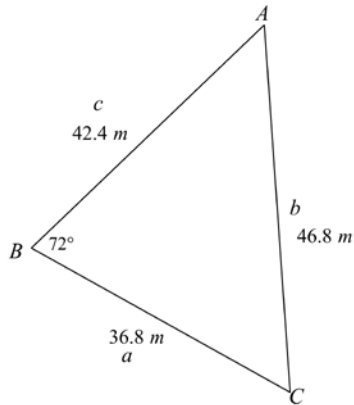
The bearing of B from A is 160° .

Mark allocation: 2 marks

- 2 marks for correct answer
- Note: if this is not achieved, 1 mark can be given for finding the angle TAB .

Question 4c.**Worked solution**

Label the triangle and determine the correct use of the cosine rule, in order to find the angle A .



We should use the following:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Substituting in our values gives:

$$\cos A = \frac{(46.8)^2 + (42.4)^2 - (36.8)^2}{2 \times 46.8 \times 42.4}$$

$$\cos A = 0.663643$$

$$A = \cos^{-1}(0.663643)$$

$$\therefore A = 48.421^\circ$$

$$A = 48^\circ$$

Mark allocation: 2 marks

- 1 mark for accurately substituting into the cosine rule
- 1 mark for working that results in an answer of 48°

Module 4 – Graphs and relations

Question 1a.i.

Worked solution

The hourly rate is \$7.50.

This can be found by looking at the slope of the equation.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 1a.ii.

Worked solution

The equation is:

$$\text{cost} = 25 + 7.50 \times \text{hours worked}$$

Substitute a *cost* of 130 into the equation:

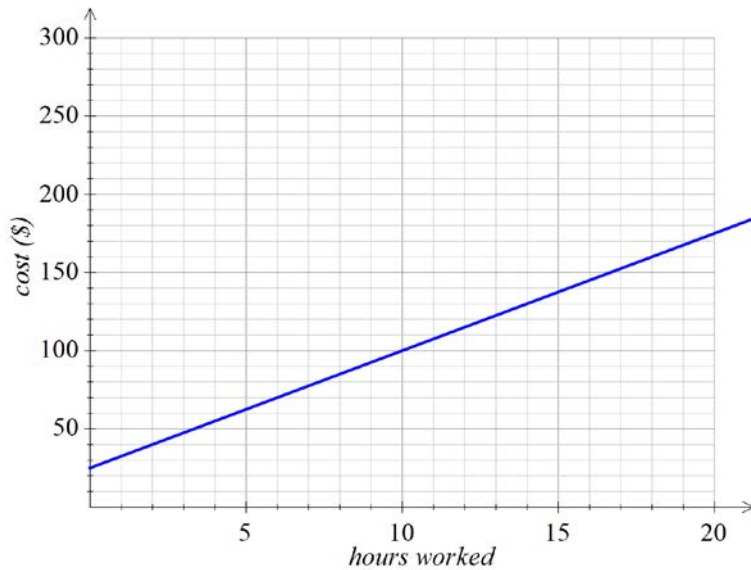
$$130 = 25 + 7.50 \times \text{hours worked}$$

Use your CAS calculator to solve for *hours worked*.

$$\text{hours worked} = 14 \text{ hours}$$

Mark allocation: 1 mark

- 1 mark for an answer of 14

Question 1b.**Worked solution****Mark allocation: 1 mark**

- 1 mark for correct graph

Question 1c.**Worked solution**

Firstly, find the *cost* of 150 *hours worked*.

$$\text{cost} = 25 + 7.50 \times \text{hours worked}$$

$$\text{cost} = 25 + 7.50 \times 150$$

$$\text{cost} = 1150$$

Secondly, find the cost of 1 ticket in order to make \$1150.

Set up an equation:

$$1150 = 300 \times t$$

Use your CAS to solve.

$$t = 3.83$$

One ticket cost is 3.83 in order to break even.

Mark allocation: 2 marks

- 2 marks for 3.83
- Note: if this is not achieved, 1 mark can be given for finding the cost of \$1150.

Question 2a.**Worked solution**

Reading from the graph, we can see that for ages between 18 and 21 employees are paid \$12 an hour.

Mark allocation: 1 mark

- 1 mark for an answer of 12

Question 2b.**Worked solution**

The graph does not begin until the age of 15, and so we assume that this is the youngest age.

Mark allocation: 1 mark

- 1 mark for 15 years old

Question 2c.**Worked solution**

$$x = 17$$

Mark allocation: 1 mark

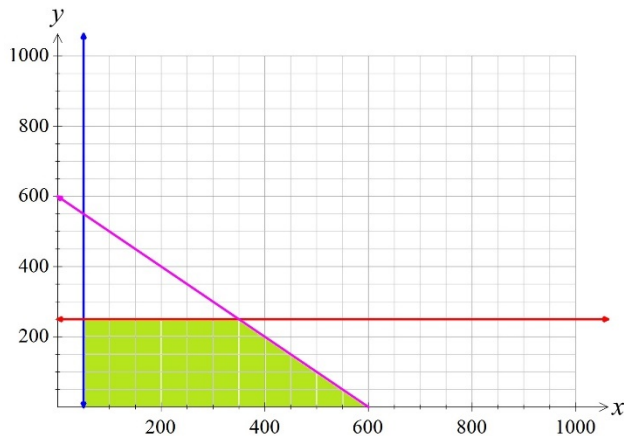
- 1 mark for 17

Question 3a.**Worked solution**

When added together, the total number of 1.5 kg and 4 kg shot-puts produced is less than or equal to 600.

Mark allocation: 1 mark

- 1 mark for statement above or similar; should include reference to the less than or equal to symbol

Question 3b.**Worked solution****Mark allocation: 2 marks**

- 1 mark for accurately showing Constraint 3
- 1 mark for correct shaded region

Question 3c.**Worked solution**

From the graph, the maximum number of shot-puts that can be produced is found at (350, 250).

Therefore, the maximum number of 1.5 kg shot-puts is 350 and the maximum number of 4 kg shot-puts is 250.

$$R = 45x + 68y$$

$$R = 45 \times 350 + 68 \times 250$$

$$R = 32\,750$$

Mark allocation: 1 mark

- 1 mark for maximum revenue of \$32 750

END OF WORKED SOLUTIONS