insight

YEAR 12 *Trial Exam Paper*

# 2014
# FURTHER MATHEMATICS

## Written examination 1

## *Worked solutions*

**This book presents:**

- correct solutions with full working
- mark allocations
- tips

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**SECTION A**

# SECTION A

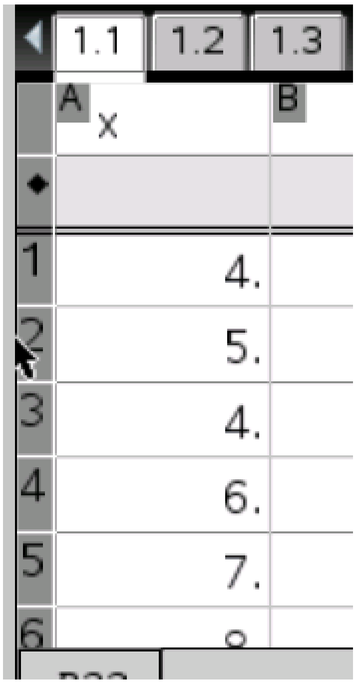
## Core

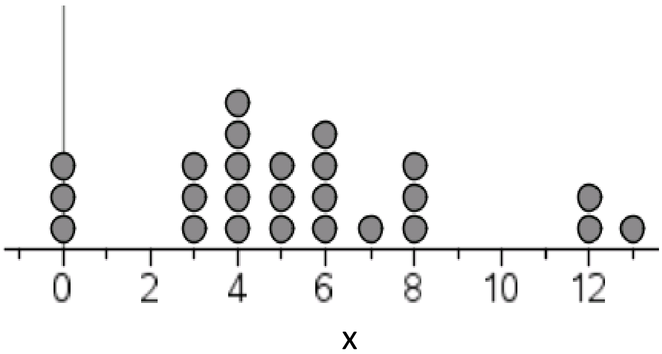
### Question 1

***Answer is B***

**Worked solution**

Enter the data as a list into a spreadsheet page and then create a dotplot. The number 4 has the highest frequency and is therefore the mode.



**Question 2**

***Answer is D***

**Worked solution**

$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR}\\ &= 3.5 - 1.5 \times 4\\ &= -2.5\end{aligned}$$

$$\begin{aligned}\text{Upper fence} &= Q_3 + 1.5 \times \text{IQR}\\ &= 7.5 + 1.5 \times 4\\ &= 13.5\end{aligned}$$

where $Q_1$ is the first quartile, $Q_3$ is the third quartile and IQR is the interquartile range.

**Question 3**

***Answer is E***

**Worked solution**

The number of students who scored above 50 in Class A is equal to the number of students who scored below 64 in class B. E is incorrect because the median of Class B is 64, not 62.

**Question 4**

***Answer is A***

**Worked solution**

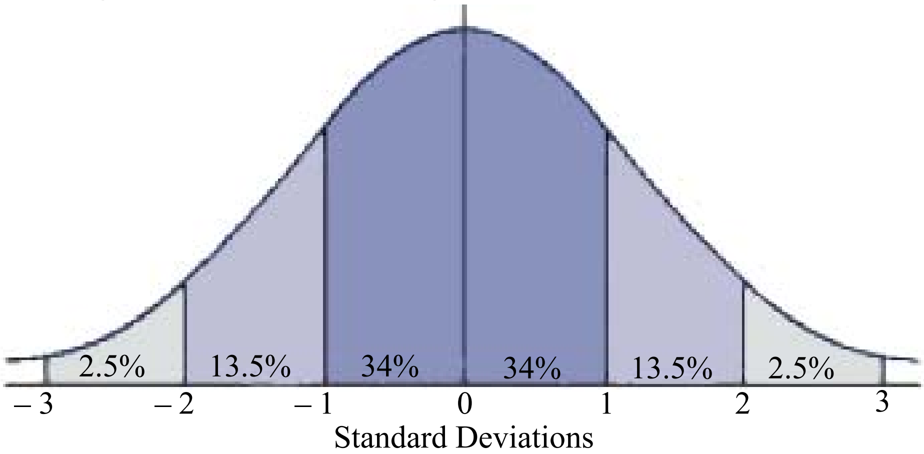
The new numbers are more spread out. The standard deviation, range and interquartile range all increase, but the mean and median remain the same.

**Question 5**

***Answer is C***

**Worked solution**

From the information given in the question, the weight range for 81.5% of empty soft drink cans is from 0.15 g below the mean to 0.30 g above the mean.



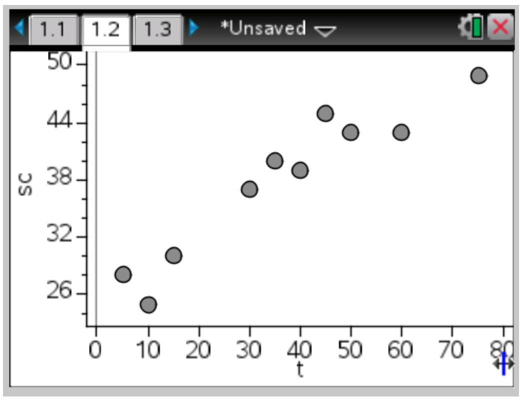

Using the graph above, 81.5% of the normal distribution lies from 1 standard deviation below the mean to 2 standard deviations above the mean. The standard deviation for the distribution of weights of empty soft drink cans is therefore 0.15 g.

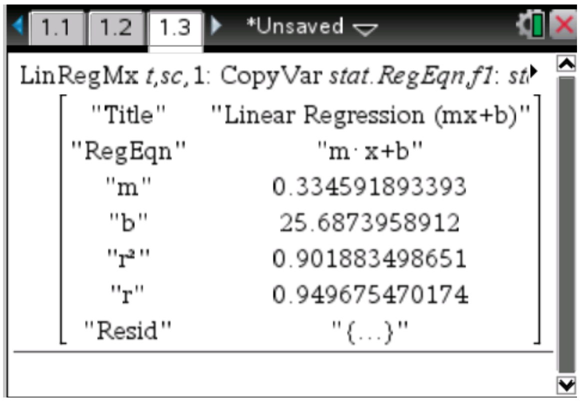
**Question 6**

***Answer is D***

**Worked solution**

Use the calculator to draw the scatterplot and calculate $r$ and $r^2$.



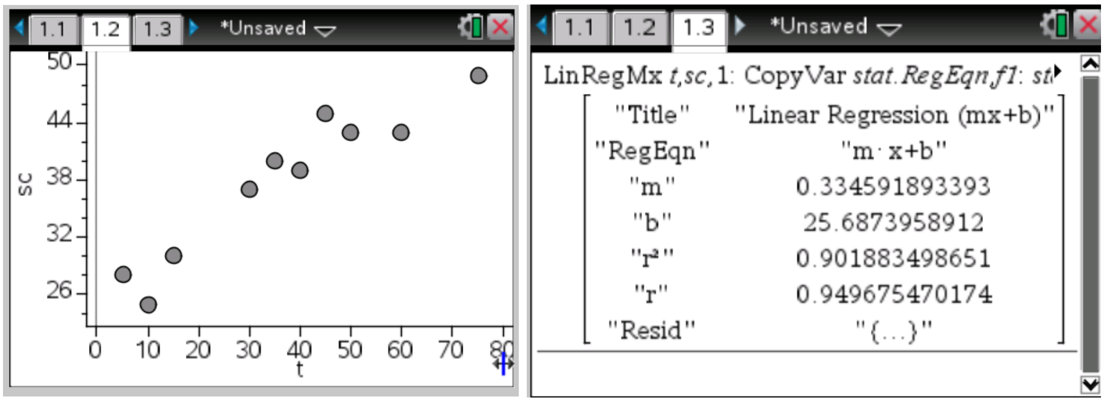

A is correct because study score depends on time spent studying (the independent variable).
B is correct because the scatterplot and $r = 0.95$ show that the relationship is strong, positive and linear.
C is correct because the scatterplot shows a positive correlation.
D is incorrect because $r^2 = 0.90$ and, therefore, only 10% of the variation in study scores can be explained by other factors.
E is correct because the regression equation can be used to predict study scores.

## Question 7

***Answer is C***

### Worked solution

Use the calculator to draw the scatterplot and calculate the equation of the least squares regression line.




The least squares regression equation for predicting study score is $m \times x + b$, where $m = 0.33$ is the regression coefficient for study time, and $b = 25.69$ is the constant.

**Tip**

- *Study time is the independent variable.*

**Question 8**

***Answer is E***

**Worked solution**

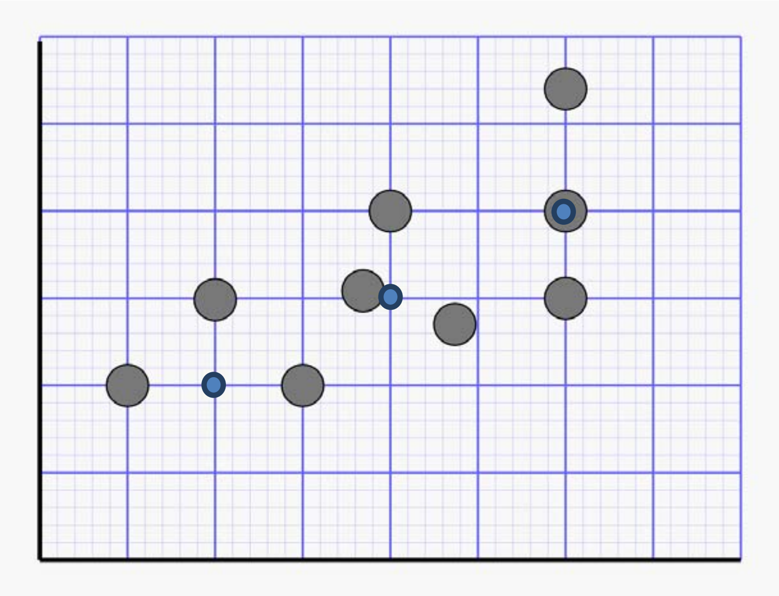
A comparison of male and female preferences must be included in the statement, and appropriate percentages need to be quoted.

**Question 9**

***Answer is A***

**Worked solution**

The 3 points are shown below. To draw the 3-median line, join the 2 outside points and move the line one-third of the way towards the middle point without changing the gradient of the line.

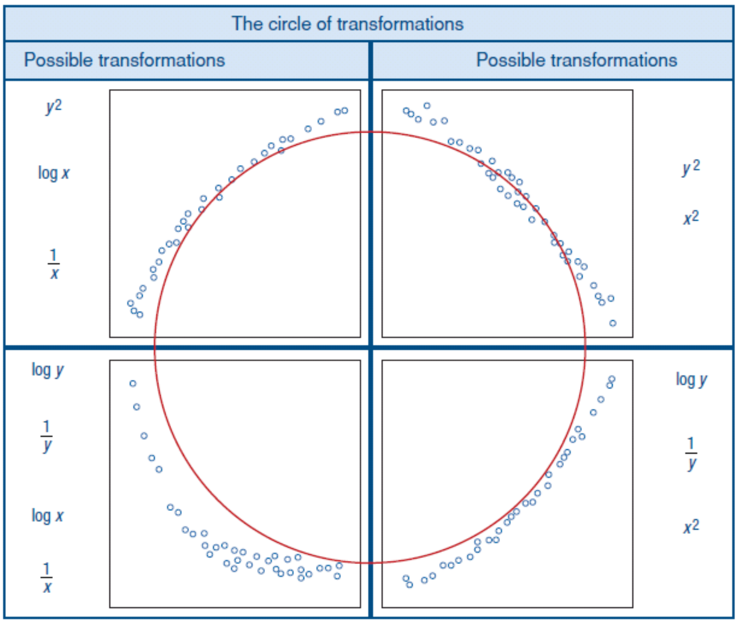


**Question 10**

***Answer is C***

**Worked solution**

Use the circle of transformations to select the best transformation to linearise the data.




**Question 11**

***Answer is B***

**Worked solution**

If there are 4 seasons, the indices must add up to 4.

$$1.25 + x + x + 0.2 + 0.99 = 4.0$$
$$x = 0.78$$

**Question 12**

***Answer is D***

**Worked solution**

Deseasonalised car sales for summer 2015 = 22 × 9 + 55
Deseasonalised car sales for summer 2015 = 253

$$\text{Deaseasonalised Value} = \frac{\text{Actual Value}}{\text{Seasonal Index}}$$

Transposing gives

$$\text{Actual Value} = \text{Deseasonalised Value} \times \text{Seasonal Index}$$
$$253 \times 1.25 = 316.25$$
$$\approx 316$$

**Question 13**

***Answer is D***

**Worked solution**

Calculate the mean of 28, 25, 27, 29, 36 (the 5 days centred on day 4) by adding them up and dividing by 5. The mean is 29.

# SECTION B

## Module 1: Number patterns

### Question 1

***Answer is C***

**Worked solution**

An arithmetic sequence increases or decreases by the same amount for each term. Because it increases by 12 from the 2nd term (9) to the 5th term (21), it must increase by 4 each time (i.e., the common difference, *d*, is 4).

The sequence is 5, 9, 13, 17, 21, 25, 29 … and the 7th term is 29.

### Question 2

***Answer is E***

**Worked solution**

The easiest way to calculate the 12th term is to use the formula where *a* is the first term, *n* is the number of the term you are looking for and *d* is the common difference.

$t_n = a + (n - 1)d$

$t_{12} = -2 + (12 - 1) \times -3$

$t_{12} = -35$

**Tip**

- *Alternatively, you could write down all 12 terms (each term is 3 lower than the previous term).*

## Question 3

***Answer is D***

**Worked solution**

$t_n = ar^{(n-1)}$

where $a$ is the first term and $r$ is the common ratio

where $a = 2a - 1$ and $r = \frac{1}{2}$

D is the only sequence where each term is obtained by multiplying the preceding term by the same number, in this case $\frac{1}{2}$.

## Question 4

***Answer is D***

**Worked solution**

This is an example of an arithmetic sequence because the next term is obtained by adding the common difference, $d$, to the previous term. The rule below can be used to calculate the sum, $S_n = 18$ of $n = 5$ terms in an arithmetic sequence when $a = 2$ is the first term. Use your solve function to find $d$ or transpose to find $d$.

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right]$$

$$18 = \frac{5}{2}\left[2 \times 2 + (5 - 1)d\right]$$

$$d = 0.8$$

## Question 5

***Answer is E***

**Worked solution**

This is a geometric sequence. The sum of a geometric sequence is given by

$$S_\infty = \frac{a}{1 - r}$$

First find $a$ (the first term) and $r$ (the common ratio).

$$a = t_1 = 2000 \times (0.6)^1 = 1200$$

$$t_2 = 2000 \times (0.6)^2 = 720$$

$$r = \frac{720}{1200} = 0.6$$

Therefore

$$\begin{aligned} S_\infty &= \frac{a}{1 - r} \\ &= \frac{1200}{1 - 0.6} \\ &= 3000 \end{aligned}$$

## Question 6

***Answer is B***

**Worked solution**

To increase last year's fish population by 40%, multiply $r_n$ by 1.4.

Then subtract 4000 due to fishing losses.

## Question 7

***Answer is B***

**Worked solution**

The sum of an arithmetic sequence is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

where $a$ is the first term and $d$ is the common difference.

$$S_5 = \frac{5}{2}(2a + 4d) = 40$$

$$S_{10} = \frac{10}{2}(2a + 9d) = 155$$

$$5a + 10d = 40$$

$$10a + 45d = 155$$

Note that $S_{10} = 155$ because the sum of terms 1 to 5 is 40 and the sum of terms 6 to 10 is 115.

Using the calculator to solve the simultaneous equations

$$\text{linSolve}\left(\begin{cases} 5\cdot a+10\cdot d=40 \\ 10\cdot a+45\cdot d=155 \end{cases}, \{a,d\}\right) \quad \{2.,3.\}$$

Therefore, the common difference is 3.

**Question 8**

***Answer is D***

**Worked solution**

The total number of chocolates eaten on a normal occasion would have been 7, 14, 21 or 28 etc. On Andy's birthday, the total number of chocolates eaten could have been 10, 17, 24 or 31 etc. Assuming that Andy and Bridgit ate whole chocolates, the first number that can be divided into the 3:1 ratio is 24 (i.e. 18:6).

**Question 9**

***Answer is D***

**Worked solution**

Because we know that $t_1 = 4$ and $t_2 = 8$ we can use $t_{n+2} = 2t_{n+1} - t_n$ to find $t_3$, $t_4$, $t_5$ and $t_6$. It is not possible to find $t_6$ without first finding $t_3$, $t_4$ and $t_5$.

$t_3 = 16 - 4 = 12$
$t_4 = 24 - 8 = 16$
$t_5 = 32 - 12 = 20$
$t_6 = 40 - 16 = 34$

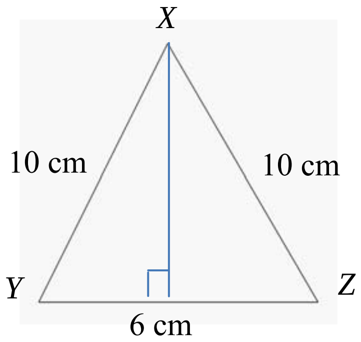
## Module 2: Geometry and trigonometry

**Question 1**

***Answer is D***

**Worked solution**

A line from $X$ to the centre of line $YZ$ will form a right-angled triangle.




$$\cos Y = \frac{\text{Adj}}{\text{Hyp}}$$

$$\cos Y = \frac{3}{10}$$

$$Y = \cos^{-1}\left(\frac{3}{10}\right)$$

$$Y = 72.54°$$

**Tip**

- *The cosine rule could also be used to find Y.*

**Question 2**

***Answer is A***

**Worked solution**

Using Heron's formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $a$, $b$ and $c$ are the lengths of the sides of the triangle, and $s = \frac{1}{2}(a+b+c)$

$$s = \frac{1}{2}(6+5+10)$$
$$s = 10.5$$

$$\text{Area} = \sqrt{10.5(10.5-6)(10.5-5)(10.5-10)}$$
$$\text{Area} = 11.40$$

**Question 3**

***Answer is E***

**Worked solution**

Use the sine rule (capital letters represent angles, lower case letters represent side lengths)

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin Q}{11} = \frac{\sin 35}{8}$$
$$\sin Q = \frac{11 \times \sin 35}{8}$$
$$Q = \sin^{-1}\left(\frac{11 \times \sin 35}{8}\right)$$
$$Q = 52.06°$$

The closest answer is 52°.

**Question 4**

***Answer is B***

**Worked solution**

Using the cosine rule

$$a^2 = b^2 + c^2 - 2bc \times \cos A$$
$$x^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos 50$$
$$x^2 = 74.728$$
$$x = 8.64 \text{ m}$$

**Question 5**

***Answer is C***

**Worked solution**

Use the rule below to calculate the area, $A$, of a non-right-angled triangle.

$$A = \frac{1}{2}ab \times \sin C$$
$$A = \frac{1}{2} \times 6 \times 7 \times \sin 88$$
$$A = 20.987$$

**Question 6**

***Answer is A***

**Worked solution**

The size ratio of triangle *XYZ*:*ABC* is 3:1
The area ratio is the size ratio squared, 9:1
The area of triangle *XYZ* is 9 times the area of triangle *ABC*. The area of triangle *ABC* is therefore 6 $m^2$.

### Question 7

***Answer is C***

**Worked solution**

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{50}{200}$$

$$\text{Slope} = \frac{1}{4}$$

### Question 8

***Answer is E***

**Worked solution**

Using Pythagoras' theorem $c^2 = a^2 + b^2$, first calculate *AE*.

$$AE^2 = AD^2 + DE^2$$

$$AE^2 = 5^2 + 5^2$$

$$AE = \sqrt{50}$$

Then use Pythagoras' theorem to calculate *AF*.

$$AF^2 = AE^2 + EF^2$$

$$AF^2 = \sqrt{50^2} + 5^2$$

$$AF^2 = 75$$

$$AF = \sqrt{75}$$

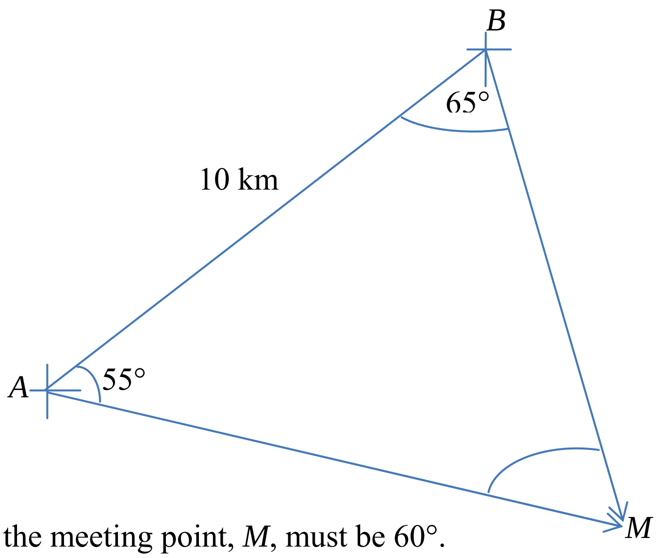
$$AF = 5\sqrt{3}$$

**Question 9**

***Answer is E***

**Worked solution**

First draw a diagram and using the bearings calculate the angles at *A* and *B*.




The angle at the meeting point, *M*, must be 60°.

Use the sine rule to calculate the distances *AM* and *BM*.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{\overrightarrow{AM}}{\sin 65} = \frac{10}{\sin 60}$$

$$AM = \frac{10}{\sin 60} \times \sin 65$$

$$AM = 10.47 \text{ km}$$

$$\frac{\overrightarrow{BM}}{\sin 55} = \frac{10}{\sin 60}$$

$$BM = \frac{10}{\sin 60} \times \sin 55$$

$$BM = 9.46 \text{ km}$$

## Module 3: Graphs and relations

### Question 1

***Answer is E***

**Worked solution**

First, find the gradient, *m*, between the 2 points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - {}^{-}4}{8 - {}^{-}2}$$

$$= \frac{1}{2}$$

Then use $y = mx + c$ to find the *y* intercept.

$y = mx + c$ substitute the point (8, 1) into *x* and *y*

$$1 = \frac{1}{2}(8) + c$$

$c = -3$ therefore $y = \frac{1}{2}x - 3$ or $2y - x + 6 = 0$

### Question 2

***Answer is C***

**Worked solution**

This could be solved manually by substitution using $y = 2x + 1$ and

$$\begin{aligned} 2y + 3x + 12 &= 0 \\ 2(2x + 1) + 3x + 12 &= 0 \\ 4x + 2 + 3x + 12 &= 0 \\ 7x &= -14 \\ x &= -2 \end{aligned}$$

Substituting back in gives *y* = –3, or use the calculator (menu, algebra, solve system of equations).

$$\mathrm{linSolve}\left(\begin{cases} y=2\cdot x+1 \\ 2\cdot y+3\cdot x+12=0 \end{cases}, \{x,y\}\right) \quad \{-2.,-3.\}$$

**Question 3**

***Answer is C***

**Worked solution**

The tide is rising when the graph has a positive gradient.

**Question 4**

***Answer is B***

**Worked solution**

The causeway can be crossed between 8 a.m. and 10 a.m. and again between 8 p.m. and 10 p.m., a total of 4 hours per day.

**Question 5**

***Answer is D***

**Worked solution**

The revenue from 400 t-shirts needs to be $5000 (read from graph) to break even. Each t-shirt must sell for

$$\frac{5000}{400} = 12.50$$

**Question 6**

***Answer is D***

**Worked solution**

Bernie always sells apples ($a \geq 0$) and bananas ($b \geq 0$), but he cannot get more than 100 apples ($a \leq 100$) and 60 bananas ($b \leq 60$).

The apples and bananas combined cannot weigh more than 10 000 g.

If apples weigh 80 g and bananas weigh 120 g, then $80a + 120b \leq 10\ 000$.

**Question 7**

***Answer is A***

**Worked solution**

First, find the equations for the lines: $x = 0$, $y = 0$, $y = 50$, $x = 80$ and $3y + 2x = 240$

Then test a point inside the shaded region and form your inequations.

For example, the point (20, 20) is inside the shaded region and
$x \le 80$, $y \le 50$, $3y + 2x \le 240$, $x \ge 0$, $y \ge 0$.

**Question 8**

***Answer is E***

**Worked solution**

The total revenue is \$500, and the total profit is \$200. Therefore, the total cost is \$300.

Cost of 100 caps sold

$100 \times \$1.50 + x = \$300$

$\$150 + x = \$300$

$x = \$150$

**Question 9**

***Answer is D***

**Worked solution**

From the original graph $y = \frac{1}{3}x^3$.

Substitute the points from each possible answer. Only (3, 9) works for this equation,

$9 = \frac{1}{3}3^3 \Leftrightarrow 9 = 9$

Therefore, the answer is D.

## Module 4: Business-related mathematics

### Question 1

***Answer is D***

**Worked solution**

When an $r$% increase has been applied (in the case of GST, $r$ = 10%), the original price

$$= \text{new price} \times \frac{100}{(100 + r)}$$

$$= 214.5 \times \frac{10}{11}$$

$$= 195$$

### Question 2

***Answer is D***

**Worked solution**

Simple interest = $\frac{Prt}{100}$ where $P$ is the principal, $r$ is the rate (as a percentage) and $t$ is time in years.

$$= \frac{4500 \times 2.7 \times 2}{100}$$

$$= 243$$

Adding the original principal gives $4743.

**Question 3**

***Answer is C***

**Worked solution**

It's a good idea to start with a hypothetical price to work with, e.g. $10 000.
After Tuesday's discount, the price is $10\ 000 \times 0.95 = 9500$.
After Wednesday's discount, the price is $9500 \times 0.94 = 8930$.
After Thursday's discount, the price is $8930 \times 0.90 = 8037$.

This is a total discount of $1963. As a percentage of the original $10 000, $1963 is

$$\frac{1963}{10\ 000} \times 100 = 19.63 \approx 20\%$$

**Tip**

- *Each percentage discount must be calculated on the previous discounted price. They are not percentage discounts off the original $10 000 and you can't just add 5% + 6% + 10% to get the correct answer.*

**Question 4**

***Answer is C***

**Worked solution**

The compound interest formula for the total value of the investment after interest is added, $A$, is

$A = P \times \left(1 + \frac{r}{n \times 100}\right)^{(n \times t)}$ where $P$ is the principal, $r$ is the interest rate, $n$ is the number of compounding periods per year and $t$ is time in years.

$$\begin{aligned} A &= 8500 \times \left(1 + \frac{6.2}{4 \times 100}\right)^{(4 \times 5)} \\ &= 11\ 561.59 \end{aligned}$$

Therefore, the interest earned is
$11\ 561.59 - 8500 = 3061.59 \approx 3062$

**Question 5**

***Answer is E***

**Worked solution**

To calculate interest payments on bank statements, a table is useful. Use the simple interest formula $I = P \times r \times t$ to calculate the interest accrued during each time period.

| Principal | Dates | Time (years) | Interest |
|---|---|---|---|
| \$800 | 1 to 8 | 8/365 | 800 × 0.02 × 8 ÷ 365 = 0.3506 |
| \$765 | 9 to 15 | 7/365 | 765 × 0.02 × 7 ÷ 365 = 0.2934 |
| \$965 | 16 to 22 | 7/365 | 965 × 0.02 × 7 ÷ 365 = 0.3701 |
| \$885 | 23 to 31 | 9/365 | 885 × 0.02 × 9 ÷ 365 = 0.4364 |
| | | 31/365 | \$1.45 |

**Tip**

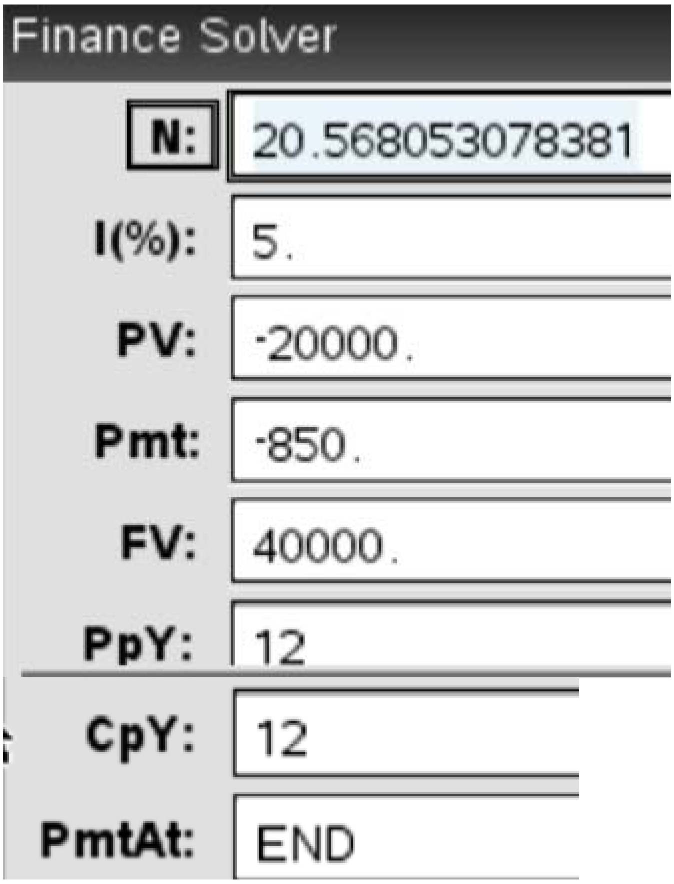
- *Check that the total time equates to the number of days in the month.*

**Question 6**

***Answer is B***

**Worked solution**

This is a situation where a compound interest account also has regular deposits and we need to use TVM solver.



**Question 7**

***Answer is E***

**Worked solution**

First calculate the total amount paid: $(500 + 200 \times 12 \times 3) = 7700$
which means he pays $3200 in interest.

**Effective interest rate**

Effective interest rate per annum, $r_e = \frac{100I}{Pt} \times \frac{2n}{(n+1)}$

where *I* = total interest paid
*P* = principal owing after the deposit has been deducted
*t* = number of years
*n* = number of payments made in total

$$\frac{100 \cdot 3200 \cdot 72}{4000 \cdot 3 \cdot 37} \qquad 51.8918918919$$

**Tip**

- *The principal in the calculation is $4000, the amount borrowed, not $4500, the total price.*

**Question 8**

***Answer is E***

**Worked solution**

Let the value of the new car be *V*.
After 5 years, it is worth $V \times (0.8)^5$ (this is the reducing balance depreciation formula)

After another 3 years, it is worth $V \times (0.8)^5 \times (0.85) = 15\ 320$

Using solve to find *V*

$$\text{solve}\left(v \cdot (0.8)^5 \cdot 0.85 = 15320, v\right)$$

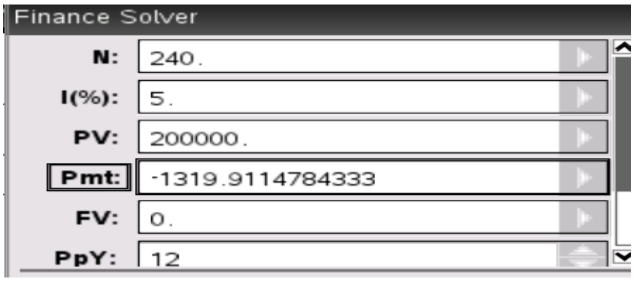
$$v = 55003.4466912$$

**Question 9**

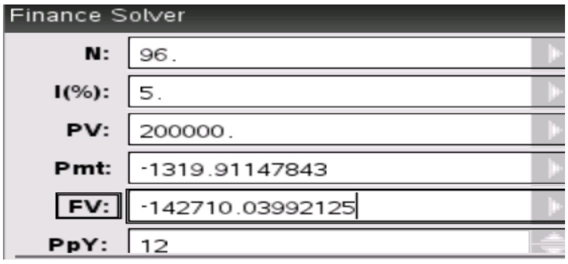
***Answer is B***

**Worked solution**

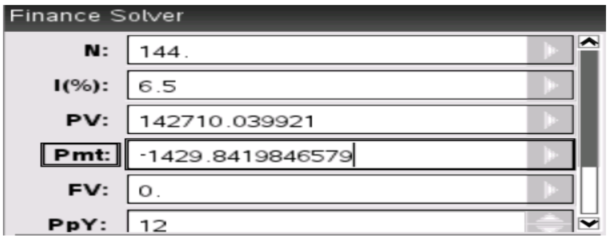
This question involves using the TVM solver.
First calculate the initial repayment.




Now calculate the amount owing after 8 years.




Now transfer the amount owing to the amount borrowed (assume it's the start of a new loan with an interest rate of 6.5%) for 12 years and find the payment.




Payment Increase

$= $ New Repayment – Old Repayment

$= 1429.84 - 1319.91$

$= 109.93$

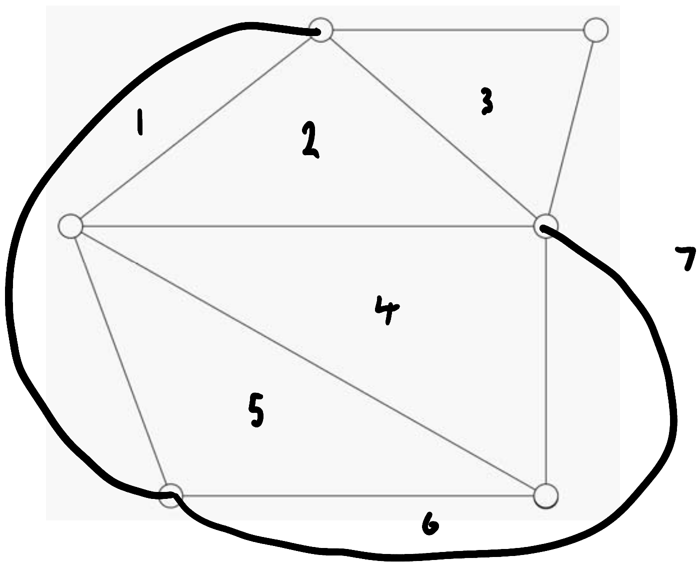
**Module 5: Networks and decision mathematics**

**Question 1**

***Answer is A***

**Worked solution**

Redraw the network so that the edges do not cross each other and then count the faces.




**Tip**

- *Don't forget to add a face for the outside (i.e. a total of 7 faces).*

**Question 2**

***Answer is B***

**Worked solution**

For a network to contain a Euler path it must contain exactly 2 odd vertices and the other vertices must be even. In **B**, only vertices *G* and *E* are odd.

**Question 3**

***Answer is E***

**Worked solution**

Possible Hamilton Circuit *A*, *B*, *D*, *E*, *F*, *C*, *G*, *A*.

**Question 4**

***Answer is D***

**Worked solution**

Football is liked by 2 boys, while the other sports are each liked by 3 boys.

**Question 5**

***Answer is C***

**Worked solution**

Subtract the lowest number in each row, from each number in that row. Write down the new table (answer A). Using this new table, subtract the lowest number in each column from each number in that column and write down the new table.

**Question 6**

***Answer is D***

**Worked solution**

The critical path, or paths, through the network are the longest paths and give us the shortest completion time for the whole project.
The critical path is A, F, D, H, I taking 21 hours.

**Question 7**

***Answer is C***

**Worked solution**

All activities except for B, C, G and J are on the new critical paths: A, F, D, H, I and A, E, K (with a total length of 14 hours).

**Question 8**

***Answer is C***

**Worked solution**

A loop is represented by a 1 in an adjacency matrix.

**Question 9**

***Answer is D***

**Worked solution**

Euler's rule can be used for a connected planar graph. Because we know that the graph has twice as many edges as it has vertices, let the number of vertices be *V* and the number of edges be 2*V*.

$v + f = 2 + e$

$V + f = 2 + 2V$

rearranging

$f = V + 2$

Therefore, the number of faces equals the number of vertices plus 2.

**Module 6: Matrices**

**Question 1**

***Answer is E***

**Worked solution**

The order of the product is the number of rows in the first matrix by the number of columns in the second. For the product to be defined, the number of columns in the first matrix must be equal to the number of rows in the second matrix.

In this question, matrix *A* is 4 × 2 and matrix *B* is 2 × 4 so the product is defined because the number of columns in the first matrix (2) = the number of rows in the second matrix (2) and the order of the product will be the number of rows in the first matrix (4) by the number of columns in the second matrix (4). The fact that column 2 of matrix *A* and row 1 of matrix *B* are made up of all zero elements does not change the order of the product matrix.

**Question 2**

***Answer is B***

**Worked solution**

This matrix calculation is easily done on the calculator.

$$\begin{bmatrix} 1 & -1 \\ 4 & -2 \end{bmatrix} - 2 \cdot \begin{bmatrix} 3 & 4 \\ -2 & -1 \end{bmatrix} \qquad \begin{bmatrix} -5. & -9. \\ 8. & 0. \end{bmatrix}$$

**Question 3**

***Answer is A***

**Worked solution**

Step 1: Interpret the text and write down the 3 equations.

$3a + 2b + c = 8.90$

$3b + 2c = 7.60$

$2a + 5b = 9.00$

Step 2: Set up the matrix equation.

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 3 & 2 \\ 2 & 5 & 0 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8.90 \\ 7.60 \\ 9.00 \end{bmatrix}$$

Step 3: Transpose the matrix equation to get the unknown variables on the left-hand side. (If necessary, in this case the step 2 version of the equation is one of the possible answers.)

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 3 & 2 \\ 2 & 5 & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} 8.90 \\ 7.60 \\ 9.00 \end{bmatrix}$$

**Question 4**

***Answer is C***

**Worked solution**

$$\begin{array}{cccc} 1\times 3 & 3\times 3 & = & 1\times 3 \\ \begin{bmatrix} B & C & S \end{bmatrix}\times & \begin{bmatrix} 0.85 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0 & 0.82 \end{bmatrix} & = & \begin{bmatrix} 0.85\mathrm{B} & 0.75C & 0.82S \end{bmatrix} \end{array}$$

**Question 5**

***Answer is E***

**Worked solution**

A is a singular matrix, not $\mathrm{A}^{-1}$.
The inverse of A does not exist.
det(A) is zero, not undefined.
$\mathrm{A}^{-1}$ is not zero; det(A) is zero.
$\mathrm{A}^{-1}$ is undefined because det(A) is zero is true.

**Question 6**

***Answer is B***

**Worked solution**

Only the first set for which the determinant = 0 does not have a unique solution. For the other 3 pairs of equations, the determinant ≠ 0.

**Question 7**

***Answer is B***

**Worked solution**

The transition matrix is $\begin{array}{cc} \begin{array}{cc} R & S \end{array} & \\ \begin{bmatrix} 0.2 & 0.9 \\ 0.8 & 0.1 \end{bmatrix} & \begin{array}{c} R \\ S \end{array} \end{array}$ and the initial state matrix is $\begin{bmatrix} 10 \\ 8 \end{bmatrix}$.

To find the Thursday state matrix find $T^3S_0$.

$$\begin{bmatrix} 0.2 & 0.9 \\ 0.8 & 0.1 \end{bmatrix}^3 \cdot \begin{bmatrix} 10 \\ 8 \end{bmatrix} \qquad \begin{bmatrix} 9.368 \\ 8.632 \end{bmatrix}$$

9.368 = 9 people

**Question 8**

***Answer is C***

**Worked solution**

Transition matrix is

$\begin{bmatrix} 0.50 & 0.25 & 0.40 \\ 0.35 & 0.10 & 0.45 \\ 0.15 & 0.65 & 0.15 \end{bmatrix}$ and initial state matrix $S_0$ is $\begin{bmatrix} 300 \\ 200 \\ 500 \end{bmatrix}$

To find the number of ants at each hole after 4 hours we need to find $S_4$.

$$\begin{bmatrix} 0.5 & 0.25 & 0.4 \\ 0.35 & 0.1 & 0.45 \\ 0.15 & 0.65 & 0.15 \end{bmatrix}^4 \cdot \begin{bmatrix} 300 \\ 200 \\ 500 \end{bmatrix} \qquad \begin{bmatrix} 392.96875 \\ 301.71875 \\ 305.3125 \end{bmatrix}$$

393 ants

**Question 9**

***Answer is D***

**Worked solution**

$$\begin{matrix} (A & \times & B) & + C \end{matrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 2 \\ x + 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$x = 2$

and $B = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

**END OF WORKED SOLUTIONS**