insight™

YEAR 12 *Trial Exam Paper*

2014

FURTHER MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- correct solutions with full workings
- mark allocations
- tips on how to approach the questions

Core

Question 1a.

Worked solution

Stem	Leaf
0	0
1	5
2	0, 5
3	0, 0, 5, 8
4	0, 5, 5, 5, 7, 8, 9
5	0, 4, 7, 8
6	
7	
8	0

Key 4|5 = 45 minutes

Mark allocation: 1 mark

- 1 mark for all data correct.

Tip

- *Stems 6 and 7 have no leaves (because there are no numbers in the 60s and 70s) but must be shown if there are numbers larger and smaller than these values in the data.*

Question 1b.

Worked solution

Summary statistic	Value
minimum	0
Q_1	30
median	45
Q_3	49.5
maximum	80
mean	40.6
standard deviation	17.5

Mark allocation: 1 mark

- 1 mark for all data correct.

Question 1c.

Worked solution

$$\text{upper fence} = Q_3 + (1.5 \times \text{IQR})$$
$$\text{upper fence} = 49.5 + (1.5 \times 19.5)$$
$$\text{upper fence} = 78.75$$

Because 80 is higher than 78.75, it is an outlier.

$$\text{lower fence} = Q_1 - (1.5 \times \text{IQR})$$
$$\text{lower fence} = 30 - (1.5 \times 19.5)$$
$$\text{lower fence} = 0.75$$

Because 0 is lower than 0.75, it is an outlier.

Mark allocation: 2 marks

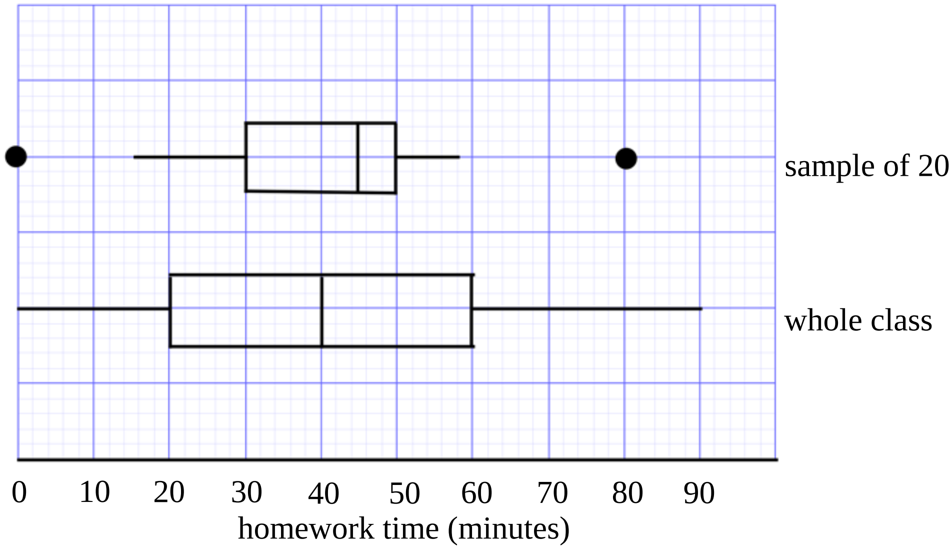
- 1 mark for both fences.
- 1 mark for both outliers.

Tip

- *Values equal to a fence are not considered to be outliers.*

Question 1d.

Worked solution



Mark allocation: 2 marks

- 2 marks for correct position of boxplot.

Question 1e.

Worked solution

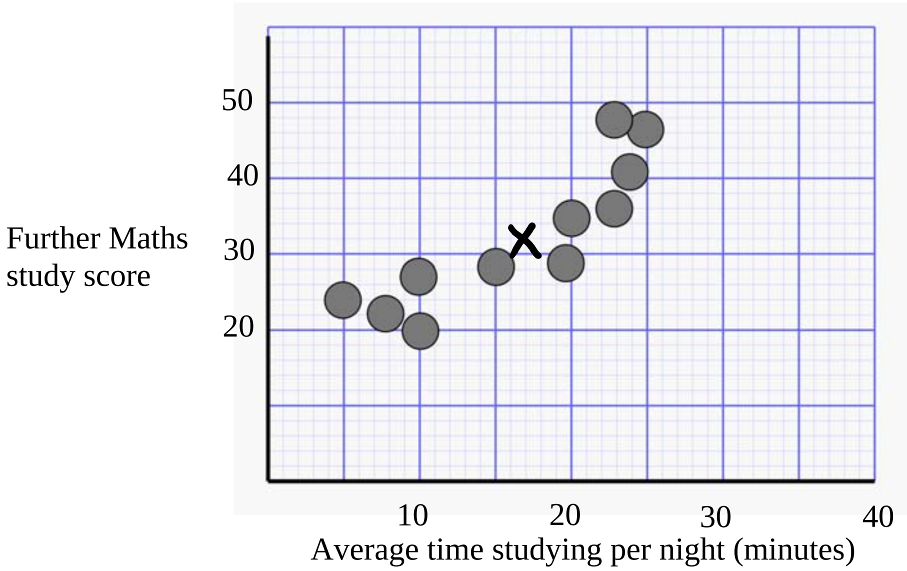
The whole class data is more widely spread with an IQR of 40 and a range of 90, while the sample of 20 students has an IQR of 19.5 and a range of 80.
The data from the sample of 20 students is negatively skewed, while the data from the whole class is approximately symmetrical.

Mark allocation: 2 marks

- 1 mark for each difference.

Question 2a.

Worked solution



Mark allocation: 1 mark

- 1 mark for placement of X.

Question 2b.

Worked solution

$r^2 = 0.785$
Of the variation in study scores, 78.5% can be explained by variation in the average minutes of study per night.

Mark allocation: 2 marks

- 1 mark for r^2.
- 1 mark for interpretation.

Question 2c.

Worked solution

$r^2 = 0.816$

Mark allocation: 1 mark

- 1 mark for correct answer.

Question 2d.

Worked solution

$$\text{solve}\left(\log_{10}(x)=0.016\cdot 30+1.228,x\right)$$
$$x=51.0504999975$$

By using the solve function, we find the predicted score to be 51 (51.05), which is clearly unrealistic because the maximum possible study score is 50. The model is only useful for study scores between 0 and 50.

Alternatively, substitute 30 into the equation for average study time to find the log of the study score and then transpose to find to find the study score.

log (study score) = 0.016(average study time) + 1.23
log (study score) = 0.016(30) + 1.23
log (study score) = 1.71
study score = 10 ^ 1.71
study score = 51

Mark allocation: 2 marks

- 1 mark for prediction.
- 1 mark for comment.

Question 2e.

Worked solution

A $\frac{1}{y}$ transformation or an x^2 transformation.

Mark allocation: 1 mark

- 1 mark for stating both types of transformation.

End of Core

Module 1: Number patterns

Question 1a.

Worked solution

His training distance increased by 12 laps over the 4 days from Tuesday to Friday, which means he increased his run by 3 laps per day. On Wednesday, he ran 16 laps.

Mark allocation: 1 mark

- 1 mark for 16 laps.

Question 1b.

Worked solution

This is an example of an arithmetic sequence because the next term is obtained by adding d, the common difference, to the previous term. The rule below can be used to calculate the sum, S_n, of n terms in an arithmetic sequence, where a is the first term (10) and d is the common difference (3). Use your solve function to find n.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$250 = \frac{n}{2}\left[2 \times 10 + (n-1)3\right]$$

$$n = 10.38$$

$$\text{solve}\left(250=\frac{n}{2}\cdot\left(2\cdot 10+(n-1)\cdot 3\right),n\right)$$

$$n=-16.0505359286 \text{ or } n=10.3838692619$$

After 10 days, Steve has run fewer than 250 laps, and after 11 days he exceeds 250 laps. Therefore, the answer is 11 February.
Alternatively, write down the sequence and add up the numbers until they exceed 250 by trial and error.

$$10+13+16+19+22+25+28+31+34+37 \qquad 235.$$

$$10+13+16+19+22+25+28+31+34+37+40 \qquad 275.$$

Mark allocation: 2 marks

- 1 mark for substitution into $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$.
- 1 mark for 11 February.

Question 1c.

Worked solution

$480 - (5 \times 30) = 330$

Mark allocation: 1 mark

- 1 mark for 330.

Question 1d.

Worked solution

$480 \div 30 = 16$
Therefore, the answer is 16 February.

Mark allocation: 1 mark

- 1 mark for 16 February.

Question 1e.

Worked solution

$m = -30$ and $T_8 = 480 + (8 \times -30)$

Mark allocation: 1 mark

- 1 mark for $m = -30$.

Question 2a.

Worked solution

$$\frac{120}{100} = \frac{144}{120} = 1.2$$

Mark allocation: 1 mark

- 1 mark for showing this equation.

Question 2b.

Worked solution

$t_n = ar^{(n-1)}$

where a is the first term and r is the common ratio

$t_7 = 100 \times 1.2^{(7-1)}$

$t_7 = 298.60$

$t_7 \approx 299$

Mark allocation: 1 mark

- 1 mark for 299 repetitions.

Question 2c.

Worked solution

$t_n = ar^{(n-1)}$ *or* $t_n = 100 \times 1.2^{(n-1)}$

Mark allocation: 1 mark

- 1 mark for this expression.

Question 2d.

Worked solution

The sum of the first n terms in a geometric sequence is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

difference = total after 14 days – total after 7 days

= $S_{14} - S_7$

$$\frac{100\cdot\left((1.2)^{14}-1\right)}{1.2-1}-\frac{100\cdot\left((1.2)^{7}-1\right)}{1.2-1}$$

4628.00192274

=4628

Mark allocation: 2 marks

- 1 mark for substitution into $S_n = \dfrac{a(r^n - 1)}{r - 1}$.
- 1 mark for 4628.

Question 3a.

Worked solution

$T_{n+2} = 0.5\left(T_n + T_{n+1}\right) - 2$

Mark allocation: 1 mark

- 1 mark for the equation.

Question 3b.

Worked solution

$T_3 = 0.5\left(T_1 + T_2\right) - 2$

$T_3 = 0.5\left(140 + 136\right) - 2$

$T_3 = 136 \text{ s}$

Mark allocation: 1 mark

- 1 mark for 136 s.

Question 3c.

Worked solution

Date	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**
Time	140	136	136	134	133	131.5	130.25	128.875

Yes, he will qualify.

Mark allocation: 2 marks

- 1 mark for finding the time for 8 February.
- 1 mark for concluding he is on track to qualify.

End of Module 1

Module 2: Geometry and trigonometry

Question 1a.

Worked solution

Using Pythagoras' theorem and the height, $h = CD = CB$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ CD^2 + CB^2 &= 70.71^2 \\ h^2 + h^2 &= 4999.90 \\ 2h^2 &= 4999.90 \\ h^2 &= 2499.95 \\ h &= 50 \text{ cm} \end{aligned}$$

Mark allocation: 2 marks

- 1 mark for substitution into Pythagoras' theorem.
- 1 mark for correct answer.

Question 1b.

Worked solution

We are looking for the angle at vertex A. The horizontal distance (100 cm) is the adjacent side of the triangle (with respect to A) and the sloping edge (112 cm) is the hypotenuse.

$$\cos X = \frac{adj}{hyp}$$

$$\cos A = \frac{100}{112}$$

$$A = \cos^{-1}\left(\frac{100}{112}\right)$$

$$A = 26.77°$$

Mark allocation: 1 mark

- 1 mark for 27°.

Tip

- *Always check your calculator settings. The angle option must be in degree mode for this module.*

Question 2a.

Worked solution

The 3 angles of a triangle add up to 180°.

angle at $C = 180 - 65 - 67$

angle at $C = 48°$

Mark allocation: 1 mark

- 1 mark for 48°.

Question 2b.

Worked solution

To find the length *CA*, we use the sine rule.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{AC}{\sin(67)} = \frac{50}{\sin(65)}$$

$$AC = \frac{50}{\sin(65)} \times \sin(67)$$

$$AC = 50.78\ \text{m}$$

$$AC \approx 51\ \text{m}$$

Mark allocation: 2 marks

- 1 mark for substitution into sine rule.
- 1 mark for correct answer.

Tip

- *If you are looking for an unknown side use the version of the sine rule with the pronumerals* a *and* b *(representing the side lengths) as the numerators. If you are looking for an unknown angle, use the version with the sin* A *and sin* B *as the numerators. It will be easier to transpose to find your unknown.*

Question 2c.

Worked solution

60 m

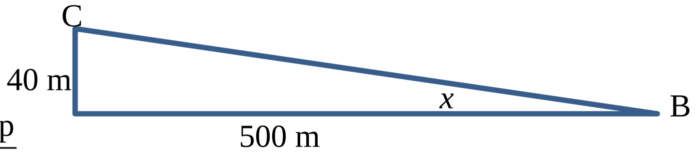
Mark allocation: 1 mark

- 1 mark for 60 m.

Question 2d.

Worked solution

The horizontal distance is 500 m and the vertical distance is 40 m.



$$\tan x = \frac{\text{opp}}{\text{adj}}$$

$$\tan x = \frac{40}{500}$$

$$x = \tan^{-1}\left(\frac{40}{500}\right)$$

$$x = 4.57°$$

Mark allocation: 2 marks

- 1 mark for substitution into tan ratio.
- 1 mark for correct answer.

Question 2e.

Worked solution

The size of the base has been increased by a factor of 2. Therefore, the area of the base will be increased by a factor of 4. Because volume is equal to the area of the base multiplied by the height (which has not changed), the volume of the larger bucket will be 4 times the volume of the smaller bucket: $4 \times 2 = 8$
A larger bucket holds 8 litres.

Mark allocation: 2 marks

- 1 mark for finding the volume ratio.
- 1 mark for correct answer.

Question 3a.

Worked solution

$u = 30°$, $v = 20°$, $w = 110°$, $x = 50°$, $y = 130°$, $z = 20°$

Mark allocation: 2 marks

- 1 mark for finding $u = 30°$ and $v = 20°$.
- 2 marks if all 6 angles correct.

Tip

- *When working with bearings, draw a compass cross at the start position. Use the bearing information clockwise from north to find angles. For example, when the cross is drawn at* O*, the bearing of* A *(060°T) and the bearing of* B *(080°T) shows us that the angle at* v *is:* $80 - 60 = 20°$.

Question 3b.

Worked solution

Using the cosine rule to find a side length

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$OD^2 = 1200^2 + 800^2 - 2 \times 1200 \times 800 \times \cos 75$$

$$OD = 1258.20 \text{ m}$$

$$OD \approx 1258 \text{ m}$$

Mark allocation: 2 marks

- 1 mark for substitution into cosine rule.
- 1 mark for correct answer.

End of Module 2

Module 3: Graphs and relations

Question 1a.

Worked solution

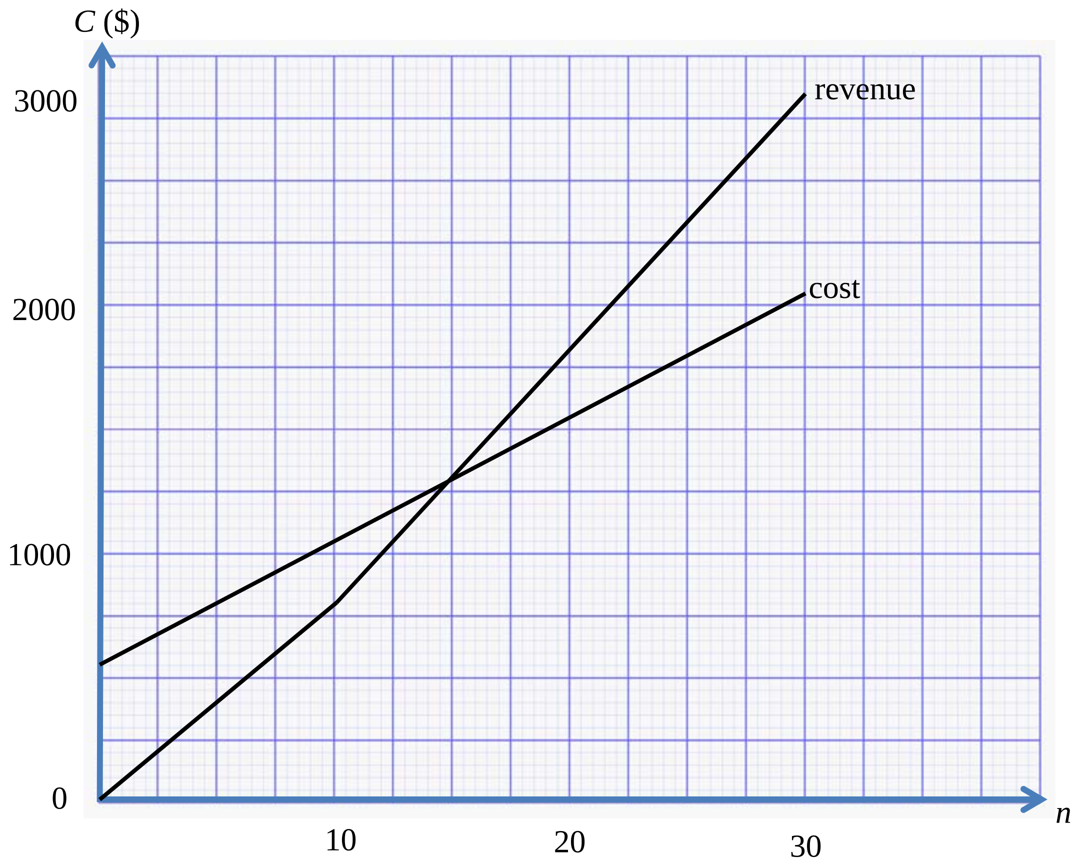
C = 550 + 50n

Mark allocation: 1 mark

- 1 mark for equation.

Question 1b.

Worked solution



Mark allocation: 1 mark

- 1 mark for correct cost equation sketched on graph.

Question 1c.

Worked solution

$$R = \begin{cases} 80n & 0 \le n \le 10 \\ 100n - 200 & 10 < n \le 30 \end{cases}$$

Mark allocation: 2 marks

- 1 mark for each equation.

Question 1d.

Worked solution

See the graph opposite.

Mark allocation: 1 mark

- 1 mark for correct revenue equation sketched on axes.

Tip

- *The –200 indicates where this graph would cut the* y *axis, not the* y *ordinate when* x = 10.

Question 1e.

Worked solution

15

Mark allocation: 1 mark

- 1 mark for 15.

Tip

- *Make sure you equate the correct revenue equation with the cost equation to find the break-even point.*

Question 2a.

Worked solution

$40x + 50y \leq 200$

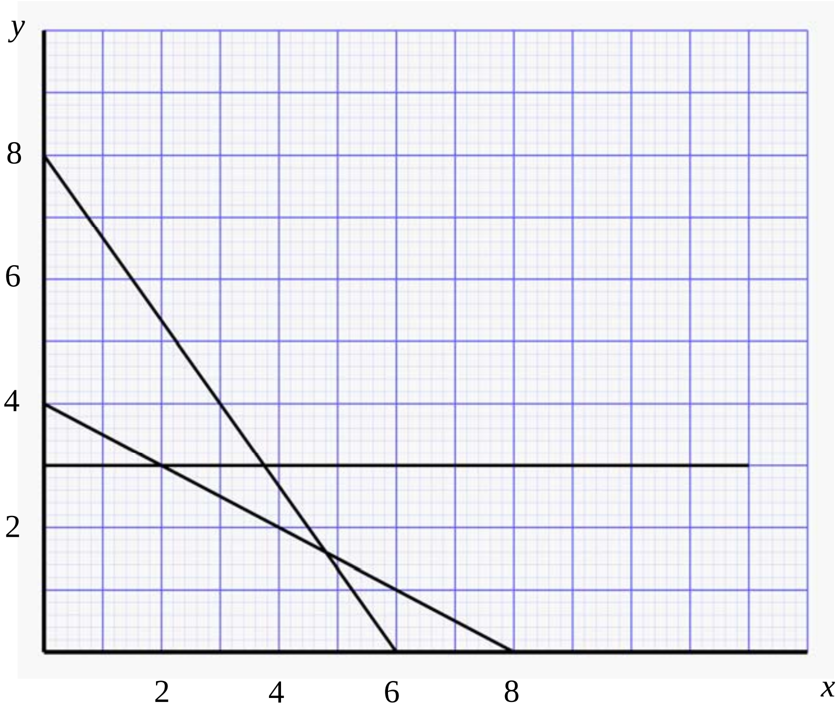
$80x + 30y \leq 240$

Mark allocation: 2 marks

- 1 mark for each constraint.

Question 2b.

Worked solution

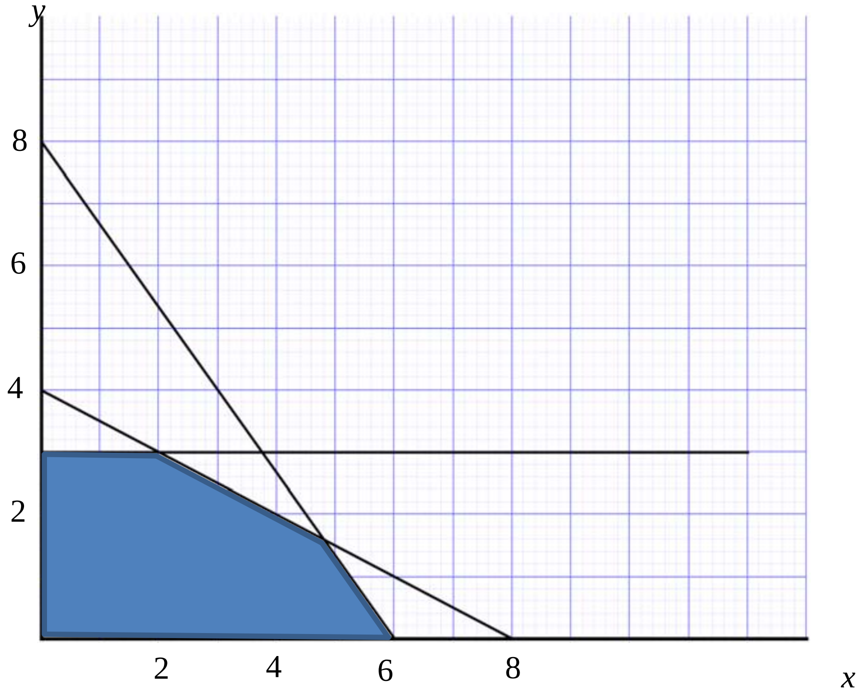


Mark allocation: 2 marks

- 1 mark for each correct line.

Question 2c.

Worked solution



Mark allocation: 1 mark

- 1 mark for correct shading.

Tip

- *A good way to discover which side of the line is in the feasible region is to test a point, such as (2, 2), to see whether it satisfies the inequation or not.*

Question 2d.

Worked solution

$P = 2x + y$

Mark allocation: 1 mark

- 1 mark for correct answer.

Question 2e.

Worked solution

$P = 2x + y$ test vertices

$(0,\ 0)\ P = 0,\ (0,\ 3)\ P = 6,\ (2,\ 3)\ P = 8,\ (4.8,\ 1.6)\ P = 8,\ (6,\ 0)\ P = 6$

The maximum performance benefit is 8.
This can be obtained by taking 2 Xanabola and 3 Yesterola tablets.
The point (4, 2) is also on the boundary of the feasible region, although not a vertex. The combination of 4 Xanabola and 2 Yesterola tablets also provides a maximum performance benefit of 8. While the point (4.8, 1.6) also provides a performance benefit of 8, these are not whole numbers of tablets. So the correct answer is 2 Xanabola and 3 Yesterola tablets or 4 Xanabola and 2 Yesterola tablets.

Mark allocation: 3 marks

- 1 mark for finding each vertex.
- 1 mark for the maximum value of *P*.
- 1 mark for numbers of each tablet.

End of Module 3

Module 4: Business-related mathematics

Question 1a.

Worked solution

original price = new price × $\frac{100}{100 + r}$ where r = % increase

original price = 825 × $\frac{100}{110}$

original price = 750

$825 - 750 = 75$

GST is $75

Mark allocation: 1 mark

- 1 mark for $75.

Question 1b.

Worked solution

5% of $825 is $825 \times 0.05 = \$41.25$

825·0.05	41.25
825–41.25	783.75
783.75–41.25	742.5
742.5–41.25	701.25
701.25–41.25	660
660–41.25	618.75
618.75–41.25	577.5

Jenny has to wait 6 months.

Mark allocation: 1 mark

- 1 mark for 6 months.

Question 1c.

Worked solution

interest earned = $\frac{Prt}{100}$

interest earned = $600 \times 9 \times \frac{8}{12} \times \frac{1}{100}$

interest earned = $36

Jenny receives $636

Mark allocation: 1 mark

- 1 mark for $636.

Question 2a.

Worked solution

interest paid = total repayments – amount borrowed
interest paid = 45 × 24 – 800
interest paid = $280

Mark allocation: 1 mark

- 1 mark for $280.

Question 2b.

Worked solution

$$R_f = \frac{I \times 100}{P \times t}$$

$$R_f = \frac{280 \times 100}{800 \times 2}$$

$R_f = 17.5\%$

Mark allocation: 1 mark

- 1 mark for 17.5%.

Question 2c.

Worked solution

$$R_e = R_f \times \frac{2n}{n+1}$$

where n is equal the total number of repayments

$$R_e = 17.5 \times \frac{48}{25}$$

$R_e = 33.6\%$

Mark allocation: 1 mark

- 1 mark for 33.6%.

Question 2d.

Worked solution

Interest is charged at a flat interest rate (17.5%) on the initial principal ($800) for the duration of the loan (24 months) even though the principal is being reduced during the loan. The effective interest rate (33.6%) takes into account the fact that the principal is being reduced while the interest charges remain unchanged and, therefore, the effective rate is higher.

Mark allocation: 1 mark

- 1 mark for a valid explanation.

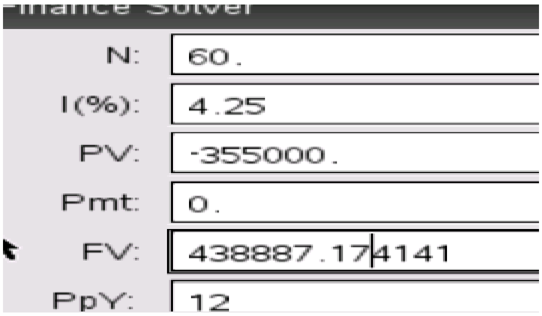
Question 3a.

Worked solution

Using the compound interest formula

$A = 355\ 000 \times \left(1 + \frac{4.25}{12 \times 100}\right)^{5 \times 12} = \$438\ 887.17$

Or use TVM solver



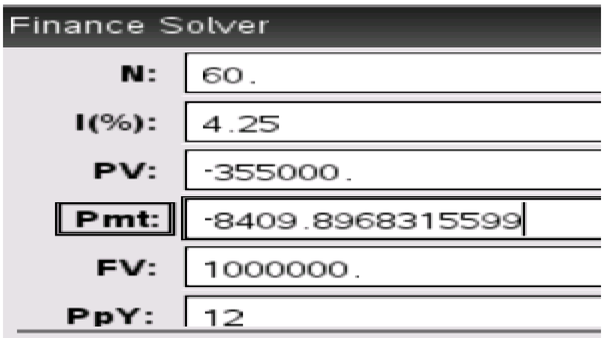
Mark allocation: 1 mark

- 1 mark for $438 887.17.

Question 3b.

Worked solution

Use TVM solver



$8409.90

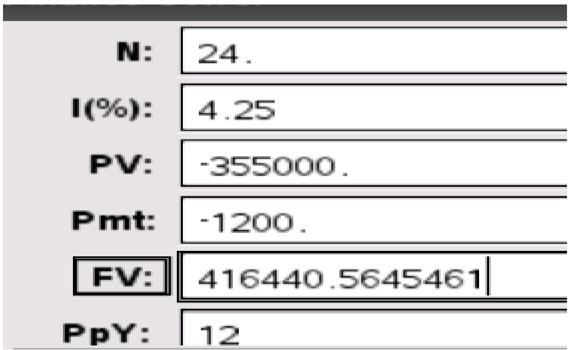
Mark allocation: 1 mark

- 1 mark for $8409.90.

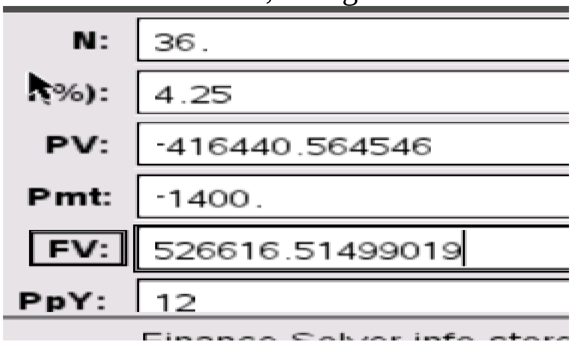
Question 3c.

Worked solution

Use TVM solver



Transfer the FV into the PV, change Pmt to 1400 and change N to 36 to find the new FV



Balance at the end of 5 years is $526 616.51

Mark allocation: 2 marks

- 1 mark for value of account after 2 years.
- 1 mark for answer.

Question 4a.

Worked solution

unit cost × number of units = depreciation
0.1 × 100 000 = $10 000

Mark allocation: 1 mark

- 1 mark for $10 000.

Question 4b.

Worked solution

$$\begin{aligned}\text{number of units} &= \frac{\text{depreciation}}{\text{unit cost}} \\ &= \frac{16\ 000}{0.08} \\ &= 200\ 000\ \text{km}\end{aligned}$$

Mark allocation: 1 mark

- 1 mark for 200 000 km.

Question 4c.

Worked solution

depreciation rate

$$\begin{aligned} &= \frac{D \times 100}{P \times t} \\ &= \frac{30\ 000 \times 100}{320\ 000 \times 1} \\ &= 9.4\% \end{aligned}$$

Mark allocation: 1 mark

- 1 mark for 9.4%.

Question 4d.

Worked solution

$$BV = P \times \left(1 - \frac{r}{100}\right)^t$$

$$30\ 000 = 320\ 000 \times \left(1 - \frac{18}{100}\right)^t$$

Using solve to find t

$$\text{solve}\left(30000=320000\cdot\left(1-\frac{18}{100}\right)^t,t\right)$$

$$t=11.9280041171$$

Therefore, the truck will be worth less than scrap value after 12 years

Mark allocation: 1 mark

- 1 mark for 12 years.

End of Module 4

Module 5: Networks and decision mathematics

Question 1a.

Worked solution

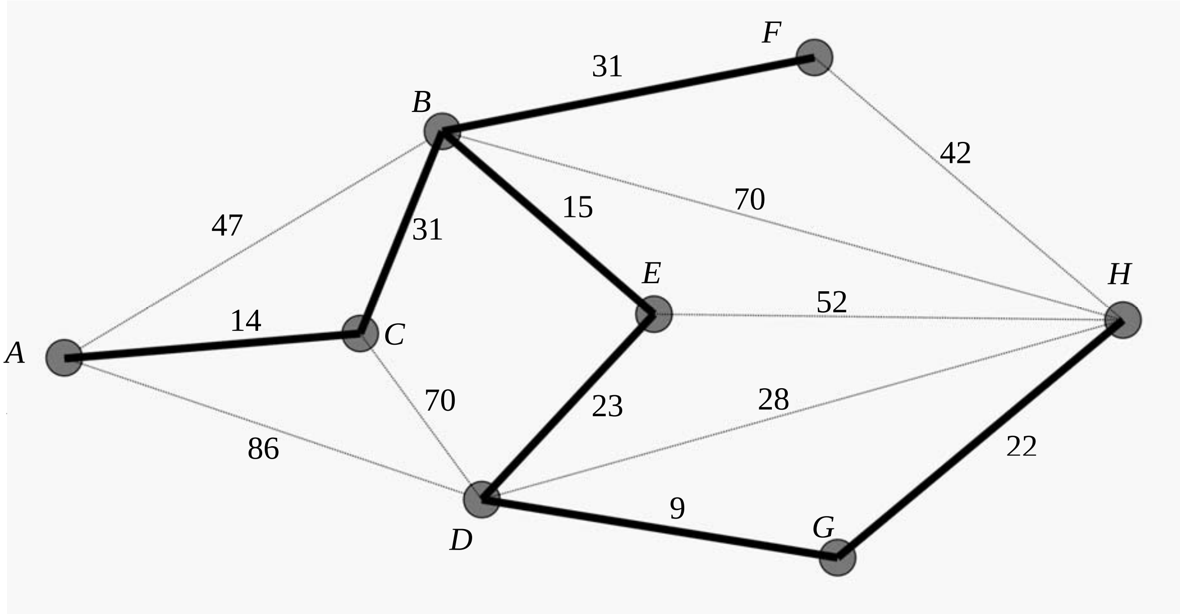
111 m (14 + 31 + 15 + 23 + 28)

Mark allocation: 1 mark

- 1 mark for 111 m.

Question 1b.

Worked solution



Mark allocation: 1 mark

- 1 mark for this plan.

Question 1c.

Worked solution

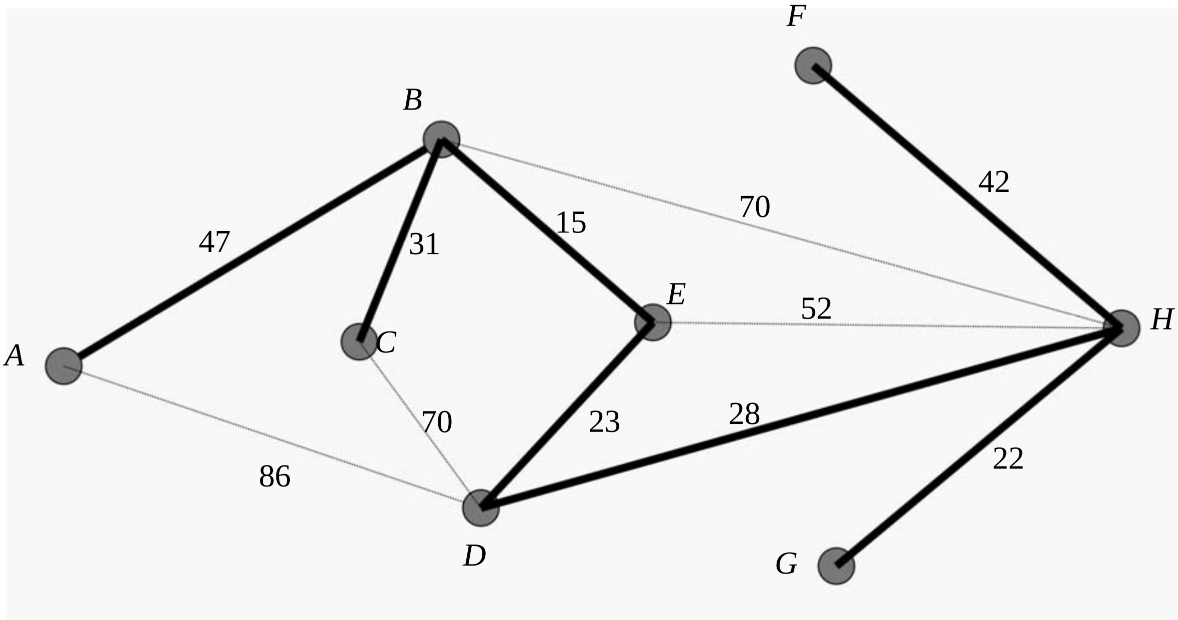
It is a minimum spanning tree.

Mark allocation: 1 mark

- 1 mark for this term.

Question 1d.

Worked solution



$47 + 31 + 15 + 23 + 28 + 22 + 42 = 208$ m.

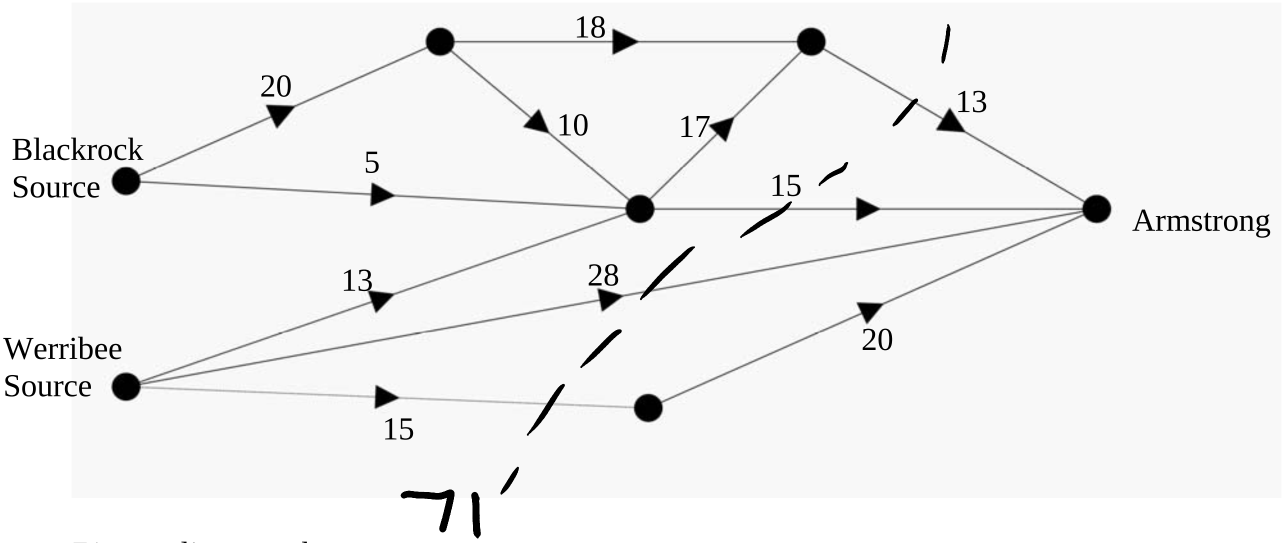
Mark allocation: 1 mark

- 1 mark for 208 metres of pipe.

Question 2a.

Worked solution

minimum cut = maximum flow



71 megalitres per hour

Mark allocation: 1 mark

- 1 mark for 71 megalitres per hour.

Question 2b.

Worked solution

The maximum flow from Blackrock to Armstrong is 25 megalitres. The maximum flow from Werribee to Armstrong is 46 megalitres. The total is 71 megalitres.
The 20 and 5 megalitres leaving the Blackrock source get through to Armstrong. The 28 and 15 leaving the Werribee source get through to Armstrong. Of the 13 megalitres leaving Werribee, only 3 megalitres can get through to Armstrong because the Blackrock flow has used the other 25 megalitres of capacity.

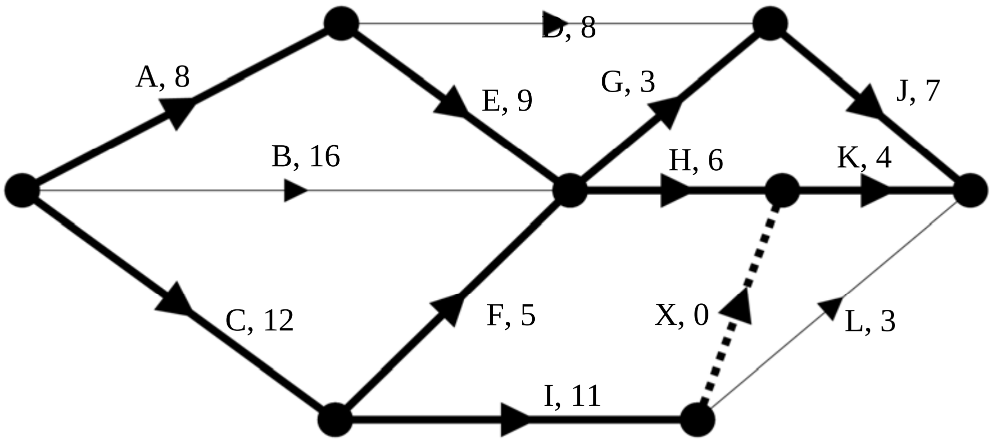
Mark allocation: 2 marks

- One mark for flow from Blackrock.
- One mark for flow from Werribee.

Question 3a.

Worked solution

First find the critical paths (i.e. the longest paths through the network). The critical paths are highlighted on the diagram below.



The length of the critical paths is 27 weeks. Critical paths are *AEGJ*, *AEHK*, *CFHK* and *CIXK*.

Mark allocation: 1 mark

- 1 mark for 27 weeks.

Tip

- *First find the critical paths (i.e. the longest paths through the network). The critical paths are highlighted on the diagram.*

Question 3b.

Worked solution

1 week
Activity *L* has 1 week slack (or float time). If activity *L* is delayed for longer than a week, the whole project will be delayed.

Mark allocation: 1 mark

- 1 mark for 1 week.

Question 3c.

Worked solution

New critical path(s) is 25 weeks.
Take 2 weeks reduction from activity A, 1 week from B and 2 weeks from C. The total cost for the reductions is
$2 \times 15\ 000 + 20\ 000 + 2 \times 25\ 000 = \$100\ 000$

Mark allocation: 2 marks

- 1 mark for shortest time.
- 1 mark for minimum cost.

Question 3d.

Worked solution

Order of activities: (*A* and *B*) equal 1st at zero weeks, *C* 3rd at 8 weeks, (*D*, *E*, *F*) equal 4th at 16 weeks, (*H* and *I*) equal 7th at 21 weeks and *G* 9th at 24 weeks.
So the answer is activities *H* and *I*.

Mark allocation: 2 marks

- 1 mark for *H*.
- 1 mark for *I*.

Question 4a.

Worked solution

Using the Hungarian algorithm, row reduction gives

	A	**B**	**C**	**D**
W	$6850	$16 600	$0	$23 600
X	$5800	$17 300	$0	$23 300
Y	$4900	$14 400	$0	$21 900
Z	$7000	$14 000	$0	$27 000

Column reduction gives

	A	**B**	**C**	**D**
W	$1950	$2600	$0	$1700
X	$900	$3300	$0	$1400
Y	$0	$400	$0	$0
Z	$2100	$0	$0	$5100

Covering the zeros gives

	A	**B**	**C**	**D**
W	$1950	$2600	$0	$1700
X	$900	$3300	$0	$1400
Y	$0	$400	$0	$0
Z	$2100	$0	$0	$5100

Further reduction gives

	A	**B**	**C**	**D**
W	$1050	$1700	$0	$800
X	$0	$2400	$0	$500
Y	$0	$400	$900	$0
Z	$2100	$0	$900	$5100

Allocation is now possible

	A	**B**	**C**	**D**
W	$1050	$1700	$0	$800
X	$0	$2400	$0	$500
Y	$0	$400	$900	$0
Z	$2100	$0	$900	$5100

Zeb does job B.

Mark allocation: 1 mark

- 1 mark for Zeb doing job B.

Tip

- *When allocating jobs, allocate any rows or columns with one zero first.*

Question 4b.

Worked solution

$8400 + 15\ 000 + 31\ 500 + 23\ 000 = \$77\ 900$

Mark allocation: 1 mark

- 1 mark for \$77 900.

End of Module 5

Module 6: Matrices

Question 1a.

Worked solution

$$\begin{bmatrix} 12.3 & 25.6 & 48.2 \\ 10.5 & 23.3 & 52.3 \\ 10.7 & 23.2 & 49.6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 86.1 \\ 86.1 \\ 83.5 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for answer.

Question 1b.

Worked solution

It is the sum of the personal best times for each athlete in the 100 m, the 200 m and the 400 m.

Mark allocation: 1 mark

- 1 mark for answer.

Question 1c.

Worked solution

$$B = \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 0.90 & 0 \\ 0 & 0 & 0.85 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for matrix.

Question 1d.

Worked solution

$$\begin{bmatrix} 12.3 & 25.6 & 48.2 \\ 10.5 & 23.3 & 52.3 \\ 10.7 & 23.2 & 49.6 \end{bmatrix} \cdot \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.85 \end{bmatrix}$$

$$\begin{bmatrix} 11.685 & 23.04 & 40.97 \\ 9.975 & 20.97 & 44.455 \\ 10.165 & 20.88 & 42.16 \end{bmatrix}$$

$$TB = \begin{bmatrix} 11.7 & 23.0 & 41.0 \\ 10.0 & 21.0 & 44.5 \\ 10.2 & 20.9 & 42.2 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for *TB*.

Question 2a.

Worked solution

x = economy price	y = ensuite price	z = deluxe price
$10x + 5y + 2z = 2400$	$25x + 2y = 2320$	$10x + 4z = 2400$

Mark allocation: 2 marks

- 1 mark for 2 correct equations.
- 2 marks for 3 correct equations.

Question 2b.

Worked solution

$$\begin{bmatrix} 10 & 3 & 4 \\ 15 & 0 & 2 \\ 12 & 8 & 0 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2675 \\ 1885 \\ 1980 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for matrix.

Tip

- *When constructing matrix equations from literal equations, be careful to include zeroes when there are only 2 terms on the left-hand side, and make sure it in the correct position, in this case* a, b *and* c.

Question 2c.

Worked solution

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 & 3 & 4 \\ 15 & 0 & 2 \\ 12 & 8 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2675 \\ 1885 \\ 1980 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 75 \\ 135 \\ 380 \end{bmatrix}$$

Standard rooms cost \$75, view rooms cost \$135 and pool rooms cost \$380.

Mark allocation: 2 marks

- 1 mark for setting up matrix equation.
- 1 mark for answer.

Tip

- *Don't forget to answer the question and state your answer in terms of the question outside the matrix. The matrix is working in this case, not the answer.*

Question 3a.

Worked solution

80

Mark allocation: 1 mark

- 1 mark for 80.

Question 3b.

Worked solution

Each year, 20% of athletes who had nominated the 5000 m as their favourite event the previous year do not nominate a favourite event – perhaps they retire.

Mark allocation: 1 mark

- 1 mark for answer.

Question 3c.

Worked solution

$0.5 \times 120 = 60$ (those still nominating the 800 m in 2002)

$0.5 \times 60 = 30$ (those still nominating the 800 m in 2003)

Answer is 30.

Mark allocation: 1 mark

- 1 mark for answer.

Question 3d.

Worked solution

$$S_1 = TS_0$$

$$S_1 = \begin{bmatrix} 0.5 & 0.0 & 0.1 \\ 0.3 & 0.6 & 0.2 \\ 0.2 & 0.4 & 0.5 \end{bmatrix} \times \begin{bmatrix} 120 \\ 80 \\ 60 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 66 \\ 96 \\ 86 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for answer.

Question 3e.

Worked solution

S_0 is 2001 therefore S_4 is 2005

$S_4 = T^4\, S_0$

$$\begin{bmatrix} 0.5 & 0 & 0.1 \\ 0.3 & 0.6 & 0.2 \\ 0.2 & 0.4 & 0.5 \end{bmatrix}^4 \cdot \begin{bmatrix} 120 \\ 80 \\ 60 \end{bmatrix} \qquad \begin{bmatrix} 24.476 \\ 80.666 \\ 88.046 \end{bmatrix}$$

A total of 81 athletes chose the 1500 m as their favourite event in 2005.

Mark allocation: 1 mark

- 1 mark for 81.

Question 3f.

Worked solution

The 2011 favourites will be given by S_{10}

$S_9 = TS_8 + B$

and

$S_{10} = TS_9 + B$

$$\begin{bmatrix} 0.5 & 0 & 0.1 \\ 0.3 & 0.6 & 0.2 \\ 0.2 & 0.4 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 55 \\ 61 \end{bmatrix} + \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 63.6 \\ 49.7 \\ 55.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 0 & 0.1 \\ 0.3 & 0.6 & 0.2 \\ 0.2 & 0.4 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 63.6 \\ 49.7 \\ 55.5 \end{bmatrix} + \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 87.35 \\ 60. \\ 60.35 \end{bmatrix}$$

60 athletes choose the 1500 m event as their favourite in 2011.

Mark allocation: 1 mark

- 1 mark for 60.

END OF WORKED SOLUTIONS