2016 VCE Further Mathematics Trial Examination 2

Kilbaha Multimedia Publishing	**Tel: (03) 9018 5376**
PO Box 2227	**Fax: (03) 9817 4334**
Kew Vic 3101	**kilbaha@gmail.com**
Australia	**http://kilbaha.com.au**

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STUDENT NUMBER									Letter

VICTORIAN CERTIFICATE OF EDUCATION 2016

FURTHER MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A - Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	9	9	36
Section B - Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to Be cleared. For approved computer based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 28 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Section A – Core

Instructions for Section A
Answer all questions in the spaces provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (10 marks)

Twelve students in each of Year 11 and Year 12 were shown 10 words and 10 pictures and then asked to write down the pictures and numbers that they remembered. The table below shows how many each student remembered correctly.

Year 11		
Student	Words	Pictures
A	7	5
B	5	4
C	6	6
D	6	7
E	8	8
F	10	1
G	9	4
H	10	6
I	4	5
J	7	8
K	3	5
L	5	4

Year 12		
Student	Words	Pictures
N	10	3
O	9	7
P	7	5
Q	6	10
R	5	5
S	8	4
T	8	5
U	5	7
V	4	8
W	6	9
X	7	4
Y	5	8

(a) What percentage of year 12 students got at least eight pictures correct?

1 mark

___

(b) What is the interquartile range for the number of words remembered by the Year 12 students?

1 mark

___

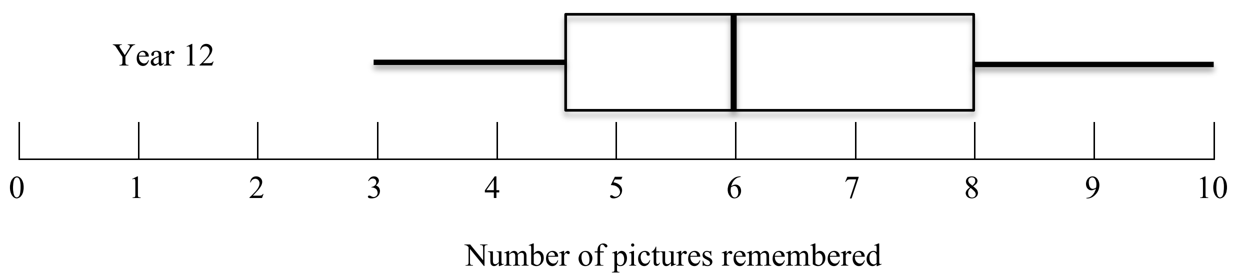
Question 1 (continued)

(c) What is the mean and standard deviation of the number of pictures remembered by the year 11 students? Give your answers to two decimal paces.

2 marks

(d) In the space provided above the boxplot for the Year 12 number of pictures remembered, draw the boxplot for the Year 11 number of pictures remembered.

2 marks



(e) Use an appropriate calculation to show whether there are any outliers for the Year 11 number of pictures remembered.

2 marks

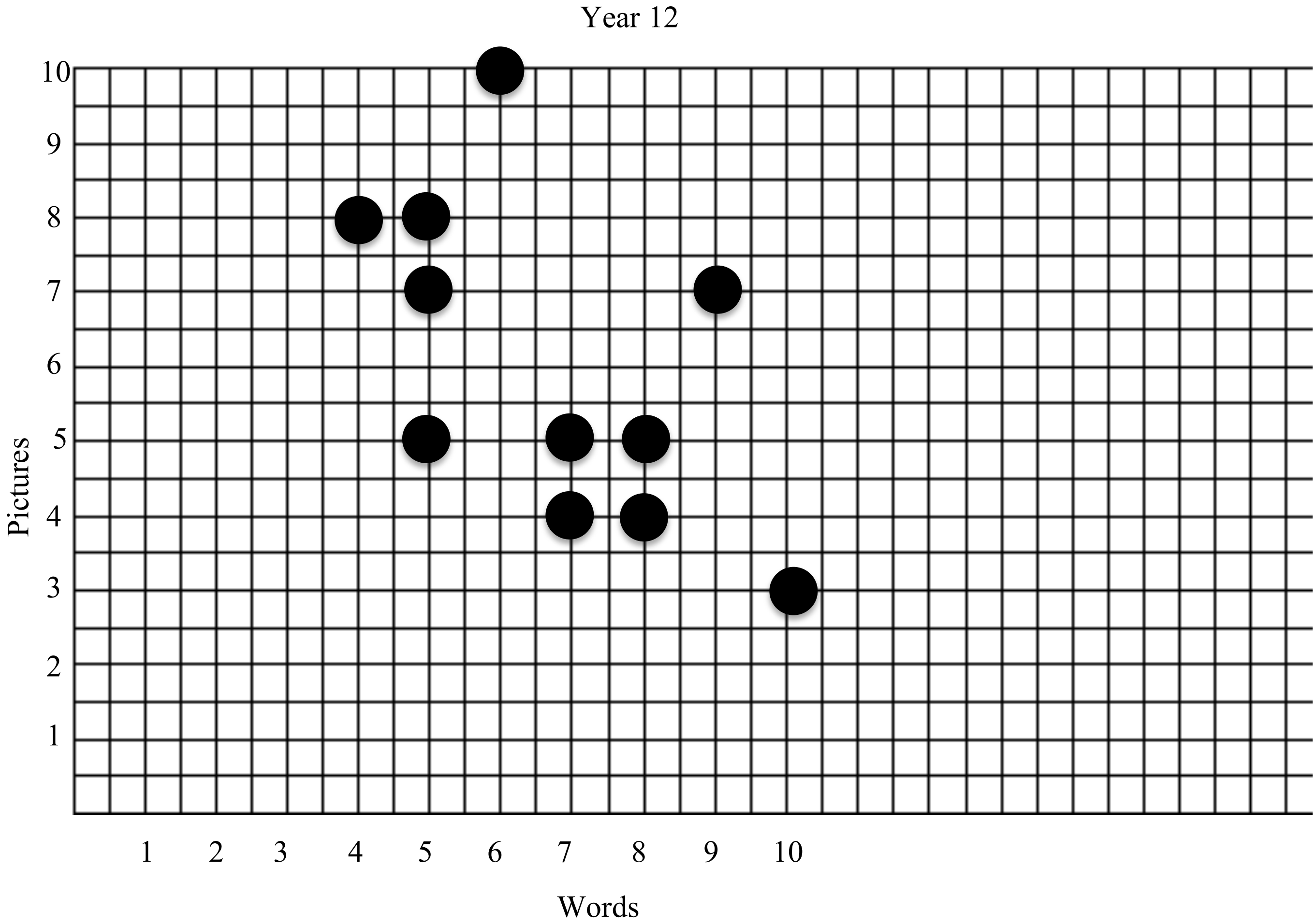
(f) Comment on the relationship between Year level and the number of pictures remembered.

2 marks

Question 2 (7 marks)

a. A scatterplot of the number of words and pictures remembered by the Year 12 students is shown below. One point is missing. Mark this point on the scatterplot.

1 mark



b. In the space below, complete the equation of the least squares regression line for this data? Give all values to one decimal place.

2 marks

Number of pictures remembered = ☐ × *number words remembered* + ☐

(c) Draw this least squares regression line on the above graph.

1 mark

Question 1 (continued)

(d) How many words would you expect a Year 12 student to remember if they remembered 6 pictures accurately?

1 mark

(e) Describe the strength of this linear relationship.

1 mark

(f) What percentage of the variation in the number of pictures remembered can be explained by the variation in the number of words remembered. Give your answer to the nearest whole number.

1 mark

Question 3 (3 marks)

Over several years, a large number of Year 12 students were shown a series of 24 objects and asked to write down the ones they remembered. It was found that the number of objects remembered was normally distributed with a mean of 15 and a standard deviation of 2.

a. What is the probability that a randomly chosen Year 12 student would remember more than 9 objects?

1 mark

____

b. What is the probability that a randomly chosen Year 12 student would remember between 11 and 21 objects?

1 mark

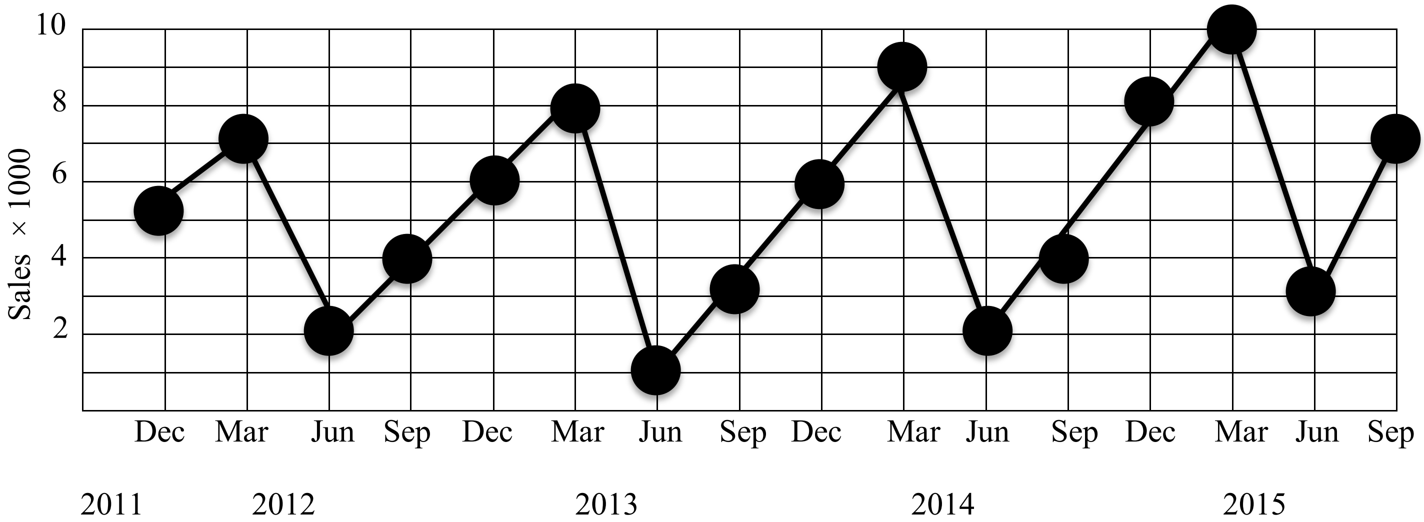
____

c. If 2000 randomly chosen Year 12 students were shown these objects, how many of the students would you expect to remember less than 17 objects?

1 mark

____

Question 4 (4 marks)



The above graph shows the number of Gidgmos sold by the Apex shop for each three - month period from December 2011 till September 2015.

a. Describe the pattern of the data.

1 mark

b. What is the 5-point moving median for December 2013?

1 mark

c. What is the seasonally adjusted value for June 2014?
Give your answer to the nearest dollar.

2 marks

Recursion and financial modelling

Question 5 (4 marks)

Koalas on an island off the coast of Australia are in decline because of habitat destruction. At present there are 1,400 koalas on the island. The recurrence relation to model this situation is given below, where K_n is the number of Koalas predicted to be on the island after n years.

$$K_n = 0.94K_{n-1}$$

a. How many Koalas will be on this island after 2 years as predicted by the above model?

___

1 mark

b. What is the percentage decrease in the number of Koalas on the island each year?

___

1 mark

c. After how many years will the koala population first be reduced to less than half of its present number?

___

1 mark

The committee for the management of Koalas on this island decides to maintain the number of Koalas on the island at 1400. The recurrence relation they use to model this situation is

$$K_n = 0.94K_{n-1} + d$$

d. What is the value of d?

___

1 mark

Question 6 (4 marks)

The committee decides to increase the number of gum trees on the island. Mr. Smith, a committee member, suggests that they increase the number of gum trees by 4% of the number of gum trees on the island at the end of each year. It is observed that 12 trees are destroyed each year by natural disasters. Mr. Smith writes the equation $T_{n+1} = aT_n + b \quad T_0 = 86$ to model this situation.

a. What are the values of a and b in this equation?

2 marks

b. What does Mr Smith's model predict for the future of this gum tree population?

1 mark

c. Mrs Nguyen suggests that they should increase the number of gum trees on the island by 20% instead of 4%.
After how many years will the number of gum trees first be greater than 200?

1 mark

Question 7 (4 marks)

The committee buys machinery to prepare the soil and help plant the trees. They decide to depreciate the machinery at a flat rate of 8% of the purchase price each year. The difference equation for the value of the machinery in any year is $V_{n+1} = V_n - 5200$

a. What is the value of V_0?

1 mark

b. What is the value of the machinery after 5 years?

1 mark

c. If the committee had decided to depreciate the machinery by 12% each year using a reducing balance method, what would be the value of the machinery after 10 years?

1 mark

d. At the end of which year will the value of the machinery first be less using the flat rate method of depreciation as compared to using the reducing balance method?

1 mark

Section B – Modules

Instructions for Section B

Select two modules and answer all questions within the selected modules. Write using blue or black pen. You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

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Module 1 - Matrices

Question 1 (4 marks)

At the Yummy Ice cream store ice cream can be bought in small, medium or large cones.
The Abercrombie family bought 2 small, 3 medium and one large cone and paid $44.
The Bartlett family bought 3 small, 4 medium and 2 large cones and paid $67.
The Chan family bought 5 small, one medium and three large cones and paid $63.

a. If the cost of a small, medium and large cone is \$$x$, \$$y$, and \$$z$, respectively, and

$$AB = \begin{bmatrix} 44 \\ 67 \\ 63 \end{bmatrix} \text{ where } B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

What is the order of matrix B?

1 mark

b. Find matrix A.

1 mark

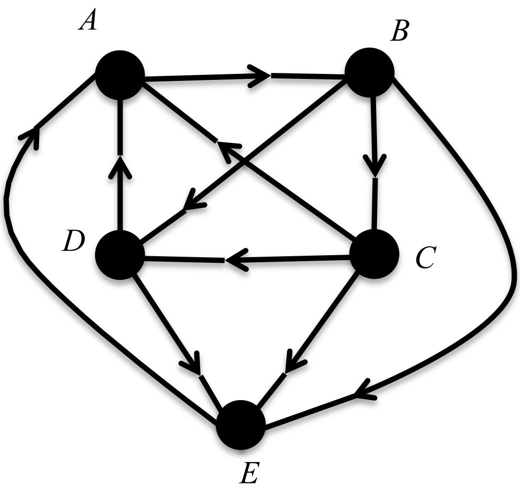
c. Find the inverse of matrix A.

1 mark

d. What is the cost of a large cone?

1 mark

Question 2 (4 marks)



The above diagram shows the results of a round robin where A defeated B.

a. Construct a one-step dominance matrix, W_1, for these results, using 1 for a win and 0 for a loss.

1 mark

b. Construct the two – step dominance matrix, W_2, for the above data.

1 mark

Question 2 (continued)

c. It is decided to weight the second order dominance matrix 40% less than the first- order dominance matrix. Write down this weighted second order dominance matrix, W_3.

1 mark

d. Name the first three place getters when the first-step dominance matrix and the second-step weighted matrix are taken into consideration.

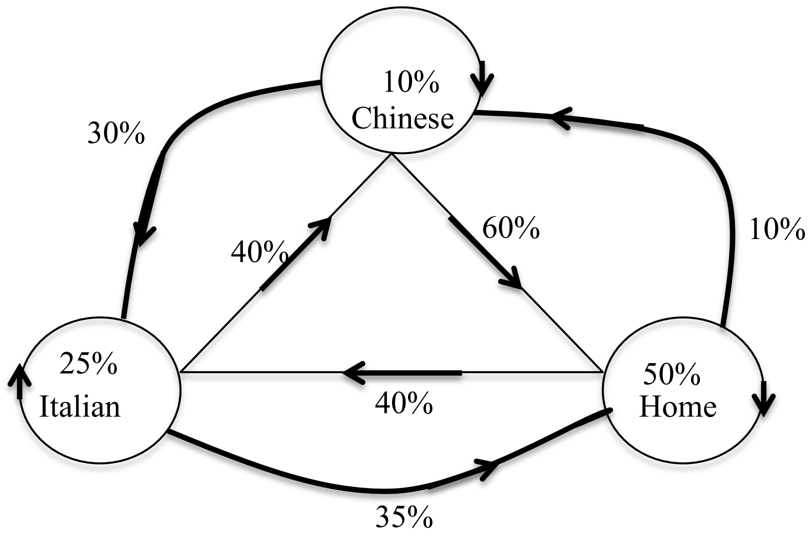
First ____

Second ____

Third ____

1 mark

Question 3 (4 marks)



It was found when a large group of people were surveyed about their nightly eating habits that they either ate at home or went to a Chinese or Italian restaurant. The results are shown in the above diagram, where it can be seen that 25% of people who go to an Italian restaurant one night go to an Italian restaurant the next night.

a. Complete the transition matrix below for the above data.

From

$$\text{To}\;\begin{matrix} I \\ C \\ H \end{matrix}\overset{\begin{matrix} I & C & H \end{matrix}}{\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}}$$

1 mark

b. On Monday night, 2000 people ate at home, 750 ate at an Italian restaurant and 1250 ate at a Chinese restaurant. Write down the initial state matrix, S_0

1 mark

Question 3 (continued)

c. How many people will be expected to eat at a Chinese restaurant on Wednesday night?

____

1 mark

d. In the long term how many people do you expect to eat at a restaurant?

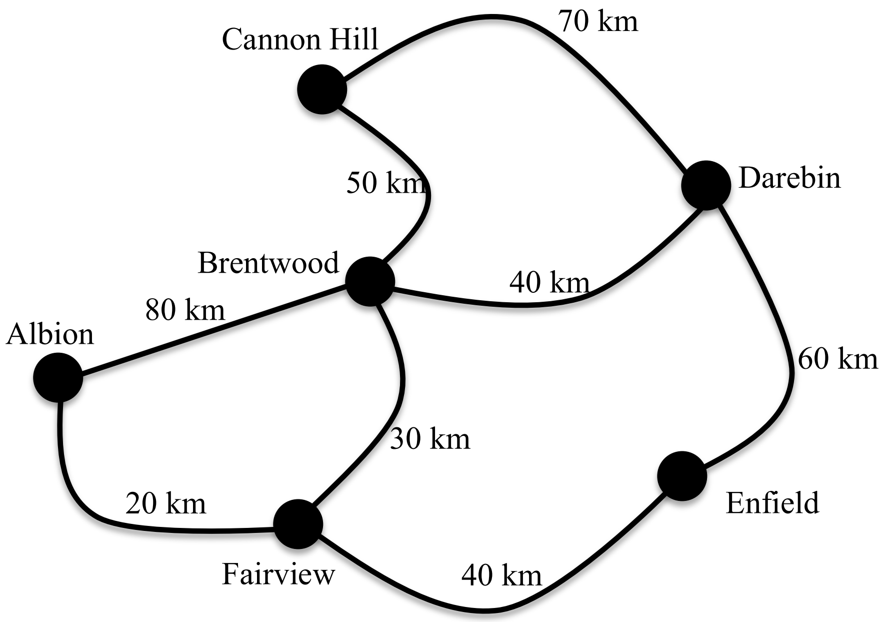
____

1 mark

End of Module 1: Matrices

Module 2: Networks and decision mathematics

Question 1 (6 marks)



The diagram above shows the network of roads between six towns: Albion, Brentwood, Cannon Hill, Darebin, Enfield and Fairview. The distances in kilometres between the connected towns are shown on the diagram.

a. What is the shortest distance from Canon Hill to Enfield?

____

1 mark

b. Jenny needs to leave parcels in each of the above towns. If she begins at Albion and has to finish in this same town, what is the shortest distance that she will have to travel?

____

1 mark

Question 1 (continued)

c. Jerome wants to travel a Hamiltonian path. What is the shortest distance he can travel?

1 mark

Bo followed an Eulerian path through the above network.

d. **i.** Write down the names of the towns where she would start and end her journey.

1 mark

ii. What is the distance that she would travel on this journey?

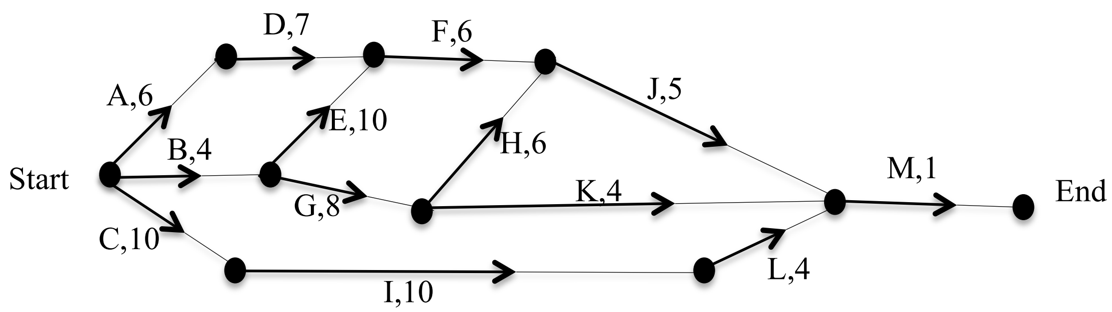
1 mark

The council intends to remove the path between Brentwood and Darebin and build a new path.

e. Between which two towns should this new path be built, so that an Eulerian circuit will exist?

1 mark

Question 2 (6 marks)



A project manager, Darren, identifies 13 activities, A to M, that must be completed before the building he is working on can be finished. Certain activities must be completed before other activities can be started. Darren draws the above diagram to show this and also includes the number of days that each activity will take to complete.

a. Which of the 13 activities must be completed before activity J can begin?

1 mark

b. What is the critical path for this project?

1 mark

c. What is the minimum number of days it will take to complete this project?

1 mark

d. What is the latest starting time for activity G?

1 mark

Question 2 (continued)

e. There is a clause in the contract that says that Darren will be charged $1000 per day for every day over 24 days that the project takes. He finds a worker who can do activity B for $150 per day for up to 3 days and reduce the time taken for this activity by the number of days he works. He also finds a worker who can do activity I for $120 per day for up to 2 days and reduce the time taken for this activity by the number of days he works. Darren wants to complete the project for the minimum cost.

i. What is the minimum number of days that the project will now take?

____

1 mark

ii. How much will Darren save by employing an extra worker/workers?

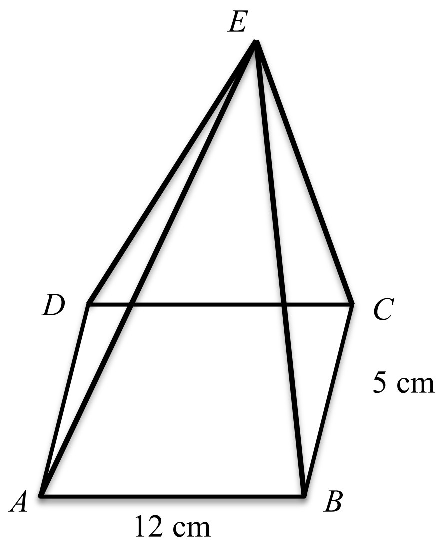
____

1 mark

End of Module 2: Networks and decision mathematics

Module 3: Geometry and measurement

Question 1 (5 marks)



The above right pyramid has a rectangular base with length 12 cm and width 5 cm. The perpendicular height of the pyramid is 20 cm.

a. i. What is the length of BD?

1 mark

ii. What is the length of BE? Give your answer to 2 decimal places.

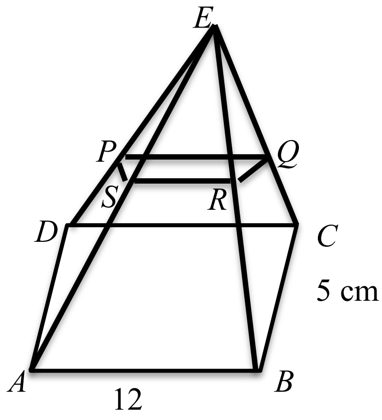
1 mark

iii. What is the size of $\angle EBD$? Give your answer to the nearest degree.

1 mark

Question 1 (continued)

A small right pyramid, *PQRSE*, is cut from the top of the given pyramid. The perpendicular height of this small pyramid is 5 cm.



b. i. What is the length of the diagonal *PR*?

___

___

___

1 mark

ii. What is the volume of this small right pyramid?

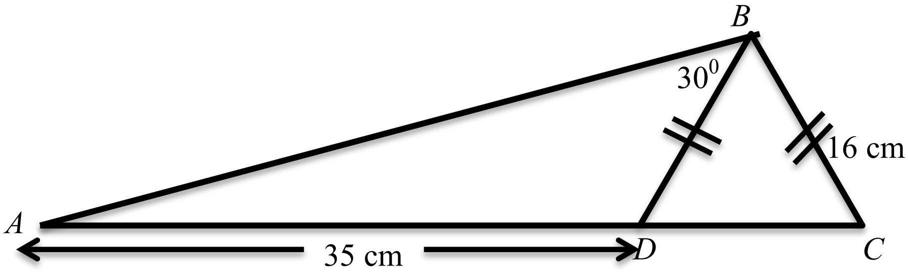
___

___

___

1 mark

Question 2 (3 marks)



In the above figure, AD = 35 cm, BC = 16 cm and $\angle ABD = 30^0$

a. What is the size of $\angle BDC$? Give your answer to 1 decimal place.

2 marks

b. What is the length of AC? Give your answer to 1 decimal place.

1 mark

Question 3 (2 marks)

a. Lisa, who lives in Melbourne, wants to ring here sister in London at 6:00 am on Monday, Melbourne time. At what time will her sister receive the call if Melbourne is 10 hours ahead of London?

1 mark

b. Two towns, A (20^0S, 30^0E) and B (x^0N, 30^0E) are 8,936 km apart along the meridian. What is the value of x?

1 mark

End of Module 3: Geometry and measurement

Module 4 : Graphs and relations

Question 1 (3 marks)

Bill's company produces phones at a fixed cost of \$12,000 per month. The cost of producing each phone is \$320.

a. Write down an equation for the cost, $\$C$, of producing n phones in a month.

____

1 mark

b. Bill sells his phones for \$480 each. Write down an equation for the revenue, R, that he receives when he sells n phones.

____

1 mark

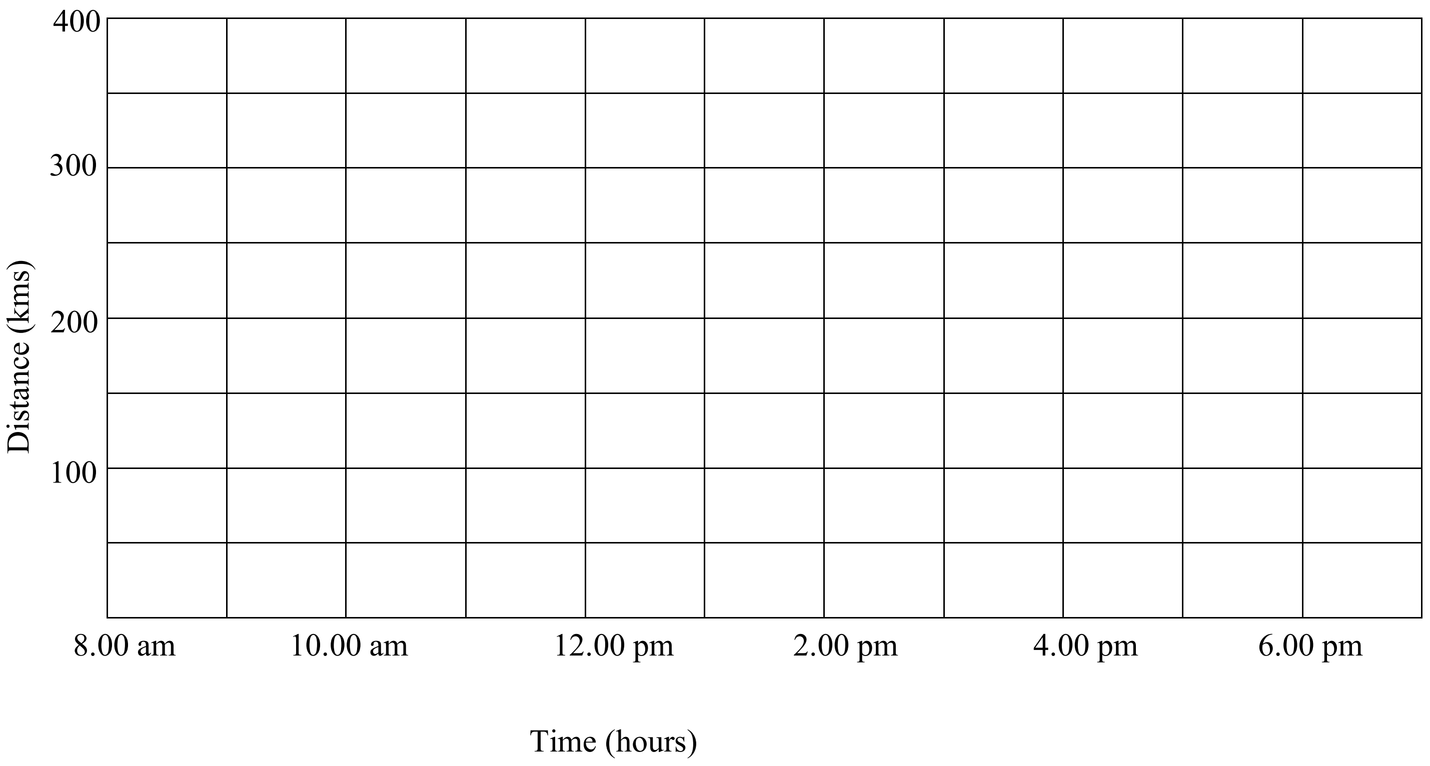
c. How many phones does Bill have to sell in a month to break even?

____

1 mark

Question 2 (3 marks)

Alice leaves home at 8:00 am and drives for 2 hours at 60 km/hr. She then continues at 85 km/hr for the next two hours and then she stops for an hour for lunch. After lunch she travels for 1 hour at 70 km/hr. At this point she takes 30 minutes to unload her supplies, after which she travels straight home at 90 km/hr.



a. At what time will she arrive home?

____

1 mark

b. On the grid above, draw the graph of Alice's travels.

1 mark

c. What was her average speed for the whole trip? Give your answer to one decimal place.

____

1 mark

Question 3 (4 marks)

Susan runs a small cottage industry making ceramic jugs and teapots. The number of items that she can produce each day is limited by the size of her kiln. This can take at least 24 pieces a day but not more than 48. The ability of her workers demands that the number of jugs produced each day is at least double the number of teapots. Previous orders require that the number of jugs produced each day is at least 26.
The profit on a teapot is $50 and the profit on a jug is $20

a. Four of the constraints for this project are listed below.
What is the missing constraint?

$x + y \geq 24$

$x + y \leq 48$

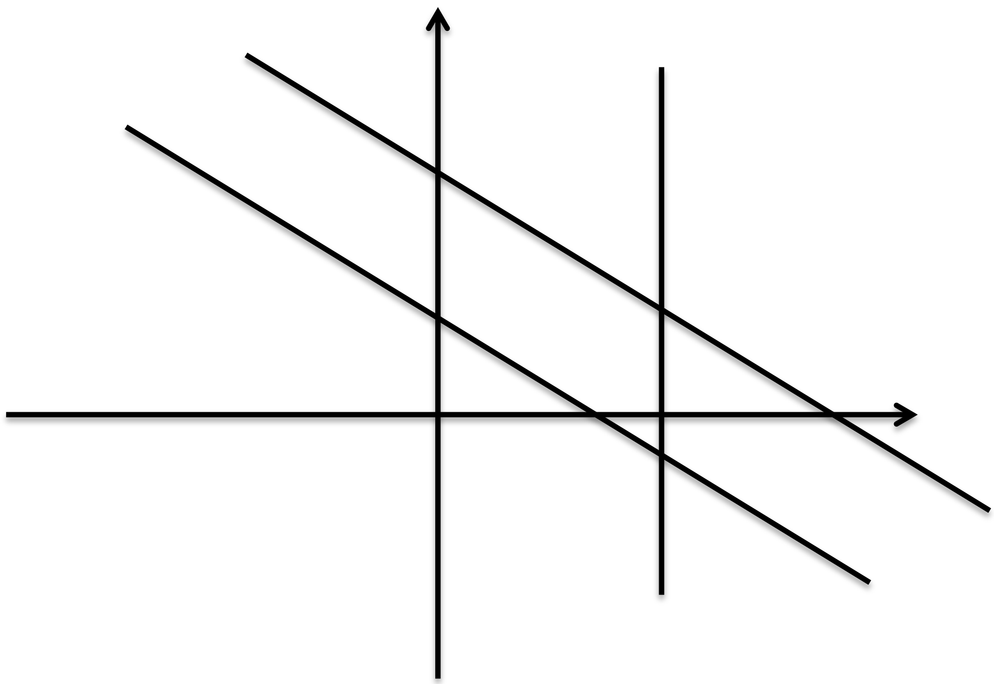
$x \geq 26$

$y \geq 0$

---

1 mark

b. Complete the graph below to show the missing line and shade the required region.



1 mark

Question 3 (continued)

c. What is the maximum profit?

____

1 mark

d. How many teapots should be produced each day to ensure a maximum profit?

____

1 mark

End of Module 4 : Graphs and relations

End of 2016 VCE Further Mathematics Trial Examination 2
Question and Answer Book

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PO Box 2227	**Fax: (03) 9817 4334**
Kew Vic 3101	**kilbaha@gmail.com**
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FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core: Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1: Matrices

determinant of a 2 × 2 matrix:	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix:	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$
recurrence relation:	$S_0 =$ initial state, $S_{n+1} = TS_n + B$

Module 2: Networks and decision mathematics

Euler's formula:	$v + f = e + 2$

Module 3: Geometry and measurement

area of a triangle:	$A = \frac{1}{2}bc\sin(\theta^0)$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$
sine rule:	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule:	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle:	$2\pi r$
length of an arc:	$r \times \frac{\pi}{180} \times \theta^0$
area of a circle:	πr^2
area of sector	$\pi r^2 \times \frac{\theta^0}{360}$
volume of a sphere:	$\frac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a prism:	area of base $\times$ height
volume of a pyramid:	$\frac{1}{3}$ $\times$ area of base $\times$ height

Module 4: Graphs and relations

gradient (slope) of a straight line:	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line:	$y = mx + c$

END OF FORMULA SHEET