2015
VCE
Further Mathematics
Trial Examination 1
Suggested Answers

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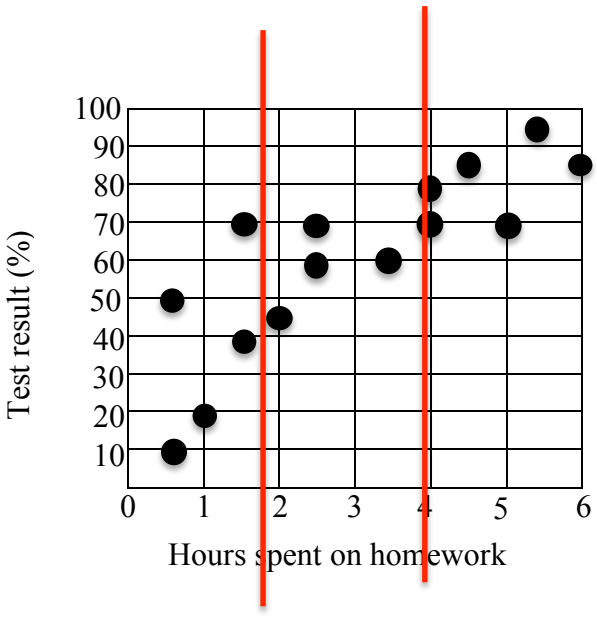
Core: Data analysis

Question 1 D Categorical data can be put in categories. Numerical data measured. Language spoken at home, Eye colour, Post codes and Birth dates are categorical data. Arm span and Time taken to travel to school are numerical data.	**Question 2 D** Median = 91.9 UQ = 93.6 Top 25% lie above the upper quartile.
Question 3 E A mean of 4.3 in ten games means a total of 43. Total at the end of 9 games $= 3 + 4 + 1 + 2 + 6 + 2 + 8 + 4 + 5 = 35$ $43 - 35 = 8$	**Question 4 B** 140 chose swimming. 60 of these were boys $\%\text{ boys} = \frac{60}{140} \times 100 = 43\%$
Question 5 D Number of students = 2 + 3 + 1 + 4 + 7 + 8 + 9 + 10 + 10 = 54	**Question 6 A** There are 54 test scores. The middle score is between the 27th and 28th scores. Theses scores occur in the column that is 9 high, which is between 70 and 80. They are the second and third scores in this group, out of 9 scores, so will be closer to 70 than to 80. The graph is negatively skewed and the spread is 100 – 10 = 90.

Question 7 C



The lower and upper median points are (1, 40) and (5, 85) respectively.

$$\text{Gradient} = \frac{85-40}{5-1} = 11.25$$

Core: Data analysis

Question 8 A

Number of accidents

$= 3.01 + 0.12 \times$ *number of hours worked*

$8 = 3.01 + 0.12 \times n$

Use solve on calculator to get

$n = 42$

Question 9 C

$b = 0.12$

$S_x = 13.18$

$S_y = 1.98$

$$r = \frac{bS_x}{S_y} = \frac{0.12 \times 13.18}{1.98} = 0.8$$

Question 10 B

25 + 5 + 15 + 20 = 65 ÷ 4 = 16.25

5 + 15 + 20 + 12.5 = 52.5 ÷ 4 = 13.125

16.25 + 13.125 = 29.375 ÷ 2

= 14.6875 thousand.

this is closest to $14,700

Question 11 A

SI for winter = 4 – (1.14 + 1.2 + 0.91) = 0.75

Deseasonalised figure = Actual figure ÷ SI

1080 ÷ 0.75 = $1440

Question 12 E

From the graph it can be seen that the correlation is negative, so not A or B.

The variation in *y* is not CAUSED by the variation in *x*, so not C.

$r = \sqrt{0.49} = 0.7$

0.7 is a moderate, not a weak correlation, so not D.

Question 13 A

$$Z = \frac{95 - 65}{10} = 3$$

0.15% lie above 3 standard deviations from the mean.

$$\frac{0.15}{100} \times 6000 = 9$$

Module 1 Number patterns

Question 1 B

This is an arithmetic sequence. The simplest way to do this is just to say
2015 salary is \$38000
2016 salary is \$38000 + 580 = \$38580
2016 salary is \$38580 + 580 = \$39160

Question 2 C

This is an arithmetic sequence because a constant amount is added each year, so not B or E.
We want the total amount earned in the five years so we want the sum of terms not a particular term, so not A.
This arithmetic sequence will have $a = 38000$, $n = 5$ and $d = 580$, so not D.

Question 3 D

$A = 800(1+0.35)^6 = 4843$

Question 4 E

To form this pattern multiply each term by 3 and then subtract 2.

$t_{n+1} = 3t_n - 2 \quad t_1 = 3$

Question 5 E

$d = -4$

$n = 7$

$S_n = 700$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$700 = \frac{7}{2}[2a + (7-1)\times -4]$

Use calculator to solve this equation and get $a = 112$.

Question 6 A

The t_n values are 12, 6, 3, 1½
If Peter keeps going in this way, the sum of these terms will be the sum to infinity of a geometric sequence with $a = 12$ and $r = ½$

$S_\infty = \frac{a}{1-r} = \frac{12}{1-\frac{1}{2}} = 24$

Module 1 Number patterns

Question 7 D

$a = f$

$r = g$

$n = 4$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_4 = \dfrac{f(1-g^4)}{1-g}$

Question 8 A

$u_5 = 3u_4 + u_3$

$195 = 3 \times 59 + u_3$

$u_3 = 18$

$u_4 = 3u_3 + u_2$

$59 = 3 \times 18 + u_2$

$u_2 = 5$

$u_3 = 3u_2 + u_1$

$18 = 3 \times 5 + u_1$

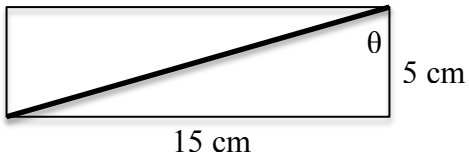
$u_1 = 3$

Question 9 C

Let the original amount of the substance be A and let the number of 25 years to half-life be n.
Decay = 1%
Amount remaining = 99% = 0.99
When half remains, then $0.5A$ remains.
$0.5A = A \times 0.99^n$
Solve on calculator to get $n = 68.97$
Total number of years = $68.97 \times 25 = 1724$ years.

Module 2 Geometry and trigonometry

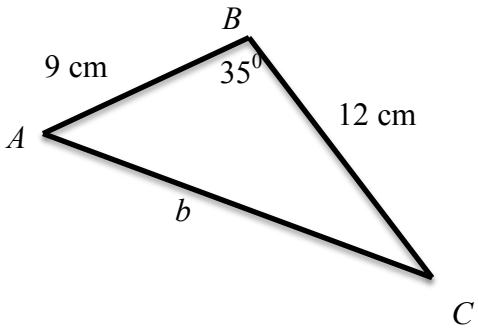
Question 1 E



$$\tan\theta = \left(\frac{15}{5}\right)$$
$$\theta = \tan^{-1}\left(\frac{15}{5}\right)$$
$$\theta = 71.56^0$$

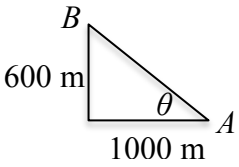
Closest to 72^0

Question 2 B



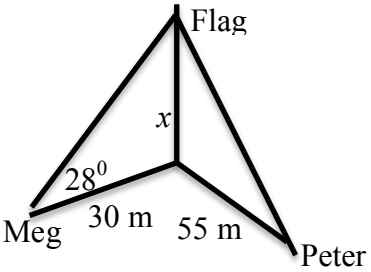
$$b^2 = 9^2 + 12^2 - 2\times 9\times 12\times\cos 35^0$$
$$b = \sqrt{9^2 + 12^2 - 2\times 9\times 12\times\cos 35^0}$$
$$AC = 7 \text{ cm}$$

Question 3 C



$$\tan\theta = \left(\frac{600}{1000}\right)$$
$$\theta = \tan^{-1}\left(\frac{600}{1000}\right) = 31^0$$

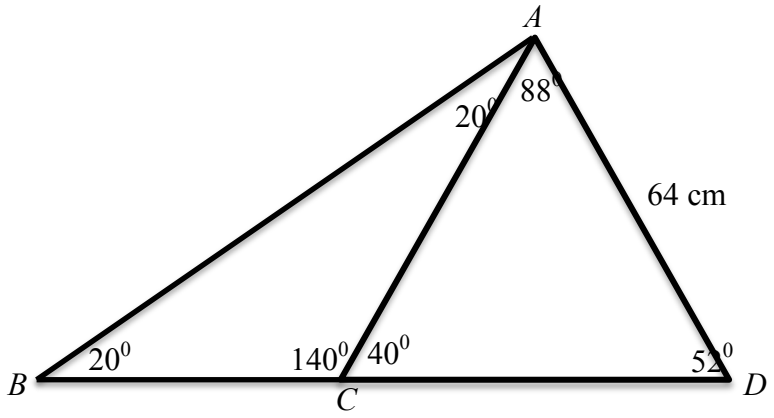
Question 4 B



$$\tan 28^0 = \frac{x}{30}$$
$$x = 15.95$$
$$\theta = \sin^{-1}\left(\frac{15.95}{55}\right) = 16^0$$

Module 2 Geometry and trigonometry

Question 5 A



Angles in a triangle add to 180^0 and angles in a straight line add to 180^0. Fill in all the angles.
Triangle ABC is isosceles, with $BC = AC$.

$$\frac{AC}{\sin 52^0} = \frac{64}{\sin 40^0}$$

$AC = 78.459$

$BC = 78.5$ cm.

Question 6 E

$$V = \frac{1}{3} \times \pi \times r^2 \times 3 + \pi \times r^2 \times 9.8 = 300$$

$r = 3$

Question 7 D

$$A = \frac{1}{2} bc \sin\theta$$

$$20 = \frac{1}{2} \times 11.3 \times 4.5 \times \sin Z$$

$$Z = 51.87^0$$

$XY =$

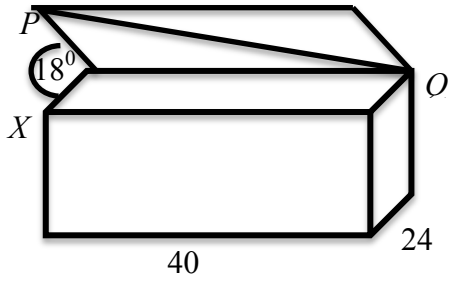
$$\sqrt{11.3^2 + 4.5^2 - 2 \times 11.3 \times 4.5 \times \cos\left(51.87^0\right)}$$

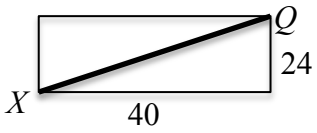
$XY = 9.2$

Perimeter = 9.2 + 11.3 + 4.5 = 25 cm.

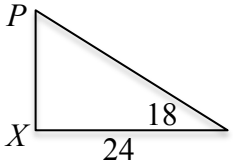
Module 2 Geometry and trigonometry

Question 8 D



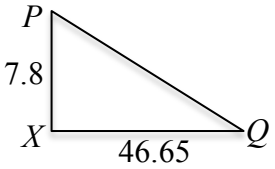


$QX = \sqrt{40^2 + 24^2} = 46.65$



$\tan 18^0 = \frac{PX}{24}$

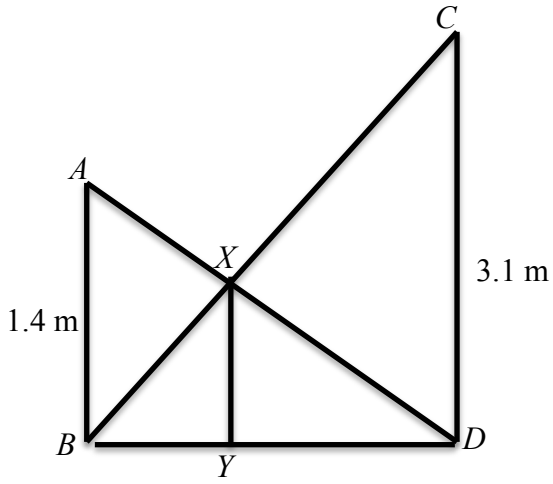
$PX = 7.8$



$\tan \angle PQX = \frac{7.8}{46.65}$

$\angle PQX = 9.49^0$

Question 9 C



Triangles ABD and XYD are similar (AAA)

$\frac{XY}{1.4} = \frac{YD}{BD} \Rightarrow XY = \frac{1.4 \times YD}{BD}$

Triangles BCD and BXY are similar (AAA)

$\frac{XY}{3.1} = \frac{BY}{BD} \Rightarrow XY = \frac{3.1 \times BY}{BD}$

$\frac{1.4 \times YD}{BD} = \frac{3.1 \times BY}{BD}$

$1.4YD = 3.1BY$ so $1.4YD = 3.1(BD - YD)$

$1.4YD = 3.1BD - 3.1YD \Rightarrow 4.5YD = 3.1BD$

$\frac{YD}{BD} = \frac{3.1}{4.5}$

$XY = \frac{1.4 \times 3.1}{4.5} = 0.96$ m

Module 3 Graphs and relations

Question 1 D
Lines parallel to the x axis have the equation y = a constant. The y value of the given point is 6 so $y = 6$.

Question 2 B
Temperatures for April, May, September, October and November are between 15^0C and 30^0C. This is 5 months.

Question 3 B
Between May and June the difference in the average maximum daily temperature was about 12.5^0C. This is the largest change.

Question 4 E
$y = mx + c$

$$m = \frac{-4-6}{-6+4} = 5$$

$y = 5x + c$

When x = -4, $y = 6$

$6 = -20 + c$

$c = 26$

$y = 5x + 26$

Question 5 A
When $q = 4, p = 200$

$$200 = \frac{k}{4}$$

$k = 800$

When $p = 1000$

$$1000 = \frac{800}{q}$$

$q = 0.8$

Question 6 E
$2y = x + 7$

$$y = \frac{1}{2}x + \frac{7}{2}$$

$$m = \frac{1}{2}$$

Gradient of perpendicular line = -2

$y = -2x + c$

When $x = 8, y = 4$

$4 = -16 + c$

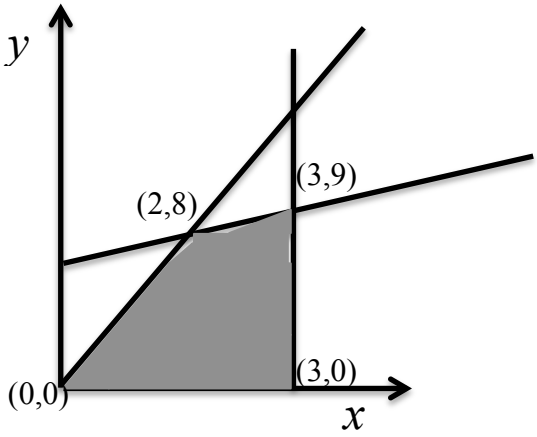
$c = 20$

$y = -2x + 20$

$2x + y - 20 = 0$

Module 3 Graphs and relations

Question 7 B



Intersection of $x = 3$ and $y = x + 6$ is (3,9)
Intersection of $y = 4x$ and $y = x + 6$
$4x = x + 6$
$x = 2$
$y = 8$

At (2,8) Z = 8 – 4 = 4
At (3,9) Z = 9 – 6 = 3
At (3,0) Z = 0 – 6 = -6
At (0,0) Z = 0 – 0 = 0
Maximum value = 4

Question 8 C

Break even is when revenue, R, equals cost of production, C.
Let x = number of urns produced and sold each week.
$R = 140x$
$C = 80x + 3000$
$140x = 80x + 3000$
$x = 50$
So, A is true.
$P = R - C$
$P = 140x - (80x + 3000) = 60x - 3000$
When $x = 80, P = 60 \times 80 - 3000 = \$1800$
So B is true and C is false.
For 100 urns $C = 8000 + 300 = 11000$
Average cost $= 11000 \div 100 = \$110$
So D is true.
$60x - 3000 = 21000$
$x = 400$
So E is true.

Question 9 C

For no solutions the lines must be parallel. Hence, y intercepts must be different and gradients must be the same.

$2y = ax - 8$

$y = \frac{a}{2}x - 4$

$3y = -x + b$

$y = -\frac{1}{3}x + \frac{b}{3}$

$\frac{a}{2} = -\frac{1}{3}$

$a = -\frac{2}{3}$

$\frac{b}{3} \neq -4$

$b \neq -12$ but b can equal any other value.

Module 4 Business-related mathematics

Question 1 D

$$I = \frac{PRT}{100}$$

$$= \frac{320 \times 4 \times 6}{100} = 76.80$$

$$A = 320 + 76.80 = \$396.80$$

Question 2 E

112½% = 5850

1% = 5850 ÷ 112½

100% = 5850 ÷ 112½ × 100 = $5200

Question 3 D

$$A = P(1 + \frac{r}{100})^n$$

$$A = 4.8 \times (1 + \frac{10}{100})^{15}$$

$$A = \$20$$

Question 4 A

Amount that Kara pays = 300 + 24 × 110 = 2940

Interest = 2940 – 2400 = 540

Amount borrowed = 2400 – 300 = 2100

$$I = \frac{PRT}{100} = \frac{2100 \times R \times 2}{100} = 540$$

$$R = 12.9\%$$

Question 5 C

Original annual pay = $78000

Tax on this = 3572 + 0.325 × (78000 – 37000) = 16897

Annual take home pay = 78000 – 16897 = $61103

New annual pay = $78000 × 1.2 = 93600

Tax on this

= 17547 + 0.37 × (93600 – 80000) = 22579

Annual take home pay

= 93600 – 22579 = $71021

Extra annual pay = 71021 – 61103 = $9918

Extra fortnightly pay = 9918 ÷ 26 = $381.46

Question 6 C

Unit cost = 0.5 ÷ 5000

Cost for 400000 = 400000 × 0.5 ÷ 5000 = $40

Book Value 620 – 40 = $580

Module 4 Business-related mathematics

Question 7 B

Use TVM solver

$N = 96$

$I = 6$

$PV = -2{,}000$

$PMT = -150$

$FV =$

$P/Y = 12$

$C/Y = 12$

This gives $FV = \$21{,}653$

Question 8 B

Jane

$$I = \frac{PRT}{100} = \frac{200 \times 3 \times 20}{100} = 120$$

$A = 200 + 120 = \$320$

James

$$A = 100\left(1 + \frac{r}{100}\right)^{20}$$

$$320 = 100\left(1 + \frac{r}{100}\right)^{20}$$

$$3.2 = \left(1 + \frac{r}{100}\right)^{20}$$

Use solve on calculator to get

$r = 6\%$

Question 9 D

$R = 1.006$

$$1 + \frac{r}{100} = 1.006$$

$$\frac{r}{100} = 0.006$$

$r = 0.6\%$ which is the monthly rate

Annual rate = $0.6 \times 12 = 7.2\%$

Module 5 Networks and decision mathematics

Question 1 C

The given diagram has vertices that are odd such as P, U, S and Q. It has vertices that are even such as T, V and R.

Connecting P and V makes P even and V odd, so 4 even and 3 odd is not traversable. Hence, not A.

Connecting T and S makes S even and T odd, so 4 even and 3 odd is not traversable. Hence, not B.

Connecting R and S makes S even and R odd, so 4 even and 3 odd is not traversable. Hence, not D.

Connecting R and T makes R odd and T odd, so 1 even and 4 odd is not traversable. Hence, not E.

Connecting U and Q makes U even and Q even, so 5 even and 2 odd is traversable. Hence, C.

Question 2 E

Need 5 vertices for the five people, so not D.

Need 4 lines from each vertex to represent the 4 people that each player plays, so E.

Question 3 A

Critical path is the longest path.

B – D is longer than A – C, so not answer B.

B – D is longer than B – E – F – G, so not D or E.

B – D – G – J is not a path, so not C.

Question 4 D

Subtract minimum in each row from every element in the row to get

40	0	15	20
55	25	0	30
0	55	70	70
35	25	25	0

From this it can be seen that

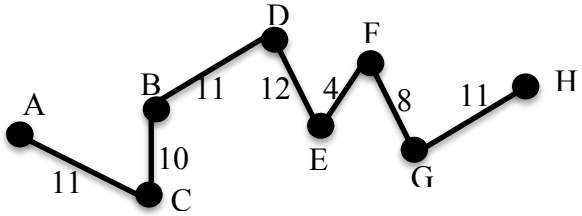
Dianna does S.

Adrian does R.

Ben does P.

Catherine does Q.

Question 5 C

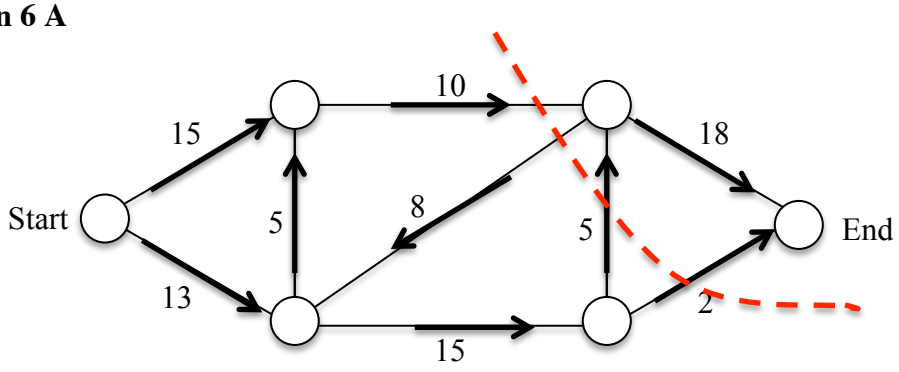


Length of cable

= 4 + 8 + 11 + 12 +11 + 10 + 11 = 67 km

Module 5 Networks and decision mathematics

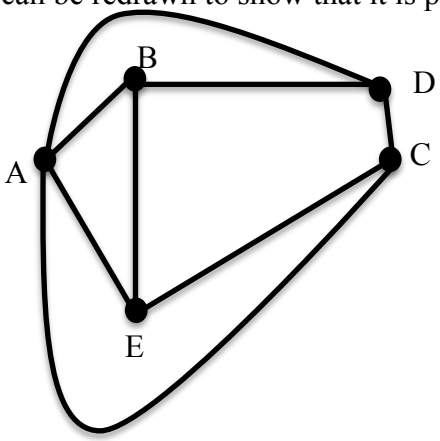
Question 6 A



Minimum cut = Maximum flow.
10 + 0 + 5 + 2 = 17 km

Question 7 D

The graph can be redrawn to show that it is planar by moving vertex D.

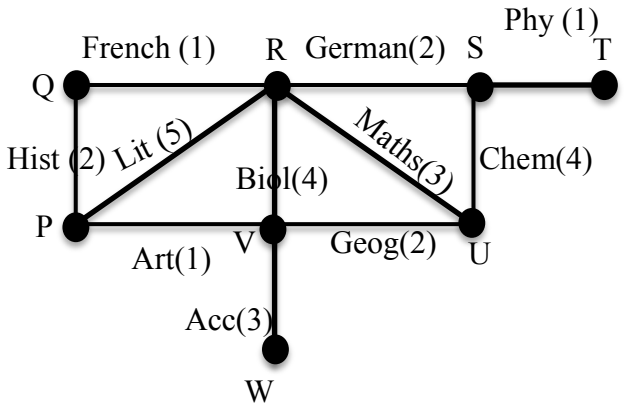


From here it can be seen that it is planar with 5 vertices and 5 faces and so A and C are true but B is false. Degenerate graphs have all vertices isolated so this graph is not degenerate. Complete graphs have every vertex linked to every other vertex, so this graph is not complete.

Question 8 B

A simple graph has no loops.
A, D and E have loops because of the non-zeros on the leading diagonal.
A simple graph has no multiple edges so all the non-zeros must be 1.
C has multiple edges.

Question 9 B



From the above diagram you can see that the minimum number of days to ensure no student has 2 exams on the one day is 5.
P has an exam on days 1, 2 and 5.
Q has an exam on days 1 and 2.
R has an exam on days 1, 2, 3, 4 and 5.
S has an exam on days 1, 2 and 4.
T has an exam on day 1.
U has an exam on days 2, 3, and 4.
V has an exam on days 1, 2, 3 and 4.
W has an exam on day 3.

Module 6 Matrices

Question 1 A

The element in the first row, fourth column is 2.

Question 2 D

Use a calculator or

$$2Y = \begin{bmatrix} 6 & -10 & 4 \\ 8 & 4 & -2 \end{bmatrix}$$

$$X - 2Y = \begin{bmatrix} 2-6 & 3+10 & 7-4 \\ 0-8 & -1-4 & 4+2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 13 & 3 \\ -8 & -5 & 6 \end{bmatrix}$$

Question 3 E

The determinant = 2 × 2 – 1 × -3 = 4 + 3 = 7
or you can use calculator.

Question 4 B

$$\begin{array}{c} \\ Tues \\ Wed \\ Thurs \\ Fri \end{array} \begin{array}{c} \begin{array}{cccc} LF & S & FF & O \end{array} \\ \begin{bmatrix} 20 & 12 & 8 & 2 \\ 14 & 16 & 10 & 5 \\ 10 & 12 & 11 & 3 \\ 24 & 16 & 8 & 1 \end{bmatrix} \end{array} \begin{bmatrix} 1.45 \\ 1.60 \\ 1.30 \\ 2.00 \end{bmatrix}$$

$$= \begin{bmatrix} 62.6 \\ 68.9 \\ 54 \\ 72.8 \end{bmatrix}$$

Thursday sales = $54.00

Question 5 C

From Q to P is 100 – (50 + 30) = 20% = 0.2
Hence, answer must be C

Question 6 E

$$\text{Let } A = \begin{bmatrix} 3 & -2 & 1 \\ -1 & 4 & -2 \\ 0 & -8 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 1 \\ -1 & 4 & -2 \\ 0 & -8 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \\ 5 \end{bmatrix}$$

$$A^{-1}A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} -6 \\ -3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} -6 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 7 \end{bmatrix}$$

$z = 7$

Module 6 Matrices

Question 7 D

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} d & b \\ c & a \end{bmatrix} = \begin{bmatrix} a-d & 0 \\ 0 & d-a \end{bmatrix}$$

Inverse does not exist if $(a-d)(d-a)=0$

$a=d$

Question 8 A

$y = 2x \Rightarrow 2x - y = 0$

$4z = x + y \Rightarrow x + y - 4z = 0$

$18x + 12y + 10z = 5940$

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & -4 \\ 18 & 12 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5940 \end{bmatrix}$$

Question 9 C

$B \times B = B$

$B \times I = B$

$\therefore B = I$

so A is true.

$CDE = m \times n \times n \times m \times m \times p$

$= m \times m \times m \times p$

$= m \times p$

so B is true.

$(B+I) \times (B+I) = B^2 + 2BI + I^2$, so C is false.

CD is an $m \times m$ matrix. $CD + F$ is an $m \times m$ matrix, which is a square matrix. Hence, D is true.

$B \times B$ is a 2×2 matrix $\times$ **B** is a 2×2 matrix and so on, so E is true.

End of Suggested Solutions 2015 VCE Further Mathematics Trial Examination 1

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