TSSM
Creating VCE Success

2016 Trial Examination

STUDENT NUMBER

Letter

Figures

Words

FURTHER MATHEMATICS
Written examination 1

Reading time: 15 minutes

Writing time: 1 hour and 30 minutes

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	24	24			24
B	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphic calculator or approved CAS calculator or CAS software and if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 30 pages.

Instructions
- Print your **name** in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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SECTION A - Core

Instructions for Section A

Answer **all** questions.

Choose the response that is correct for the question.
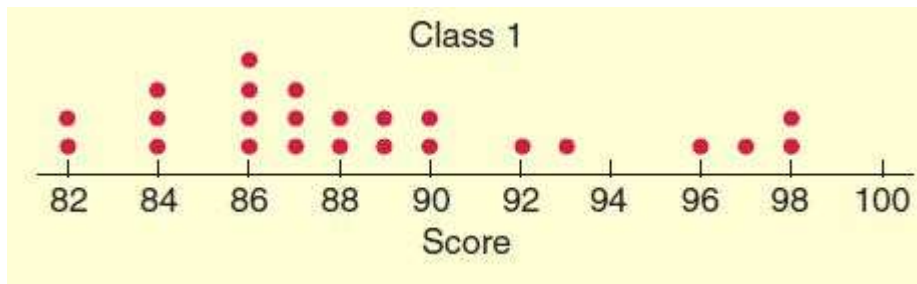
A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Data analysis

The following information relates to Questions 1 and 2

The dot plot below shows the scores (in %) of students in Class 1.

Question 1

The mean score, as a percentage, of this class is:

A. 87
B. 88
C. 89
D. 90
E. 92

Question 2

The percentage of students in class 1 who had scores more than 90% is:

A. 50%
B. 24%
C. 6%
D. 25%
E. 20%

Question 3
For data that is symmetrically distributed:

A. Mean, median and mode are the same
B. Mean is more than median
C. Mean is less than median
D. Mode is more than mean
E. Mode is more than median

The following information relates to Questions 4 and 5

The heights of 80 individuals follow a normal distribution with mean height 167 cm and standard deviation 15 cm.

Question 4
The expected number of individuals with height of at least 182 cm is closest to:

A. 12
B. 13
C. 14
D. 15
E. 16

Question 5
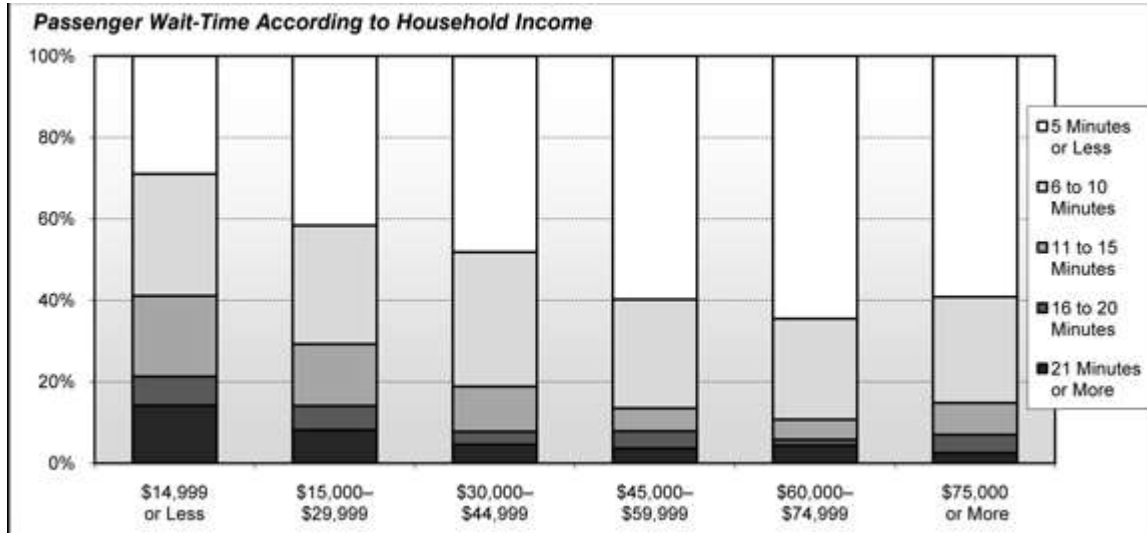An individual has standardised height as $z = -1.2$.
The actual height of this individual is:

A. 167 cm
B. 154 cm
C. 110 cm
D. 111 cm
E. 149 cm

Question 6

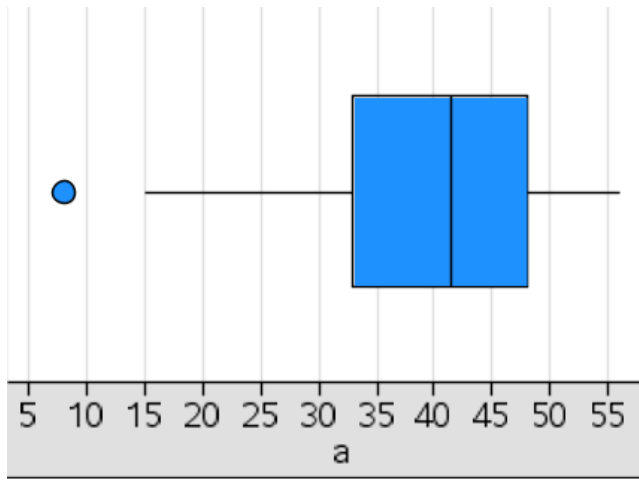The following segmented bar chart shows the wait-time of passengers relative to their salary.

If 4000 people in the sample had salaries between $45000 and $59999, how many people had waited for more than 6 minutes?

A. 200
B. 400
C. 800
D. 1600
E. 3200

Question 7

The following box plot represents data from a sample.

The range of data is closest to:

A. 49
B. 41
C. 15
D. 52
E. 40

Question 8

For a set of bivariate data that involves the variables x and y:

$$\bar{x} = 61195, \quad \bar{y} = 4999, \quad S_X = 50989, \quad S_Y = 4079, \quad r = -0.641$$

Given the information above, the least squares regression line predicting y from x is closest to:

A. $y = 8137 - 0.05128x$
B. $y = 8137 + 0.05128x$
C. $x = 8137 - 0.05128y$
D. $y = 8137 - 0.5128x$
E. $y = 8138x - 0.05128$

SECTION A - continued

Question 9
The coefficient of determination between two numerical variables is 0.8276. The slope of the least squares regression line was found to be negative. The value of the coefficient of correlation is closest to:

A. 82.76
B. ± 0.9097
C. 0.9097
D. -0.9097
E. -0.8276

Question 10

The value of the log of the variable where the actual value of the variable is 0.001 is:

A. 3
B. -3
C. 1000
D. 10000
E. $\frac{1}{3}$

The following information relates to Questions 11 and 12

The least-squares regression equation of de-seasonalised sales of ice-creams is

$$Deseasonalised\ Sales(in\ 100's\ of\ \$) = 2400 + 2.56 \times Time\ (quarter\ number)$$

Question 11
From this equation, it can be concluded that:

A. actual sales increases by $256 each quarter
B. initially the sales was $2400
C. actual sales decreases by $256 each quarter
D. deseasonalised sales increases by $256 each quarter
E. deseasonalised sales increases by $2.56 each quarter

Question 12

The seasonal indices of each quarter are given below:

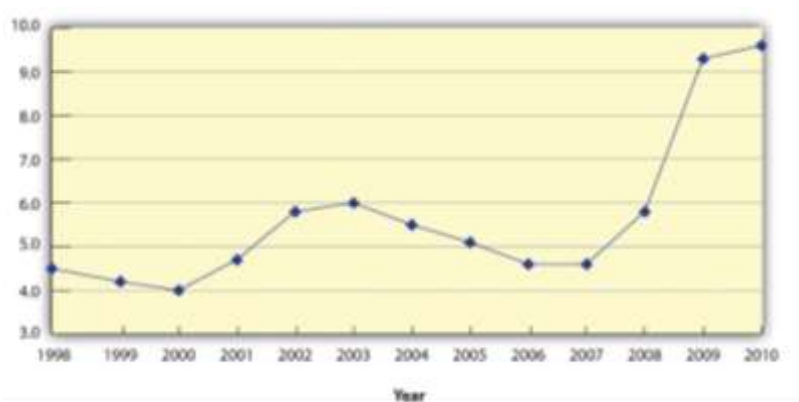| Quarter	1	2	3	4
Seasonal Index	1.53	0.56	0.87	1.04

The actual sales figure in quarter number 7 is closest to:

A. $2779
B. $2103
C. $2400
D. $2515
E. $1900

Question 13
The time series plot below charts the unemployment rate over a 13-year period

Using five-median smoothing, the smoothed value in 2006 was closest to:
A. 4.0
B. 4.5
C. 5.5
D. 4.8
E. 5.0

The following information relates to Questions 14 and 15

The following data shows the relationship between number of cigarettes smoked per month and the life expectancy of a particular town.

Number of cigarettes smoked	Life Expectancy
5	80
23	78
25	60
48	53
17	85
8	84
4	73
26	79
11	81
19	75
14	68
35	72
29	58
4	92
23	65

Question 14
The least squares regression line to predict life expectancy is given by

A. $Life\ expectancy = 85.72 + 0.63 \times Number\ of\ cigarettes$
B. $Number\ of\ cigarettes = 78.98 - 0.81 \times life\ expectancy$
C. $Numebr\ of\ cigarettes = 85.72 - 0.63 \times life\ expectancy$
D. $Life\ expectancy = 85.72 - 0.63 \times Number\ of\ cigarettes$
E. $Life\ expectancy = 78.98 - 0.81 \times Number\ of\ cigarettes$

Question 15
Jeremy smokes 18 cigarettes in a particular month.
Using the above linear model, his life expectancy is closest to

A. 75
B. 74
C. 65
D. 64
E. 50

Question 16

The mean score in a Further Maths practice exam for a group of students at a school is 82%. If Sam's scored a standardised result of -1.5 in this exam and her raw score was 78.7%.
The standard deviation of this group of students in this exam is closest to

A. 2.7
B. 2.1
C. 2.2
D. 1.02
E. 1.1

Recursion and Financial Modelling

Question 17

An appliance repairman charges $80 to turn up and $55 per minutes (or part thereof) working time to make home visits. The recurrence relation that models this situation is:
A. $A_0 = 80, \quad A_{n+1} = A_n + 55$
B. $A_0 = 55, \quad A_{n+1} = A_n + 80$
C. $A_0 = 80, \quad A_{n+1} = 0.55A_n$
D. $A_0 = 80, \quad A_{n+1} = 1.55A_n$
E. $A_0 = 80, \quad A_{n+1} = A_n - 55$

The following information relates to Questions 18 and 19

Kelly invested $3000 in a term deposit that earns 4% per annum compounded quarterly.

Question 18

The recurrence relation that models this investment is:
A. $V_0 = 3000, \quad V_{n+1} = 1.04V_n$
B. $V_0 = 3000, \quad V_{n+1} = 0.01V_n$
C. $V_0 = 3000, \quad V_{n+1} = 0.04V_n$
D. $V_0 = 3000, \quad V_{n+1} = 4V_n$
E. $V_0 = 3000, \quad V_{n+1} = 1.01V_n$

Question 19

After 2 years, Kelly has:
A. $3216.40
B. $3248.60
C. $3000.00
D. $4105.70
E. $3947.80

The following information relates to Questions 20, 21 and 22

Andrew takes a personal loan of $5000 for a year which charges interest every month. The recurrence relation that models his loan is given below.

$$V_0 = 5000, \quad V_{n+1} = 1.0125V_n - 450$$

Question 20

The interest rate charged per annum is:
A. 1.25%
B. 12%
C. 15%
D. 1.5%
E. 10.125%

Question 21

The amount of the loan after 4 months is:
A. $3822.90
B. $4220.15
C. $5000.00
D. $3420.70
E. $3013.45

SECTION A - continued
TURN OVER

Question 22

The interest charged in this loan is closest to:
A. $330.35
B. $450
C. $400
D. $416.60
E. $433.30

Question 23
A couple takes a home loan of $450 000 from a bank. The bank charges interest at the rate of 8.7% per annum, compounded monthly.
The loan is repaid in 25 years.
The total amount of interest paid by the couple is:

A. $92100
B. $655311
C. $450000
D. $405000
E. $607200

Question 24

The recurrence relation $A_0 = 1200, \ A_{n+1} = 1.2A_n + 300$ models an investment over time.
Which of the following is not true about this investment?

A. The initial investment is $1200.
B. The investment after 3 periods of time is $3165.60.
C. $300 is added after each period of time.
D. The investment doubles at the end of the second period.
E. The investment is not a linear model.

END OF SECTION A

SECTION B

Instructions for Section B

Select **two** modules and answer **all** questions within the modules.
Choose the response that is **correct** or **best answers** the question and mark this response on the multiple-choice response sheet.
A correct answer scores 1, an incorrect answer scores 0.
No mark will be given if more than one answer is completed for any question.
Marks will not be deducted for incorrect answers.

Module	**Page**

Module 1: Matrices

The following information relates to Questions 1 and 2

The following matrix shows the number of coloured poster papers left in each of the staffrooms in a school.

$$A = \begin{array}{c} \\ \end{array} \begin{array}{ccccc} Red & Blue & Green & Yellow & Orange \\ \begin{bmatrix} 12 & 34 & 5 & 20 & 18 \\ 23 & 31 & 17 & 34 & 41 \\ 18 & 42 & 20 & 28 & 33 \end{bmatrix} & & & & \end{array} \begin{array}{c} Science \\ Math \\ Art \end{array}$$

Question 1

The number of coloured posters in the Art staff room is:

A. 18

B. 33

C. 140

D. 141

E. 92

Question 2

The element in row i and column j of matrix A is a_{ij}. The element a_{23} represents:

A. the number of blue poster papers in Math staff room

B. the number of blue posters in Art staff room

C. the number of green posters in Math staff room

D. the number of green posters in Art staff room

E. the number of yellow posters in Math staff room

SECTION B - MODULE 1 - continued

Question 3

Matrices *A, B, C* and *D* are defined as follows-

$$A = \begin{bmatrix} 1 & 6 & 3 \\ 4 & 0 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 7 \\ 6 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 1 \\ 6 & 0 \\ 0 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & 4 \\ 3 & 8 \\ 0 & 1 \end{bmatrix}$$

Which of the following matrix product is not defined?

A. $C \times D$

B. $D \times B$

C. $C \times B$

D. $A \times D$

E. $D \times A$

Question 4

For the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, AA^T is equal to:

A. $\begin{bmatrix} -1 \\ 7 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 2 \\ 2 & 25 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 10 \\ 2 & 25 \end{bmatrix}$

D. $\begin{bmatrix} 13 & 10 \\ 10 & 17 \end{bmatrix}$

E. $\begin{bmatrix} 4 & 1 \\ 9 & 16 \end{bmatrix}$

Question 5

If $AXB = CD$, the matrix X can be found by finding the matrix product:

A. $CD(B^{-1}A^{-1})$

B. $AB(CD)^{-1}$

C. $B^{-1}(A^{-1}CD)$

D. $A^{-1}(BCD)$

E. $A^{-1}(CDB^{-1})$

SECTION B - MODULE 1 – continued
TURN OVER

Question 6

Let the matrix $P = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$.

The inverse of the matrix $k \times P$, where k is a scalar is:

A. $\dfrac{1}{2k}\begin{bmatrix} 7 & -3 \\ -4 & 2 \end{bmatrix}$

B. $\dfrac{1}{2}\begin{bmatrix} 7k & -3k \\ -4k & 2k \end{bmatrix}$

C. $\dfrac{1}{2k}\begin{bmatrix} 7k & -4 \\ -4k & 1 \end{bmatrix}$

D. $\dfrac{1}{2}\begin{bmatrix} 7 & -3 \\ -4 & 2 \end{bmatrix}$

E. $-\dfrac{1}{2}\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$

The following information relates to Questions 7 and 8

The following transition matrix shows the expected change from a choice of tea and coffee for morning breakfast.

$$T = \begin{array}{c} \\ \\ \\ \end{array} \begin{matrix} & This\ week \\ & Tea\ \ Coffee \\ \begin{bmatrix} 0.65 & 0.30 \\ 0.35 & 0.70 \end{bmatrix} & \begin{matrix} Tea \\ Coffee \end{matrix} \end{matrix} Next\ week$$

In week 1, 150 customers drink tea and 160 customers drink coffee.

Question 7

The number of customers who drink coffee in week 3 is:

A. 167

B. 143

C. 166

D. 144

E. 53

Question 8

After 3 weeks, 20 customers who preferred coffee in week 3 left the town.
The number of customers who preferred coffee in week 6 is:

A. 156

B. 143

C. 134

D. 144

E. 167

END OF MODULE 1

Module 2: Networks and decision mathematics

Question 1
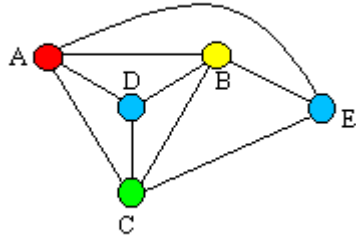
In the graph above, the number of vertices of odd degree is:
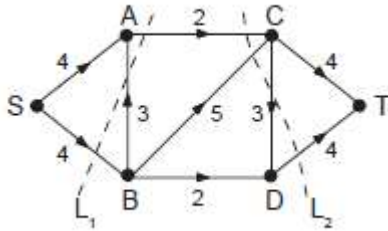
A. 0

B. 1

C. 2

D. 3

E. 4

Question 2

A planar graph has five vertices and nine edges.

The number of faces is:

A. 2

B. 4

C. 5

D. 6

E. 14

SECTION B - MODULE 2 - continued
TURN OVER

Question 3

L_1 and L_2 are two cuts shown as a dotted line above.
The capacity of cut L_1 is:

A. 4
B. 6
C. 8
D. 9
E. 11

Question 4
The following graph shows a network of schools in a region working together on Numeracy programs.
An edge here represents the presence of resource sharing between schools.
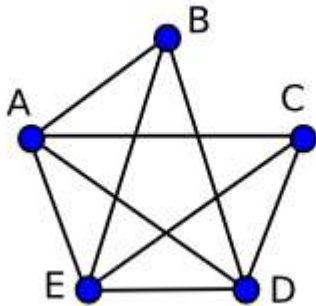
Which of the following is not true from the above graph?

A. The above graph is not a planar graph.
B. School B is working with three other schools in the region.
C. School E has a partnership with schools A and B.
D. Each school has a partnership with every other school in the region.
E. There are five schools in the region.

SECTION B - MODULE 2 - continued

Question 5

A company has three employees A, B and C and three tasks to be completed in set time frame.
The table below shows the number of hours that each employee takes to finish each of the tasks.

	1	2	3
A	17	10	12
B	9	8	10
C	14	4	7

If each person is allocated a different task, the total minimum time, in hours, that these employees
will take to finish all three tasks is:

A. 4

B. 9

C. 12

D. 22

E. 25

Question 6

The following graph shows the distances between seven towns.
A traveller decides to start his journey from Bodhgaya and travel to Lucknow via Kushinagar.
From Lucknow, he travels to Delhi via Jaipur and then finally reaches Kolkata via Kanpur.
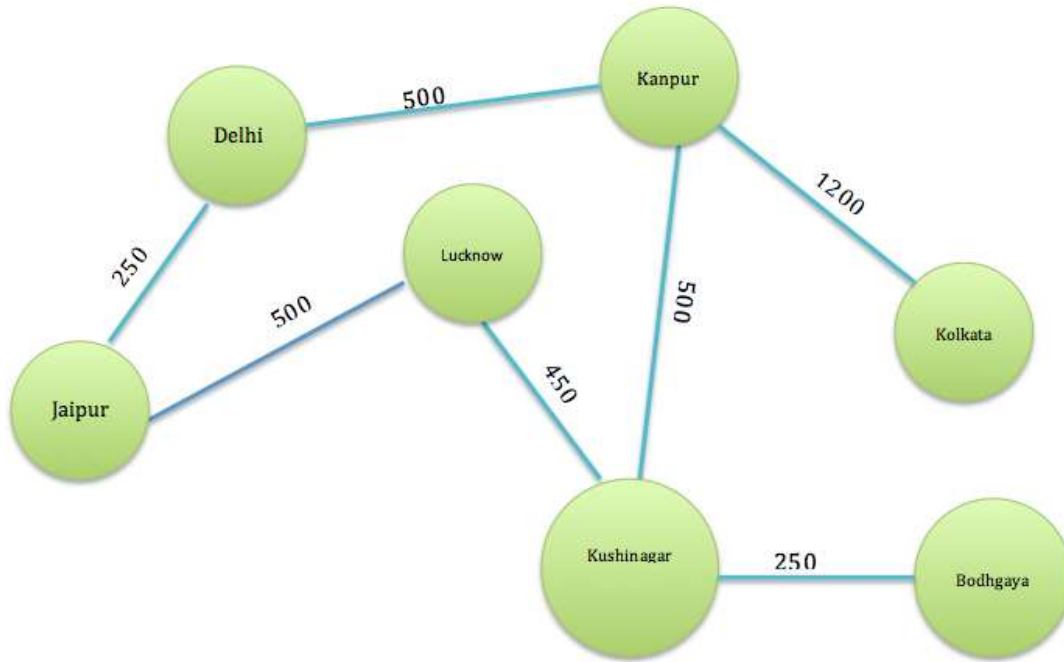
The mathematical term for this route is:

A. a loop

B. an Eulerian Circuit

C. an Eulerian trail

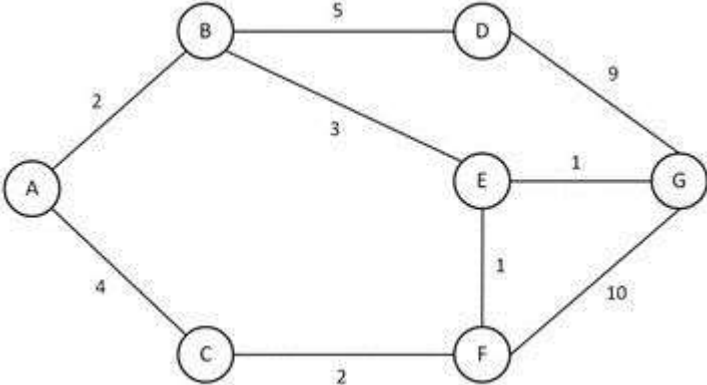**D.** a Hamiltonian path

E. a Hamiltonian cycle

Question 7
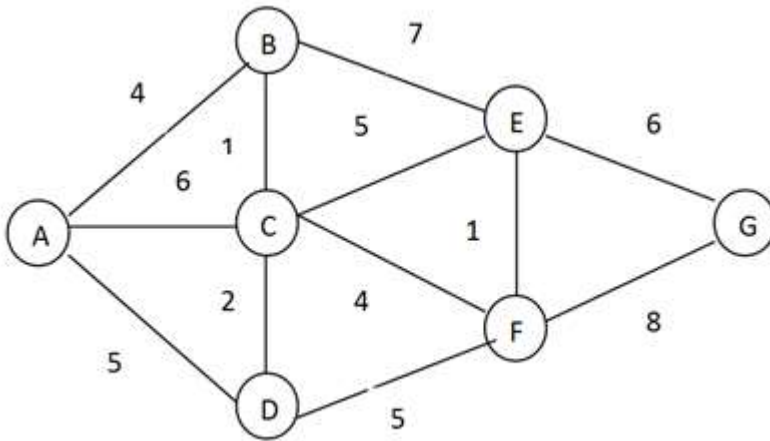
The length of the minimum spanning tree for the network shown above is:

A. 17
B. 18
C. 19
D. 20
E. 21

Question 8

The length of the shortest path from A to G in the network shown above is:

A. 8
B. 7
C. 16
D. 6
E. 10

END OF MODULE 2
TURN OVER

Module 3: Geometry and measurement

Question 1
PQRS is a trapezium with SR = 48cm, PQ = 40cm and QR = 17cm.
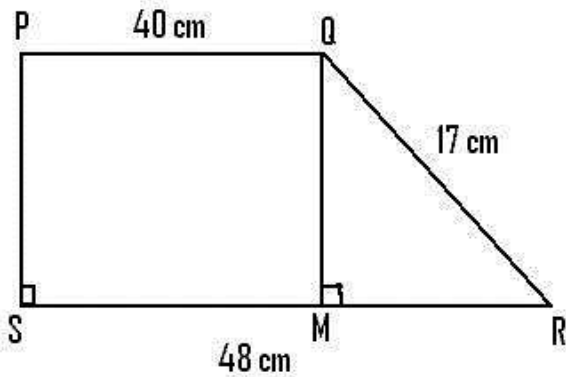
The area of this trapezium, in square centimetres, is:

A. 660

B. 748

C. 1920

D. 960

E. 830

Question 2
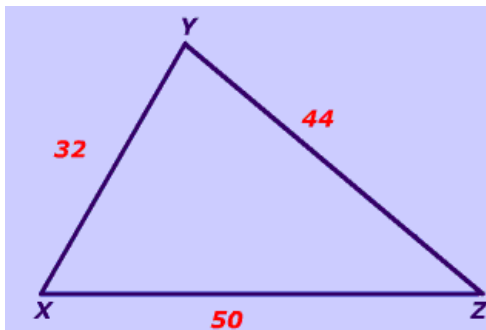
The angle opposite to the largest side in the above triangle can be found using:

A. $\dfrac{32^2+44^2-50^2}{2816}$

B. $\cos^{-1}\left(\dfrac{44}{50}\right)$

C. $\sin^{-1}\left(32 \times \dfrac{44}{50}\right)$

D. $\cos^{-1}\left(\dfrac{32^2+44^2-50^2}{2816}\right)$

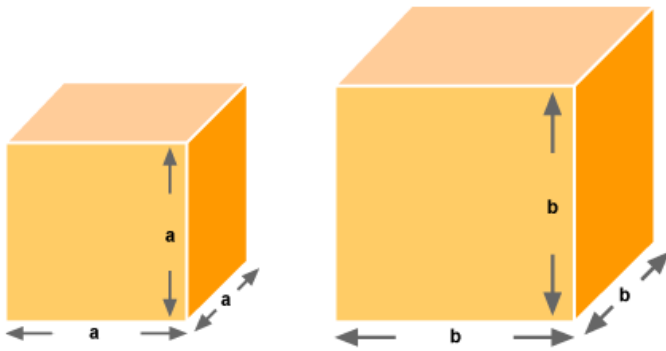**E.** $\sqrt{32^2 + 44^2 - 2 \times 32 \times 44}$

TURN OVER

Question 3
There are three towns A, B and C. Town B is NE of Town A and Town C is north of Town B.
The bearing of Town C from A is:

A. $between\ 000°\ and\ 045°$
B. $045°$
C. $between\ 045°\ and\ 090°$
D. $between\ 090°\ and\ 180°$
E. $between\ 180°\ and\ 270°$

Question 4
The following diagram shows two similar cubes.

If the smaller cube has a volume of 1 cubic centimetre and the larger cube has a volume of 64
cubic centimetres, which of the following is true?

A. $b = a$
B. $b = 4a$
C. $b = \frac{1}{4}a$
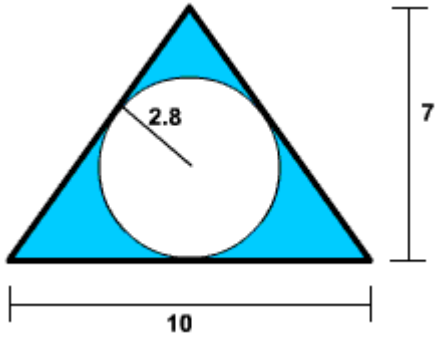**D.** $b = 64a$
E. $b = 8a$

Question 5

The length of a great circle on a sphere with radius 48cm is closest to:

A. 250 cm
B. 48 cm
C. 300 cm
D. 150 cm
E. 3500 cm

SECTION B - MODULE 3 - continued

Question 6
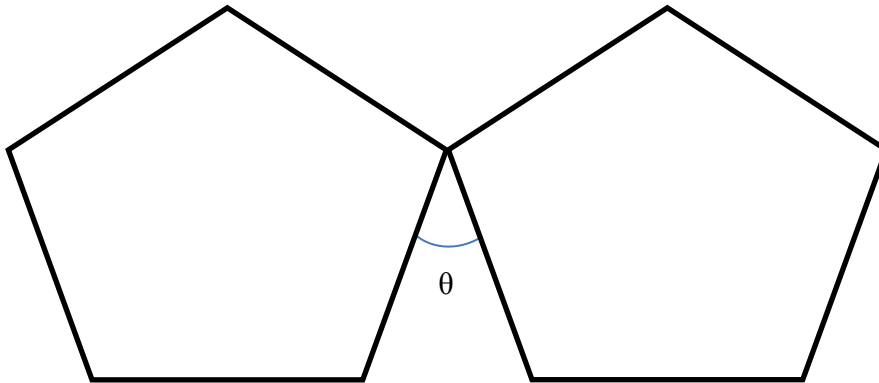
In the diagram below, all measurements are in centimetres.

The area, in cm^2, of the shaded region in the above diagram is closest to:

A. 11

B. 10

C. 5

D. 30

E. 18

Question 7

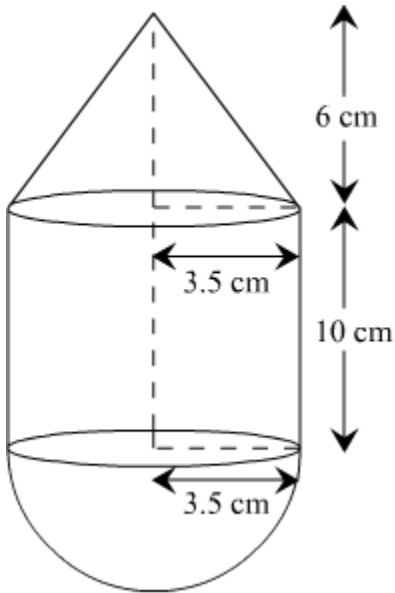Two identical pentagons are joined together as shown below.

The angle θ is:

A. 36°

B. 45°

C. 72°

D. 40°

E. 44°

Question 8

A tank filled with oil has a hemispherical base with a cone mounted on a cylinder with dimensions as shown in the diagram below.

The volume of oil, in litres, in the tank is closest to:
A. 0.71
B. 0.64
C. 130
D. 0.6
E. 550
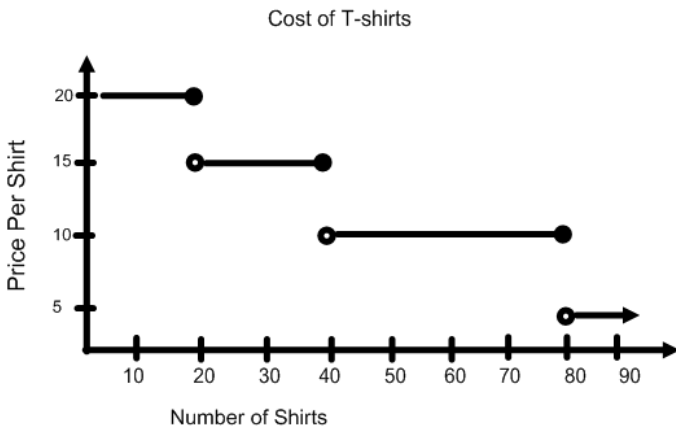
END OF MODULE 3

Module 4: Graphs and Relations

Question 1
A point that satisfies the inequality $2y - 5x \geq 8$ is:

A. (1, 4)
B. (-2, -1)
C. (0, 1)
D. (-1, -3)
E. (1, 1)

Question 2
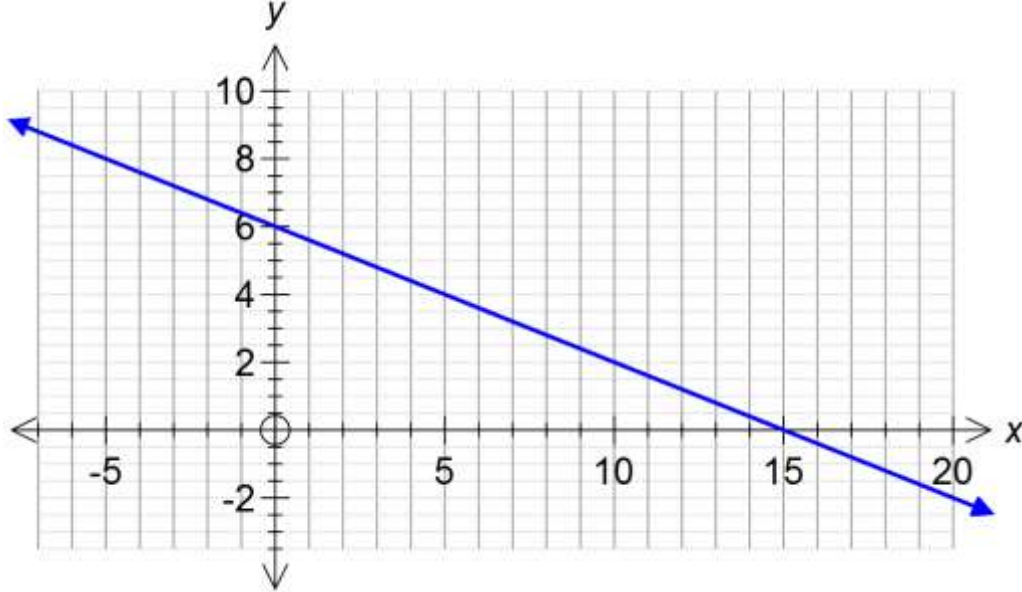The graph below shows the price per shirt depending on the number of shirts ordered.

Cost of T-shirts

The cost per shirt for a retailer who orders 40 shirts is:
A. $5
B. $10
C. $15
D. $20
E. $60

SECTION B - MODULE 4 - continued
TURN OVER

Question 3

The following graph shows a straight line.

The equation of the above straight line is:

A. $2x - 5y = 30$
B. $5y - 2x = 30$
C. $6x + 15y = 1$
D. $2x + 5y = 30$
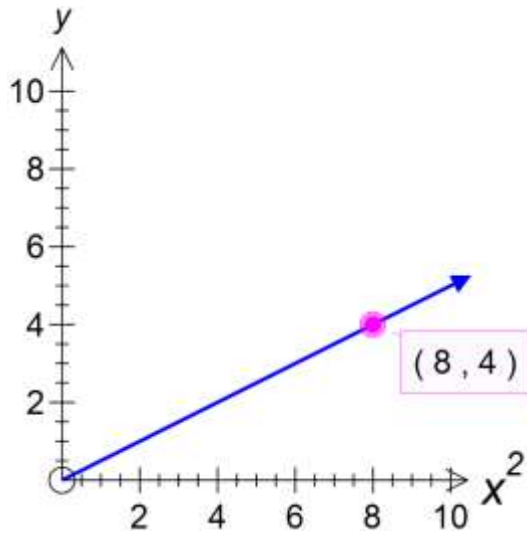E. $3x + 5y = 30$

Question 4

There are at least 16 more red cars than blue cars in a car park.
If x is the number of red cars and y is the number of blue cars, the inequality for this constraint is:

A. $y \geq x + 16$
B. $x + y \geq 16$
C. $x \leq 16 + y$
D. $x \geq 16 + y$
E. $x > 16 + y$

Question 5

The following graph shows the relationship between y and x^2.

 The value of y when $x^2 = 16$ is:

A. 8

B. 16

C. 4

D. 128

E. 64

Question 6

Year 12s organise a fund raising event at a community event. They rent a stall for $180 and sell show bags for $45. They buy each show bag for $18.
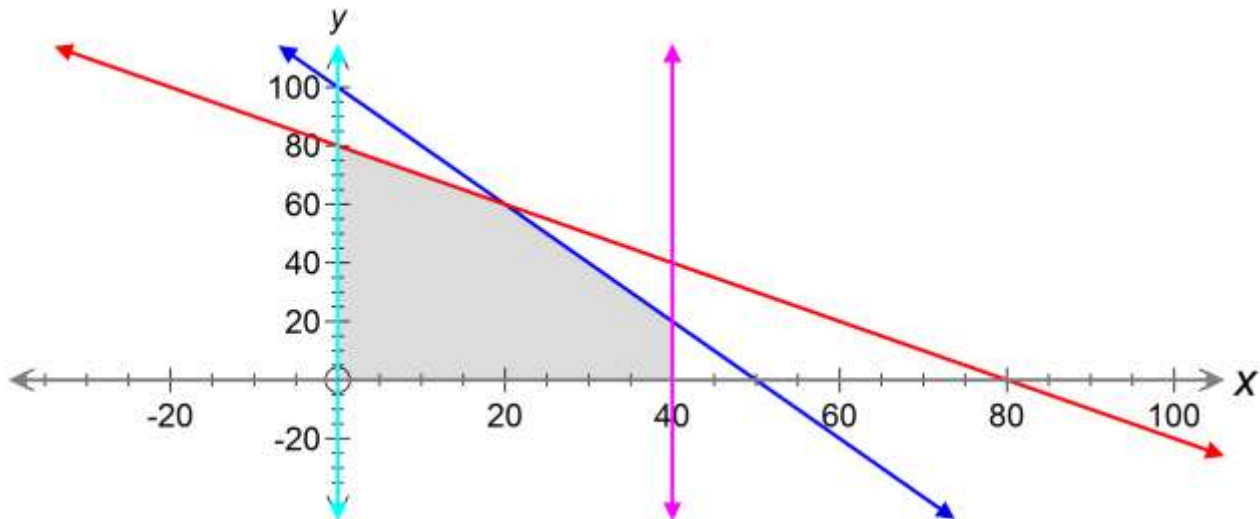
To make a profit of $3000, the minimum number of show bags to be sold is:

A. 117

B. 118

C. 50

D. 51

E. 167

SECTION B - MODULE 4 - continued

TURN OVER

Use the following sketch to answer Questions 7 and 8

The shaded area in the graph below shows the feasible region for a linear programming problem.

Question 7

The inequalities that represent the feasible region above are:

A. $2x + y \leq 100, \quad x + y \geq 80, \quad 0 \leq x \leq 40, \quad y \geq 0, \quad y \geq -x + 1$

B. $2x + y \geq 100, \quad x + y \leq 80, \quad 0 \leq x \leq 40, \quad y \geq 0$

C. $2x + y \leq 100, \quad x + y \leq 80, \quad 0 \leq x \leq 40, \quad y \geq 0$

D. $2x + y \leq 100, \quad x \leq 80 + y, \quad 0 \leq x \leq 40, \quad y \geq 0$

E. $2x \leq 100 + 2x, \quad x + y \leq 80, \quad 0 \leq x \leq 40, \quad y \geq 0$

Question 8
The point that maximises the objective function $z = 3x + 2y$ is:

A. $(0, \ 80)$

B. $(40, \ 0)$

C. $(40, 20)$

D. $(20, 60)$

E. $(60, \ 20)$

END OF MULTIPLE-CHOICE QUESTION BOOK