VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2013

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

FURTHER MATHEMATICS

Written examination 2

Monday 4 November 2013

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 41 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

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Core 4

Module

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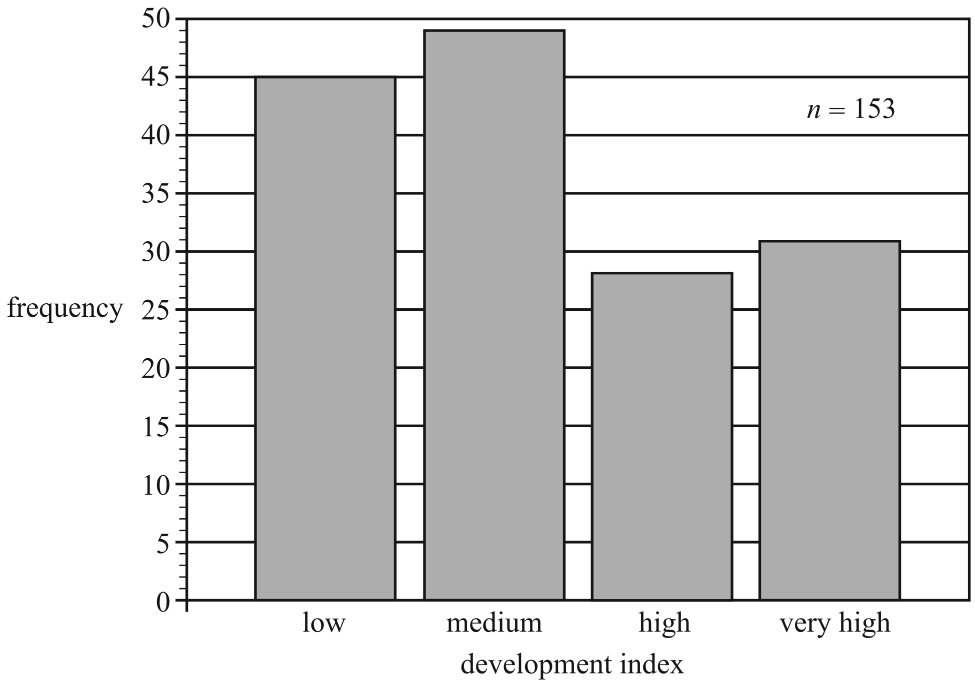
CONTINUES OVER PAGE

Core

Question 1 (2 marks)

A development index is used as a measure of the standard of living in a country.

The bar chart below displays the development index for 153 countries in four categories: low, medium, high and very high.



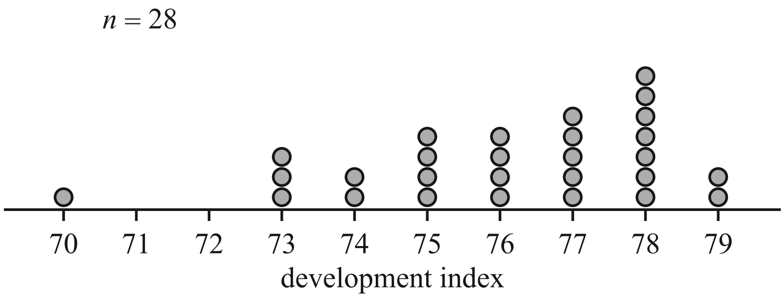
a. How many of these countries have a very high development index? 1 mark

b. What percentage of the 153 countries has either a low or medium development index?
Write your answer, correct to the nearest percentage. 1 mark

Question 2 (3 marks)

The development index for each country is a whole number between 0 and 100.

The dot plot below displays the values of the development index for each of the 28 countries that has a high development index.



a. Using the information in the dot plot, determine each of the following. 1 mark

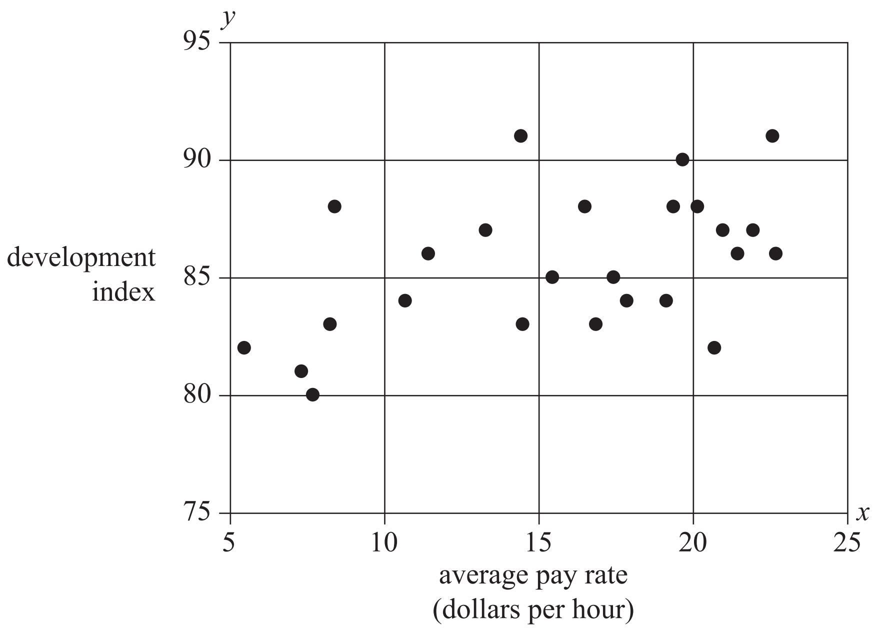
the mode

the range

b. Write down an appropriate calculation and use it to explain why the country with a development index of 70 is an outlier for this group of countries. 2 marks

Question 3 (5 marks)

The development index and the average pay rate for workers, in dollars per hour, for a selection of 25 countries are displayed in the scatterplot below.



The table below contains the values of some statistics that have been calculated for this data.

Statistic	Average pay rate (x)	Development index (y)
mean	$\bar{x} = 15.7$	$\bar{y} = 85.6$
standard deviation	$s_x = 5.37$	$s_y = 2.99$
correlation coefficient	$r = 0.488$	

a. Determine the standardised value of the development index (z score) for a country with a development index of 91.

Write your answer, correct to one decimal place. 1 mark

b. Use the information in the table to show that the equation of the least squares regression line for a country's development index, y, in terms of its average pay rate, x, is given by 2 marks

$$y = 81.3 + 0.272x$$

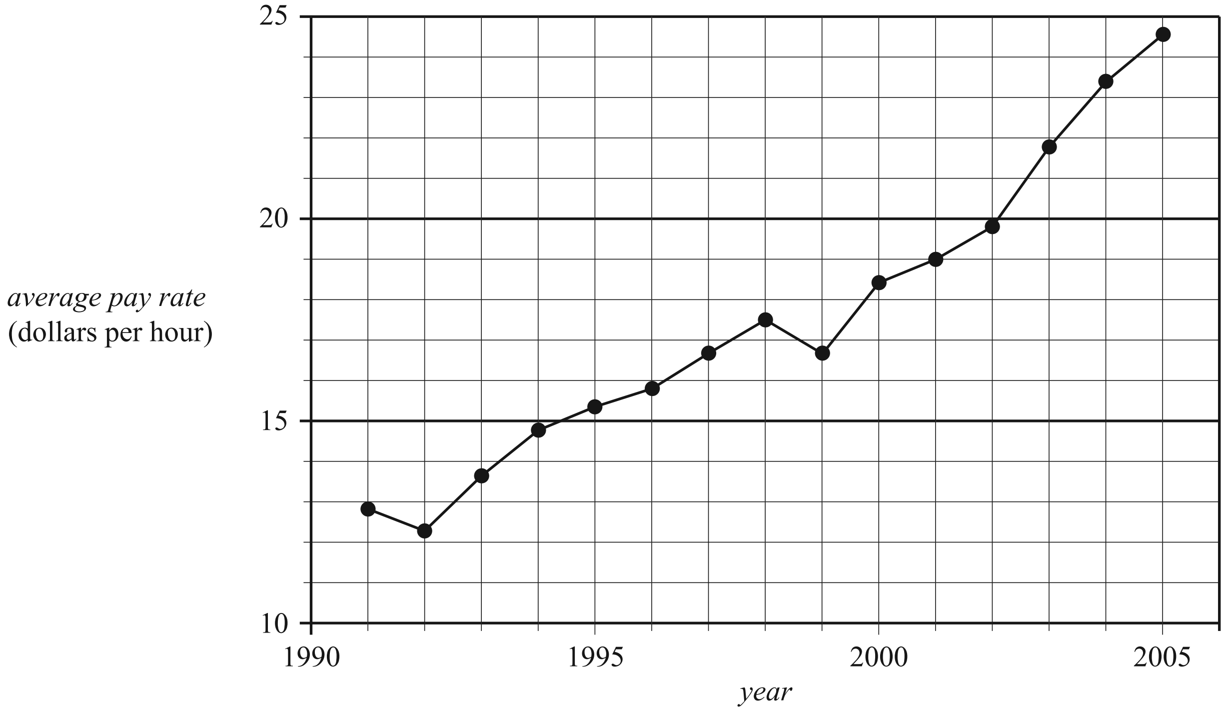
c. The country with an average pay rate of $14.30 per hour has a development index of 83.

Determine the residual value when the least squares regression line given in **part b.** is used to predict this country's development index.

Write your answer, correct to one decimal place. 2 marks

Question 4 (5 marks)

The time series plot below shows the average pay rate, in dollars per hour, for workers in a particular country for the years 1991 to 2005.



A three median line will be used to model the increasing trend in the average pay rate shown in this time series.

The independent variable to be used is *year*.

a. Three medians will be used to draw the three median line.

i. On the time series plot above, mark the location of each of the three medians with a cross (X). 2 marks

ii. Draw the three median line on the time series plot. 1 mark

b. Calculate the slope of the three median line.
Write your answer, correct to one decimal place. 1 mark

c. Interpret the slope of the three median line in terms of the relationship between the variables *average pay rate* and *year*. 1 mark

END OF CORE
TURN OVER

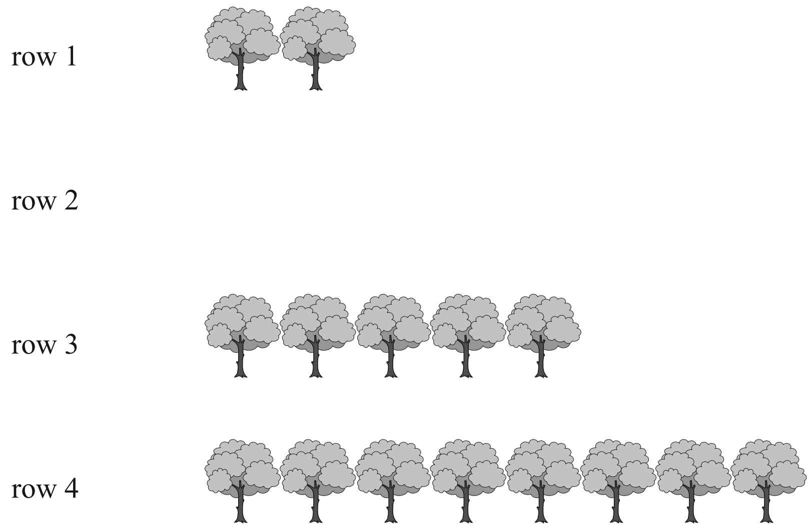
Module 1: Number patterns

Question 1 (2 marks)

Cherry trees are planted in rows in an orchard.

The number of trees planted in successive rows of the orchard follows a Fibonacci-related sequence, as shown in the diagram below.

The trees planted in row 2 are missing from the diagram.



a. How many trees should be drawn in row 2? 1 mark

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b. Assume that the pattern of tree planting follows this Fibonacci-related sequence.
How many trees would be planted in the sixth row? 1 mark

Question 2 (3 marks)

The weight of the cherries that are produced in the orchard each year follows a geometric sequence with common ratio $r = 1.2$

The first two terms of this sequence are shown in the table below.

Year	1	2
Weight of cherries (tonnes)	8	9.6

a. How many tonnes of cherries will the orchard produce in the fifth year?
Write your answer, correct to one decimal place. 1 mark

b. During which year will the orchard first produce more than 30 tonnes of cherries? 1 mark

c. Find the total weight of the cherries produced in the orchard from the start of the first year to the end of the 10th year.
Write your answer, correct to the nearest tonne. 1 mark

Question 3 (4 marks)

Pear trees are grown in another orchard.

At the end of each year, new pear trees are always planted.

It is known that 250 new pear trees are planted at the end of the third year.

The total number of new pear trees, P_n, planted at the end of the nth year, is modelled by the difference equation

$$P_{n+1} = P_n + 50 \qquad P_1 = c$$

a. **i.** Determine the value of c. 1 mark

ii. The difference equation above generates a sequence.

What is the mathematical name given to this type of sequence? 1 mark

The total number of new pear trees planted at the end of the nth year can also be found from the rule $P_n = a + b \times n$, where a and b are constants.

b. Determine the values of a and b. 2 marks

$a =$ $\square$ $b =$ $\square$

Question 4 (3 marks)

Apple trees are growing in a third orchard.

Over time, some of the trees stop producing enough apples and are removed at the end of the year in which this first occurs. Immediately afterwards, a fixed number of new apple trees will be planted.

The total number of apple trees growing in the orchard at the end of the *n*th year, A_n, immediately after the planting of the new apple trees for that year, is modelled by the difference equation

$$A_{n+1} = 0.8 \times A_n + k \qquad A_1 = 18\,000$$

where k is the number of new apple trees planted at the end of each year.

a. What percentage of apple trees will be removed at the end of each year? 1 mark

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b. Assume 100 new apple trees are planted at the end of each year.

Determine how many apple trees will be growing in the orchard at the end of the third year, immediately after the planting of the new apple trees for that year. 1 mark

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c. Determine the number of new apple trees, k, that needs to be planted at the end of each year so that there will always be 18 000 apple trees growing in the orchard. 1 mark

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Question 5 (3 marks)

The total weight of apples produced in a section of the orchard is expected to decrease in successive years according to a geometric sequence.

Let L_n be the total weight of apples produced in this section of the orchard, in tonnes, during year n.

In 2012, this section of the orchard produced 20 tonnes of apples, that is, $L_{2012} = 20$.

In 2013, this section of the orchard produced 18 tonnes of apples, that is, $L_{2013} = 18$.

a. Write a difference equation in terms of L_n and L_{n+1} for the total weight of apples, in tonnes, that this section of the orchard is expected to produce in successive years. 1 mark

The income that is generated by this section of the orchard, per tonne of apples, depends on the age of the trees.

In 2012, the income, per tonne of apples, was \$1500.

The income, per tonne of apples, is expected to decrease by \$200 each year due to the ageing trees.

All the apple trees in this section of the orchard will be removed in the year in which the **total** income they generate falls below \$10 000.

b. Determine the year in which all the apple trees in this section of the orchard will be removed. 2 marks

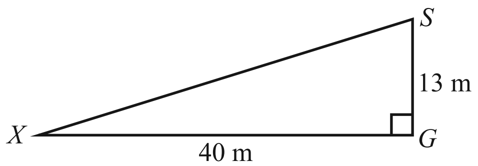
END OF MODULE 1

CONTINUES OVER PAGE

Module 2: Geometry and trigonometry

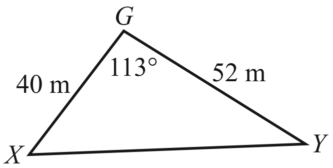
Question 1 (4 marks)

A spectator, *S*, in the grandstand of an athletics ground is 13 m vertically above point *G*.
Competitor *X*, on the athletics ground, is at a horizontal distance of 40 m from *G*.



a. Find the distance, *SX*, correct to the nearest metre. 1 mark

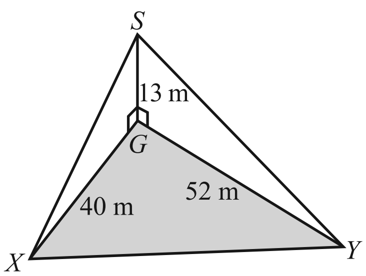
Competitor *X* is 40 m from *G* and competitor *Y* is 52 m from *G*.
The angle *XGY* is 113°.



b. **i.** Calculate the distance, *XY*, correct to the nearest metre. 1 mark

ii. Find the area of triangle *XGY*, correct to the nearest square metre. 1 mark

c. Determine the angle of elevation of spectator *S* from competitor *Y*, correct to the nearest degree.
Note that *X*, *G* and *Y* are on the same horizontal level. 1 mark



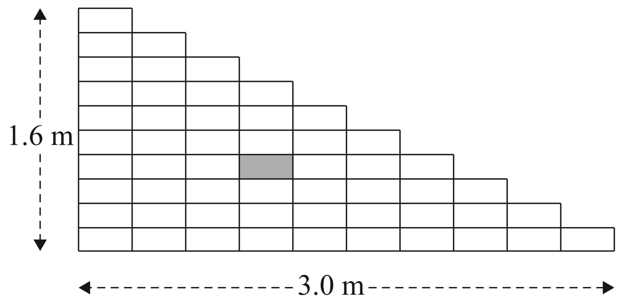
Question 2 (3 marks)

A concrete staircase leading up to the grandstand has 10 steps.

The staircase is 1.6 m high and 3.0 m deep.

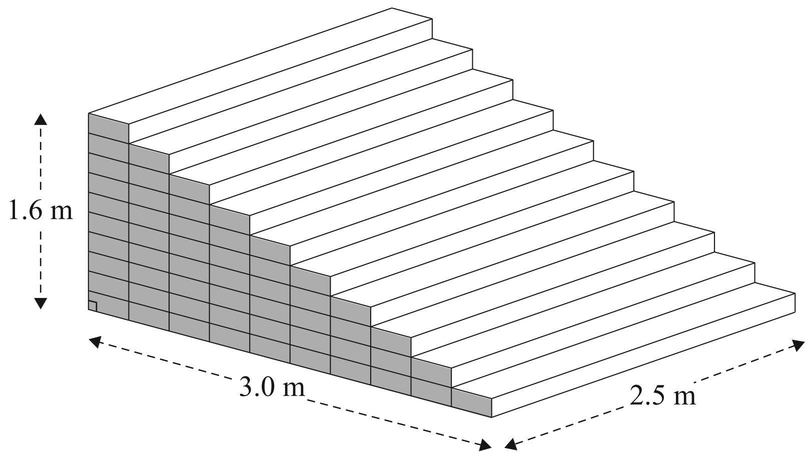
Its cross-section comprises identical rectangles.

One of these rectangles is shaded in the diagram below.



a. Find the area of the shaded rectangle in square metres. 1 mark

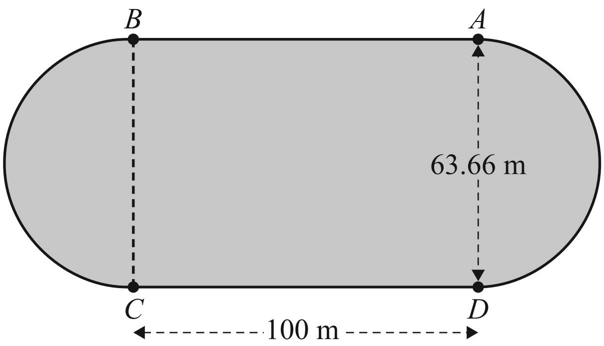
The concrete staircase is 2.5 m wide.



b. Find the volume of the solid concrete staircase in cubic metres. 2 marks

Question 3 (4 marks)

A grassed region in the athletics ground is shown shaded in the diagram below.



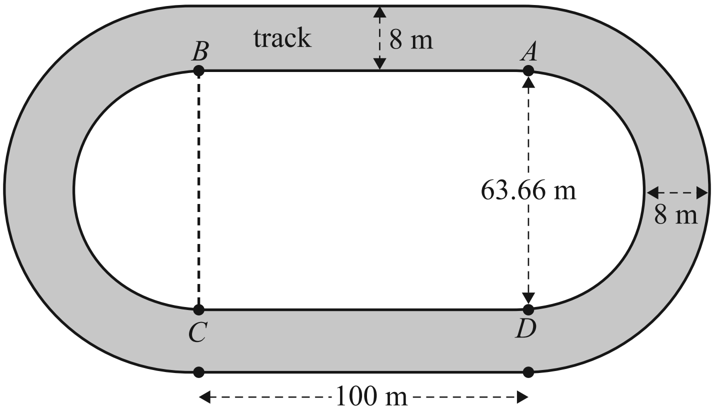
The perimeter of the grassed region comprises two parallel lines, *BA* and *CD*, each 100 m in length, and two semi-circles, *BC* and *AD*.

In total, the perimeter of the grassed region is 400 m.

a. The diameter of the semi-circle *AD* is 63.66 m, correct to two decimal places.
Show how this value could be obtained. 1 mark

b. Determine the area of the grassed region, correct to the nearest square metre. 1 mark

A running track, shown shaded in the diagram below, surrounds the grassed region. This running track is 8 m wide at all points.



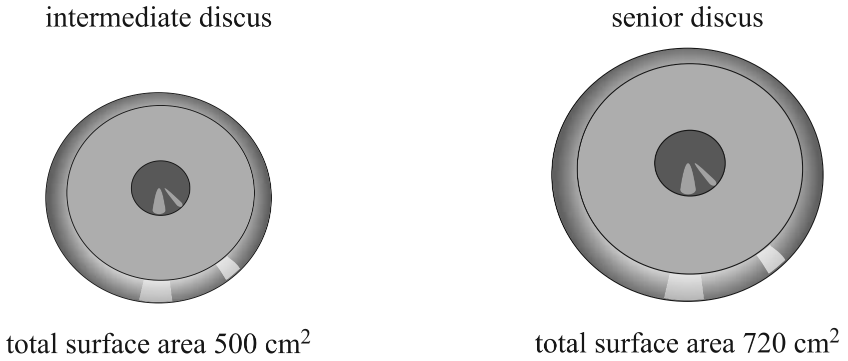
c. The running track is to be resurfaced with special rubber material that is 0.1 m deep.

Find the volume of rubber material that is needed to resurface the running track.

Write your answer, correct to the nearest cubic metre. 2 marks

Question 4 (2 marks)

Competitors in the intermediate division of the discus use a smaller discus than the one used in the senior division, but of a similar shape. The total surface area of each discus is given below.



By what value can the volume of the intermediate discus be multiplied to give the volume of the senior discus?

Question 5 (2 marks)

Daniel threw a javelin a distance of 68.32 m on a bearing of 057° on his first throw.

On his second throw from the same point, he threw the javelin a distance of 72.51 m.

The second throw landed at a point on a bearing of 125°, measured from the point where the first throw landed.

Determine the distance between the point where Daniel's first throw landed and the point where his second throw landed.

Write your answer in metres, correct to one decimal place.

END OF MODULE 2

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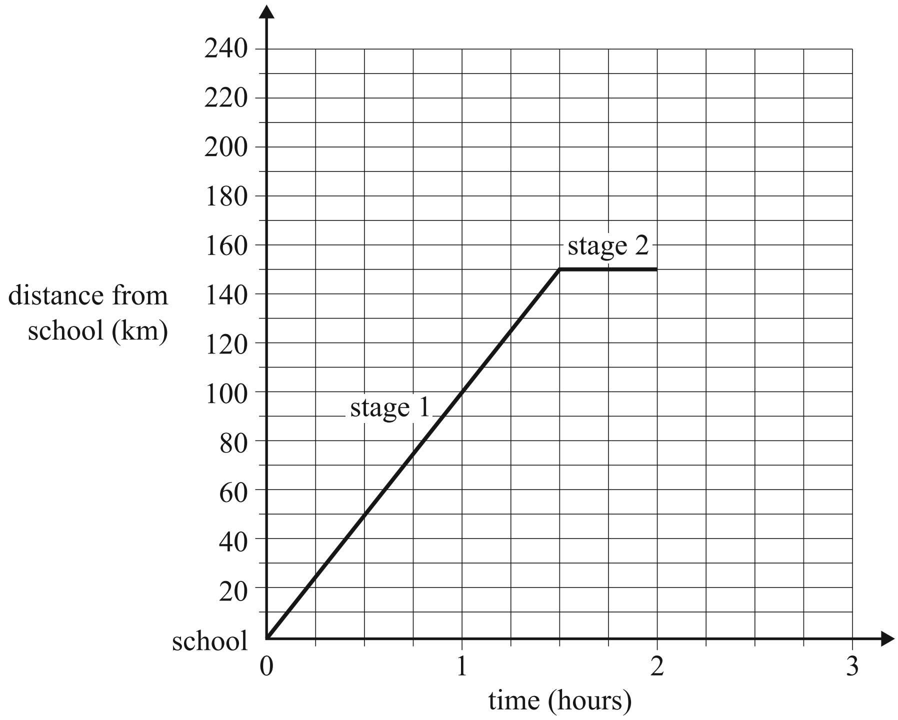
Module 3: Graphs and relations

Question 1 (5 marks)

The distance-time graph below shows the first two stages of a bus journey from a school to a camp.



a. At what constant speed, in kilometres per hour, did the bus travel during stage 1 of the journey? 1 mark

b. For how many minutes did the bus stop during stage 2 of the journey? 1 mark

The third stage of the journey is missing from the graph.

During stage 3, the bus continued its journey to the camp and travelled at a constant speed of 60 km/h for one hour.

c. Draw a line segment on the graph above to represent stage 3 of the journey. 1 mark

d. Find the average speed of the bus over the three hours.
Write your answer in kilometres per hour. 1 mark

The distance, D km, of the bus from the school t hours after departure is given by

$$D = \begin{cases} 100t & 0 \le t \le 1.5 \\ 150 & 1.5 \le t \le 2 \\ 60t + k & 2 \le t \le 3 \end{cases}$$

e. Determine the value of k. 1 mark

Question 2 (2 marks)

Students at the camp can participate in two different watersport activities: canoeing and surfing.

The cost of canoeing is $30 per hour and the cost of surfing is $20 per hour.

The budget allows each student to spend up to $200, in total, on watersport activities.

The way in which a student decides to spend the $200 is described by the following inequality.

$$30 \times \textit{hours canoeing} + 20 \times \textit{hours surfing} \leq 200$$

a. Hillary wants to spend exactly two hours canoeing during the camp.

Calculate the maximum number of hours she could spend surfing. 1 mark

b. Dennis would like to spend an equal amount of time canoeing and surfing.

If he spent a total of $200 on these activities, determine the maximum number of hours he could spend on each activity. 1 mark

Question 3 (2 marks)

A rock-climbing activity will be offered to students at the camp on one afternoon.

Each student who participates will pay \$24.

The organisers have to pay the rock-climbing instructor \$260 for the afternoon. They also have to pay an insurance cost of \$6 per student.

Let n be the total number of students who participate in rock climbing.

a. Write an expression for the profit that the organisers will make in terms of n. 1 mark

b. The organisers want to make a profit of at least \$500.

Determine the minimum number of students who will need to participate in rock climbing. 1 mark

Question 4 (6 marks)

The school group may hire two types of camp sites: powered sites and unpowered sites.

Let x be the number of powered camp sites hired
y be the number of unpowered camp sites hired.

Inequality 1 and inequality 2 give some restrictions on x and y.

inequality 1 $x \leq 5$
inequality 2 $y \leq 10$

There are 48 students to accommodate in total.

A powered camp site can accommodate up to six students and an unpowered camp site can accommodate up to four students.

Inequality 3 gives the restrictions on x and y based on the maximum number of students who can be accommodated at each type of camp site.

inequality 3 $ax + by \geq 48$

a. Write down the values of a and b in inequality 3. 1 mark

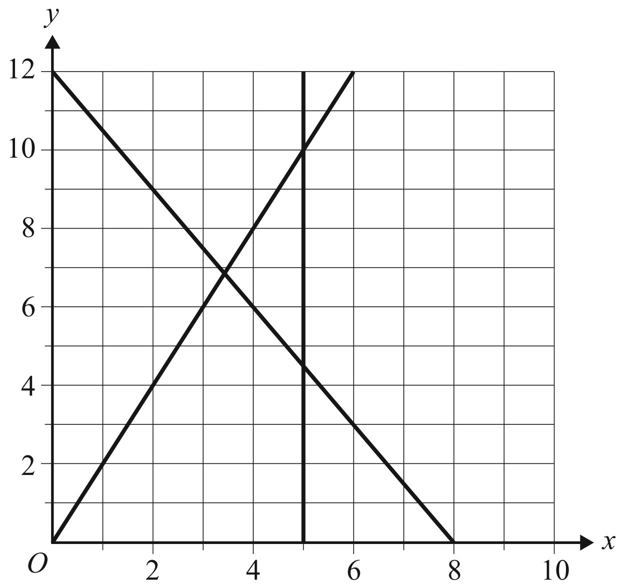
$a =$ ☐ $b =$ ☐

School groups must hire at least two unpowered camp sites for every powered camp site they hire.

b. Write this restriction in terms of x and y as inequality 4. 1 mark

inequality 4 ______________________

The graph below shows the three lines that represent the boundaries of inequalities 1, 3 and 4.



c. On the graph above, show the points that satisfy inequalities 1, 2, 3 and 4. 1 mark

d. Determine the minimum number of camp sites that the school would need to hire. 1 mark

e. The cost of each powered camp site is \$60 per day and the cost of each unpowered camp site is \$30 per day.

i. Find the minimum cost per day, in total, of accommodating 48 students. 1 mark

School regulations require boys and girls to be accommodated separately.

The girls must all use one type of camp site and the boys must all use the other type of camp site.

ii. Determine the minimum cost per day, in total, of accommodating the 48 students if there is an equal number of boys and girls. 1 mark

END OF MODULE 3
TURN OVER

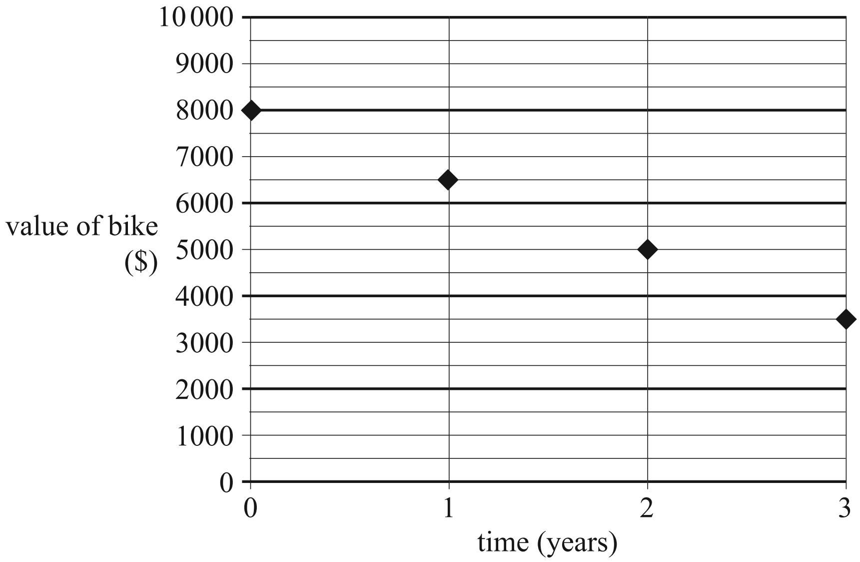
Module 4: Business-related mathematics

Question 1 (4 marks)

Hugo is a professional bike rider.

The value of his bike will be depreciated over time using the flat rate method of depreciation.

The graph below shows his bike's initial purchase price and its value at the end of each year for a period of three years.



a. What was the initial purchase price of the bike? 1 mark

b. **i.** Show that the bike depreciates in value by $1500 each year. 1 mark

ii. Assume that the bike's value continues to depreciate by $1500 each year.
Determine its value five years after it was purchased. 1 mark

The unit cost method of depreciation can also be used to depreciate the value of the bike.

In a two-year period, the total depreciation calculated at $0.25 per kilometre travelled will equal the depreciation calculated using the flat rate method of depreciation as described above.

c. Determine the number of kilometres the bike travels in the two-year period. 1 mark

Question 2 (3 marks)

Hugo won \$5000 in a road race and invested this sum at an interest rate of 4.8% per annum compounding monthly.

a. What is the value of Hugo's investment after 12 months?

Write your answer in dollars, correct to the nearest cent. 1 mark

b. i. Suppose instead that at the end of each month Hugo added \$200 to his initial investment of \$5000.

Find the value of this investment immediately after the 12th monthly payment of \$200 is made.

Write your answer in dollars, correct to the nearest cent. 1 mark

ii. Assume Hugo follows the investment that is described in **part b.i.**

Determine the total interest he would earn over the 12-month period.

Write your answer in dollars, correct to the nearest cent. 1 mark

Question 3 (6 marks)

Hugo paid $7500 for a second bike under a hire-purchase agreement.

A flat interest rate of 8% per annum was charged.

He will fully repay the principal and the interest in 24 equal monthly instalments.

a. Determine the monthly instalment that Hugo will pay.

Write your answer in dollars, correct to the nearest cent. 2 marks

b. Find the effective rate of interest per annum charged on this hire-purchase agreement.

Write your answer as a percentage, correct to two decimal places. 1 mark

c. Explain why the effective interest rate per annum is higher than the flat interest rate per annum. 1 mark

d. The value of his second bike, purchased for $7500, will be depreciated each year using the reducing balance method of depreciation.

One year after it was purchased, this bike was valued at $6375.

Determine the value of the bike five years after it was purchased.

Write your answer, correct to the nearest dollar. 2 marks

Question 4 (2 marks)

Hugo took out a reducing balance loan of $25 000 to compete in road races overseas.

Interest was charged at a rate of 12% per annum compounding quarterly.

His loan is to be repaid fully in four years with equal quarterly payments.

After two years, how much of the $25 000 will Hugo have repaid?

Write your answer, correct to the nearest dollar.

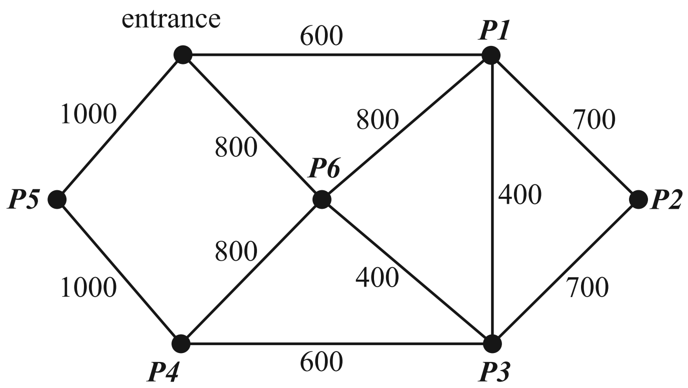
END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1 (5 marks)

The vertices in the network diagram below show the entrance to a wildlife park and six picnic areas in the park: *P1*, *P2*, *P3*, *P4*, *P5* and *P6*.

The numbers on the edges represent the lengths, in metres, of the roads joining these locations.



a. In this graph, what is the degree of the vertex at the entrance to the wildlife park? 1 mark

b. What is the shortest distance, in metres, from the entrance to picnic area *P3*? 1 mark

c. A park ranger starts at the entrance and drives along every road in the park once.

i. At which picnic area will the park ranger finish? 1 mark

ii. What mathematical term is used to describe the route the park ranger takes? 1 mark

d. A park cleaner follows a route that starts at the entrance and passes through each picnic area once, ending at picnic area *P1*.

Write down the order in which the park cleaner will visit the six picnic areas. 1 mark

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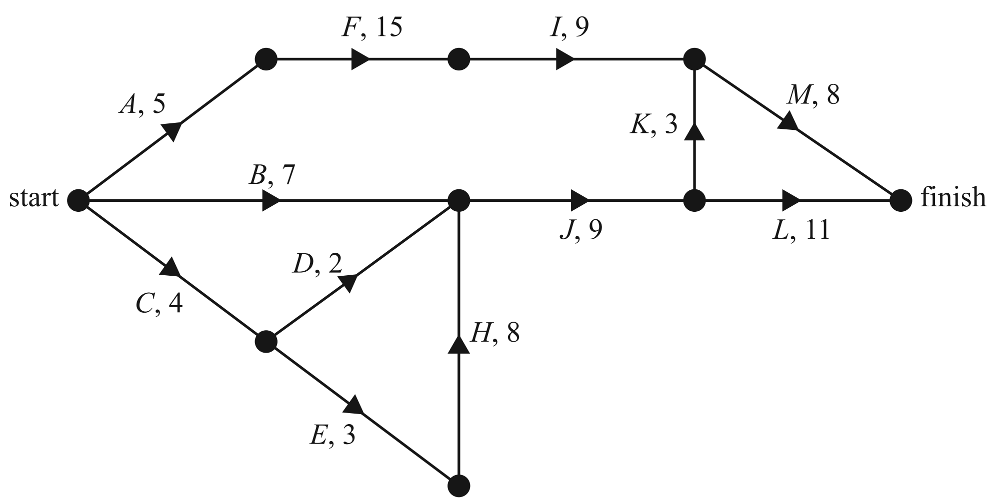
Question 2 (6 marks)

A project will be undertaken in the wildlife park. This project involves the 13 activities shown in the table below. The duration, in hours, and predecessor(s) of each activity are also included in the table.

Activity	Duration (hours)	Predecessor(s)
A	5	–
B	7	–
C	4	–
D	2	*C*
E	3	*C*
F	15	*A*
G	4	*B*, *D*, *H*
H	8	*E*
I	9	*F*, *G*
J	9	*B*, *D*, *H*
K	3	*J*
L	11	*J*
M	8	*I*, *K*

Activity *G* is missing from the network diagram for this project, which is shown below.



a. Complete the network diagram above by inserting activity *G*. 1 mark

b. Determine the earliest starting time of activity *H*. 1 mark

c. Given that activity G is not on the critical path

i. write down the activities that are on the critical path in the order that they are completed 1 mark

ii. find the latest starting time for activity D. 1 mark

d. Consider the following statement.

'If just one of the activities in this project is crashed by one hour, then the minimum time to complete the entire project will be reduced by one hour.'

Explain the circumstances under which this statement will be true for this project. 1 mark

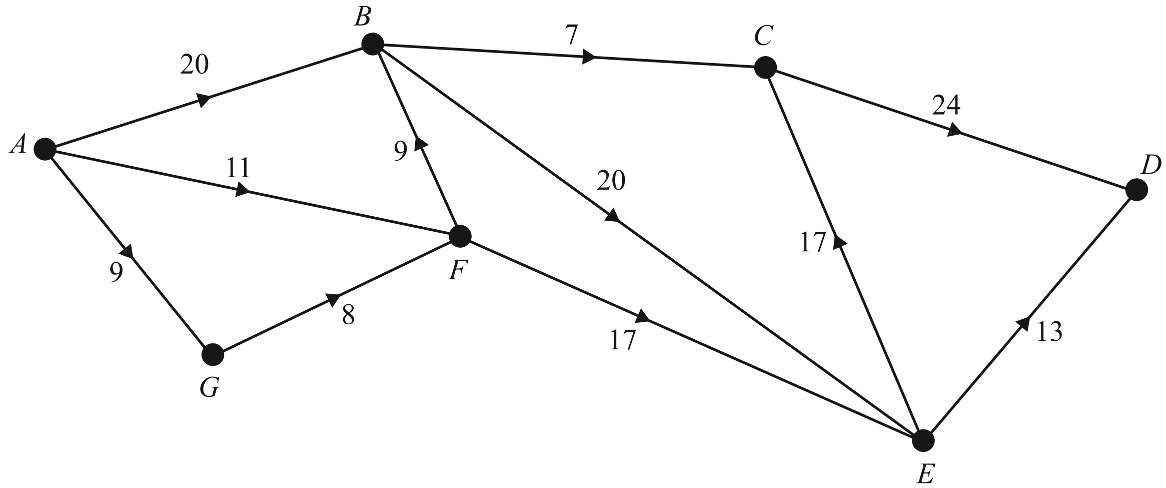
e. Assume activity F is crashed by two hours.

What will be the minimum completion time for the project? 1 mark

Question 3 (4 marks)

The rangers at the wildlife park restrict access to the walking tracks through areas where the animals breed.

The edges on the directed network diagram below represent one-way tracks through the breeding areas. The direction of travel on each track is shown by an arrow. The numbers on the edges indicate the maximum number of people who are permitted to walk along each track each day.



a. Starting at *A*, how many people, in total, are permitted to walk to *D* each day? 1 mark

One day, all the available walking tracks will be used by students on a school excursion.

The students will start at *A* and walk in four separate groups to *D*.

Students must remain in the same groups throughout the walk.

b. i. Group 1 will have 17 students. This is the maximum group size that can walk together from *A* to *D*.

Write down the path that group 1 will take. 1 mark

ii. Groups 2, 3 and 4 will each take different paths from *A* to *D*.

Complete the six missing entries shaded in the table below. 2 marks

Group	Maximum group size	Path taken from *A* to *D*
1	17	answered in **part b.i.**
2		
3		
4		

END OF MODULE 5

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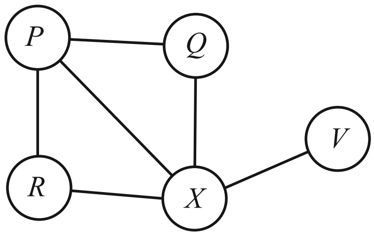
Module 6: Matrices

Question 1 (3 marks)

Five trout-breeding ponds, P, Q, R, X and V, are connected by pipes, as shown in the diagram below.



The matrix W is used to represent the information in this diagram.

$$W = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccccc} P & Q & R & X & V \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} P \\ Q \\ R \\ X \\ V \end{array}$$

In matrix W

- the 1 in column 1, row 2, for example, indicates that a pipe directly connects pond P and pond Q
- the 0 in column 1, row 5, for example, indicates that pond P and pond V are not directly connected by a pipe.

a. Find the sum of the elements in row 3 of matrix W. 1 mark

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b. In terms of the breeding ponds described, what does the sum of the elements in row 3 of matrix W represent? 1 mark

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The pipes connecting pond P to pond R and pond P to pond X are removed.

Matrix N will be used to show this situation. However, it has missing elements.

c. Complete matrix N below by filling in the missing elements in row 1 and column 1. 1 mark

$$N = \begin{array}{c} \begin{array}{ccccc} P & Q & R & X & V \end{array} \\ \begin{bmatrix} 0 & \_ & \_ & \_ & \_ \\ \_ & 0 & 0 & 1 & 0 \\ \_ & 0 & 0 & 1 & 0 \\ \_ & 1 & 1 & 0 & 1 \\ \_ & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} P \\ Q \\ R \\ X \\ V \end{array}$$

Question 2 (12 marks)

10 000 trout eggs, 1000 baby trout and 800 adult trout are placed in a pond to establish a trout population.

In establishing this population

- eggs (*E*) may die (*D*) or they may live and eventually become baby trout (*B*)
- baby trout (*B*) may die (*D*) or they may live and eventually become adult trout (*A*)
- adult trout (*A*) may die (*D*) or they may live for a period of time but will eventually die.

From year to year, this situation can be represented by the transition matrix *T*, where

$$T = \begin{array}{c} \textit{this year} \\ \begin{array}{cccc} E & B & A & D \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{array}\right] \end{array} \begin{array}{c} E \\ B \\ A \\ D \end{array} \ \textit{next year}$$

a. Use the information in the transition matrix *T* to

i. determine the number of eggs in this population that die in the first year 1 mark

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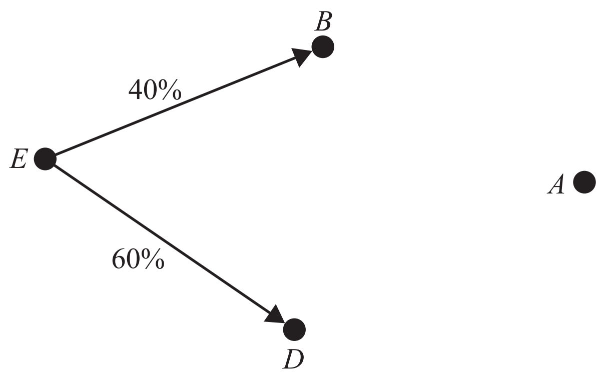
ii. complete the transition diagram below, showing the relevant percentages. 2 marks



The initial state matrix for this trout population, S_0, can be written as

$$S_0 = \begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} \begin{matrix} E \\ B \\ A \\ D \end{matrix}$$

Let S_n represent the state matrix describing the trout population after n years.

b. Using the rule $S_n = T\,S_{n-1}$, determine each of the following.

i. S_1 1 mark

ii. the number of adult trout predicted to be in the population after four years 1 mark

c. The transition matrix T predicts that, in the long term, all of the eggs, baby trout and adult trout will die.

i. How many years will it take for all of the adult trout to die (that is, when the number of adult trout in the population is first predicted to be less than one)? 1 mark

ii. What is the largest number of adult trout that is predicted to be in the pond in any one year? 1 mark

d. Determine the number of eggs, baby trout and adult trout that, if added to or removed from the pond at the end of each year, will ensure that the number of eggs, baby trout and adult trout in the population remains constant from year to year. 2 marks

The rule $S_n = T\,S_{n-1}$ that was used to describe the development of the trout in this pond does not take into account new eggs added to the population when the adult trout begin to breed.

e. To take breeding into account, assume that 50% of the adult trout lay 500 eggs each year.

The matrix describing the population after one year, S_1, is now given by the new rule

$$S_1 = T\,S_0 + 500\,M\,S_0$$

where $$T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.25 & 0.50 & 0 \\ 0.60 & 0.75 & 0.50 & 1.0 \end{bmatrix},\ M = \begin{bmatrix} 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } S_0 = \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix}$$

i. Use this new rule to determine S_1. 1 mark

This pattern continues so that the matrix describing the population after n years, S_n, is given by the rule

$$S_n = T\,S_{n-1} + 500\,M\,S_{n-1}$$

ii. Use this rule to determine the number of eggs in the population after two years. 2 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1-r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a+b+c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET