Neap:

Trial Examination 2012

VCE Further Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
Core	3	3	15
Section	Number of modules	Number of modules to be answered	Number of marks
Modules	6	3	45

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied
Question booklet of 26 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions
Detach the formula sheet from the centre of this booklet during reading time.
Please ensure that you write your **name** and your **teacher's name** in the space provided on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2012 VCE Further Mathematics Units 3 & 4 Written Examination 2.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

Page

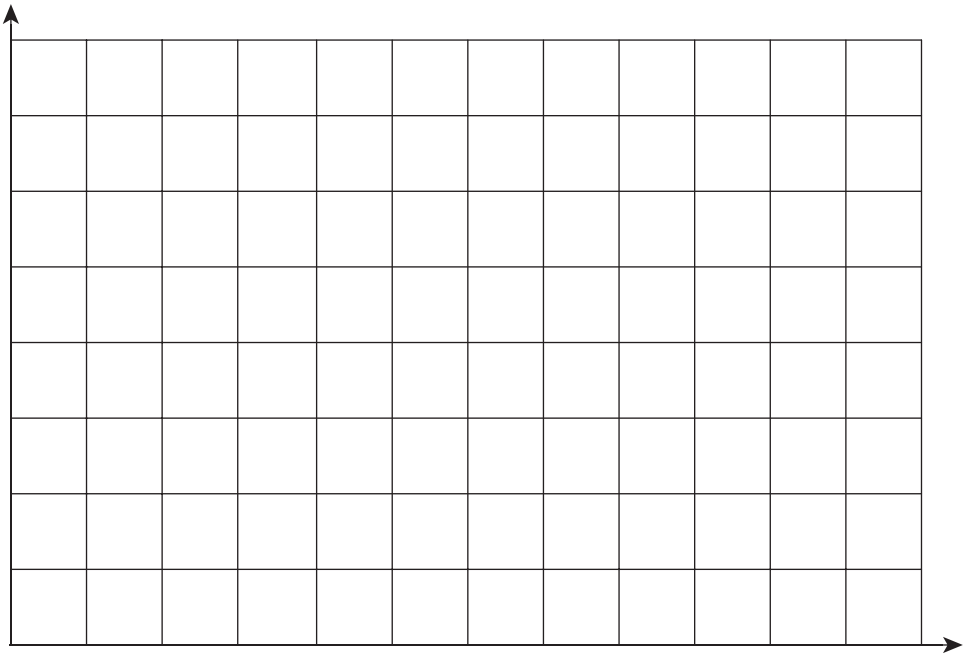
Core

Question 1

In 'New Mathsland' the Minister for Tourism is given the following set of data for visitors per quarter over the last three years.

Year	1st quarter	2nd quarter	3rd quarter	4th quarter	Total
2009	24 000	38 500	58 750	31 890	153 140
2010	26 500	37 600	67 900	43 750	175 750
2011	19 800	42 000	65 460	51 430	178 690
Total	70 300	118 100	192 110	127 070	

a. On the axis below draw a time-series plot.



2 marks

b. Comment on any trend or pattern you see in the time series.

_____

1 mark

c. Calculate the seasonal index for the second quarter to one decimal place.

_____

_____

_____

2 marks

Question 2

The figures for the first quarter trouble the Minister so she invests heavily in promotion of New Mathsland for the beginning of 2012. The number of weekly visitors for the first three months in 2012 is recorded below.

Week	Visitors
1	4 500
2	4 700
3	7 900
4	3 600
5	3 700
6	3 100
7	1 850
8	6 200
9	5 800
10	3 900
11	4 700
12	5 050

a. Calculate, to the nearest whole number, the information required to complete the following:

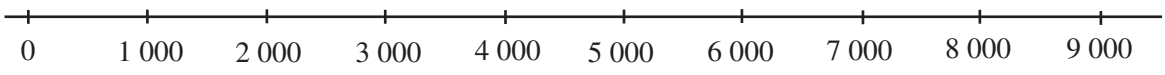
The mean number of visitors per week is ☐

The standard deviation of the number of visitors per week is ☐

The IQR is ☐

3 marks

b. Draw a box plot to represent the data.



1 mark

c. Use at least one calculated statistic to comment on the effectiveness of the promotional campaign compared to the previous three years (refer back to the data given in Question 1).

_____

_____

_____

1 mark

Question 3

a. The weather in New Mathsland is very consistent, a mean maximum daily temperature of 28°C with a standard deviation of 4°C. As a promotional exercise, a 50% discount is offered on accommodation charges if the maximum for a particular day is below 20°.

On what percentage of days will guests be able to claim this discount?

_____

_____

_____

_____

1 mark

b. A correlation coefficient of 0.75 is found to exist between the variable number of tourists and sales at restaurants.

Calculate the coefficient of determination and comment on this value in terms of the variables.

_____

_____

_____

2 marks

c. The following data is recorded for sales compared to the number of tourists at the Infinity Restaurant on New Mathsland.

Number of tourists per day	180	275	190	310	85
Sales per day ($)	10 235	15 760	19 560	21 650	4 600

i. Use linear regression techniques to complete the following table to 1 decimal place.

sales per day = [ ] + [ ] × number of tourists

ii. The correlation coefficient for the linear relationship is only 0.84 so an x^2 transformation is done.

What is the value of r after the transformation?

1 + 1 = 2 marks

Total 15 marks

END OF CORE

MODULES

Module 1: Number patterns

Question 1

Glorietta productions are a small animation company. In their first month of operations they were able to produce two hours of animation but this increased to four hours in the second month. The company wishes to make predictions for their third month.

a. Manager Mary thinks that the hours produced will increase by the same amount every month.

i. What type of sequence is this?

_____

1 mark

ii. How many total hours would be produced in the first eight months if Mary is correct?

_____

_____

_____

_____

2 marks

b. IT manager Jason thinks that the sequence is geometric.

i. How many hours would be produced in the third month?

_____

_____

2 marks

ii. How many total hours would be produced in the first eight months if Jason is correct?

_____

_____

_____

_____

2 marks

c. Elmo, the HR director, says that the studio is limited to making 40 hours of video each month. He counts the number of hours that production is below capacity (i.e. 36 hours in the second month) and calls it "usage deficit". He predicts that this deficit in the second month onward will form a geometric sequence with a common ratio 0.5.

i. Which of the three models – that of Elmo, Jason or Mary – predicts the most production and the least production in the sixth?

_____

_____

_____

_____

_____

_____

3 marks

ii. What is the limit to the total usage deficit (sum of all such deficits) if Elmo is correct?

_____

_____

_____

_____

_____

_____

1 mark

d. Elmo considers other options and decides to use a difference equation of the form:

$$t_{n+1} = at_n + b$$

She does not know the correct values of a and b. She does know that usage should be found by

- reducing usage deficit by 10% compared with the previous month and then
- increasing usage by one hour.

i. Show that $a = 0.90$ and that $b = 5$.

_____

_____

_____

_____

_____

2 marks

ii. Determine the third month for this sequence if the first month has two hours usage but the second month is unknown.

_____

_____

_____

_____

2 marks
Total 15 marks

END OF MODULE 1

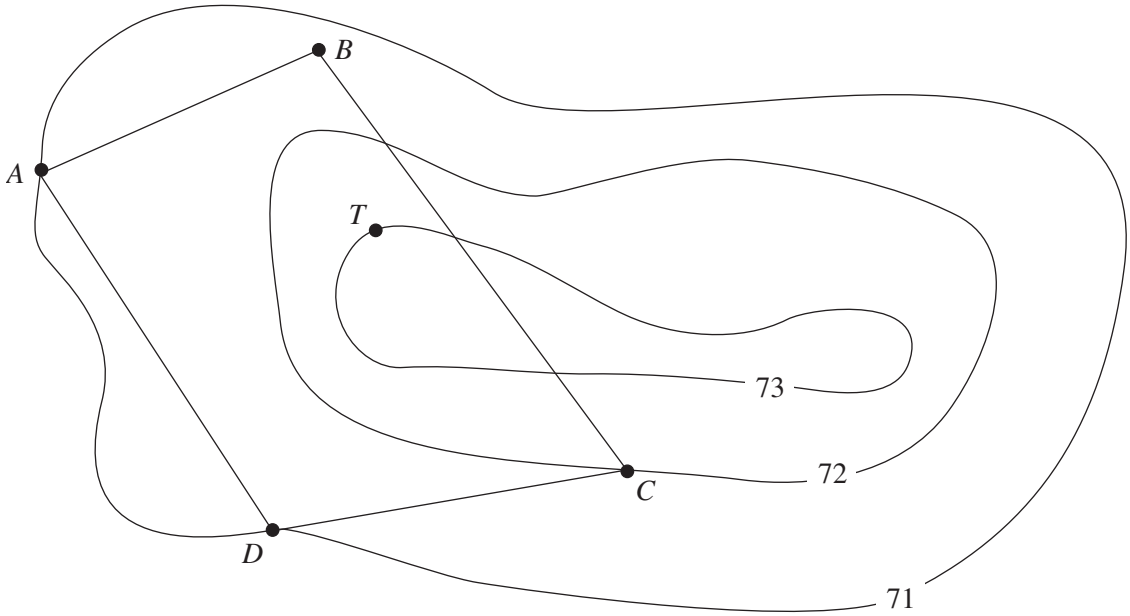
Module 2: Geometry and trigonometry

Question 1

Stu and Nelly purchased a block of land *ABCD*.

The contour map below shows the block of land and contours drawn at one metre intervals.

Point *T* represents a point on the ground.



a. What is the difference in height between point *A* and point *C*?

_____

_____

1 mark

b. The average slope between point *A* and point *T* is 0.05.

Calculate the horizontal distance between point *A* and point *T*.

_____

_____

_____

1 mark

c. The scale used on the contour map is 1 : 500.

Calculate the length of a straight line on the block of land that is represented by 12 cm on the map.

_____

_____

_____

1 mark

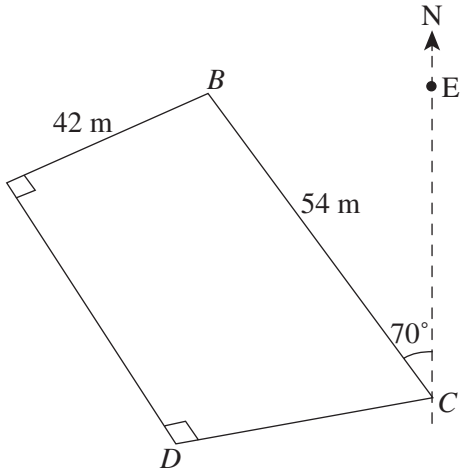
Question 2

The diagram below shows the boundaries of the block of land *ABCD*.

Boundary *AB* is 42 metres.

Boundary *BC* is 54 metres.

Angle *BCE* is 70°.



a. Determine the bearing of point *B* from point *C*.

_____

_____

1 mark

b. The bearing of point *C* from point *D* is 080°.

Determine the bearing of point *D* from point *C*.

_____

_____

1 mark

c. Show that distance *AD* is 27 metres.

_____

_____

_____

_____

2 marks

d. Calculate the area *ABCD*.

Write your answer correct to the nearest square metre.

_____

_____

_____

_____

_____

2 marks

Question 3

Stu and Nelly have designed a house to build on their block of land.

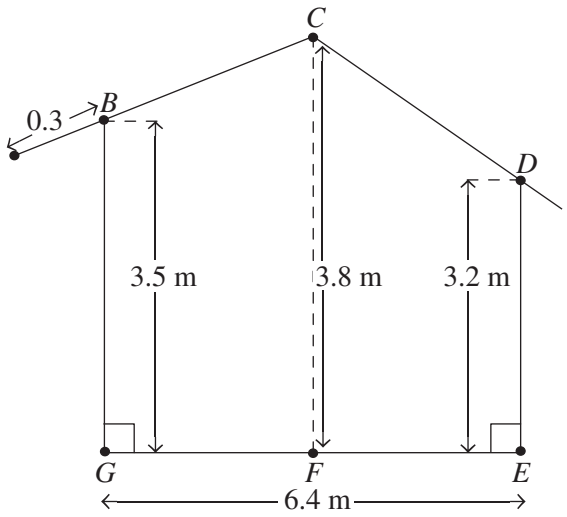
The maximum height of the house, point *C*, is 3.8 metres.

The height of the wall *BG* is 3.5 metres.

The height of the wall *DE* is 3.2 metres.

The length of *EG* is 6.4 metres.

Point *F* is the midpoint of *EG*.



a. Calculate the distance *AC*. Write your answer correct to one decimal place.

_____

_____

_____

1 mark

b. Calculate the angle of elevation of Point *C* from Point *D*. Write your answer correct to the nearest degree.

_____

_____

_____

1 mark

c. Calculate the area *EGBCD*. Write your answer correct to one decimal place.

_____

_____

_____

_____

2 marks

Question 4

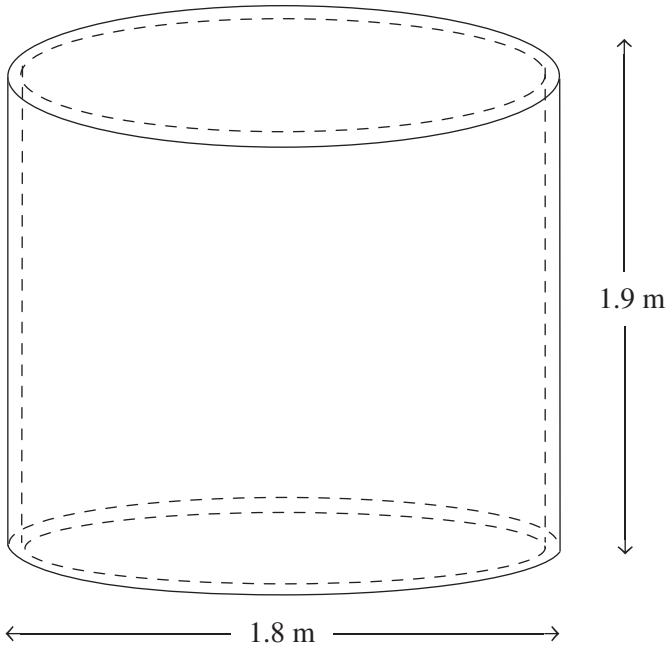
Nelly purchased a water tank.

Part of the water tank is an open cylinder with an internal height of 1.9 metres.

The external diameter is 1.8 metres.

The maximum volume inside the open cylinder is 4.73 m^3.

Assuming that the wall has a constant thickness, calculate the thickness of the wall. Write your answer correct to the nearest cm.



_____

_____

_____

_____

_____

_____

_____

2 marks
Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

Kathy works in a shop that sells souvenirs. There are two main types of souvenirs, postcards and commemorative spoons.

The postcards are made in the shop and the materials and equipment cost $50. In addition to that, the staff can print 100 per hour. Each staff member gets paid $25 per hour to do the printing. These are the only costs.

The equation giving cost to make n postcards is $\text{cost} = a \times n + b$.

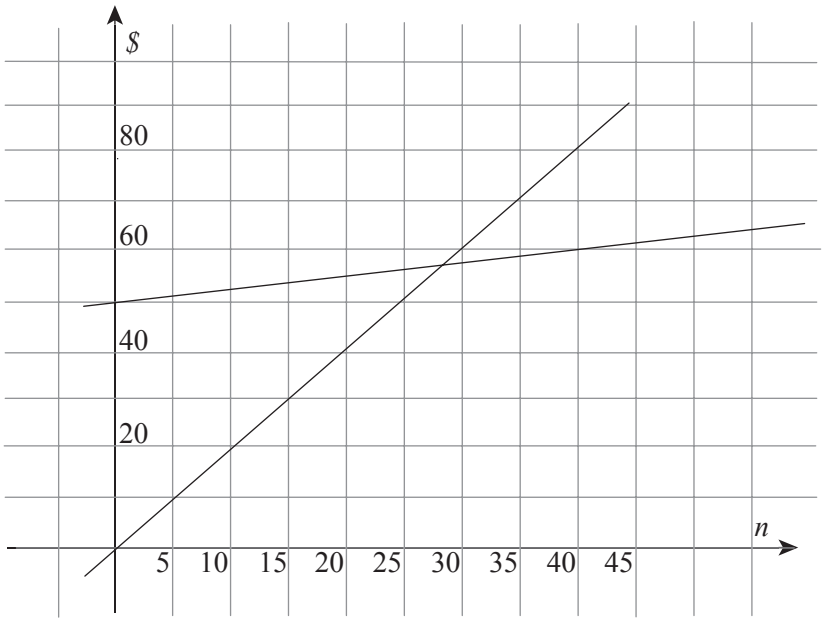
a. Determine the values of a and b.

_____

_____

1 mark

The following graph shows the cost and revenue associated with producing and selling n postcards.



b. Determine the selling price of each postcard.

_____

_____

1 mark

c. Determine the approximate break-even point for postcard sales.

How many must be sold (to the nearest whole number)?

_____

_____

1 mark

d. For the spoons, the cost equation is $\text{cost} = 4n + 100$ while the revenue equation is $R = 10n$.

Algebraically determine the minimum number to be sold in order to make a profit.

_____

_____

_____

_____

2 marks

Question 2

Kathy is opening a new workshop and needs to determine the correct staffing requirements. Junior staff members are able to produce 5 spoons per hour and can also make 2 deliveries to the store per day. Senior staff can make 10 spoons per hour and 3 deliveries per day. The store requires at least 42 deliveries per day and the minimum number of spoons to be made per day is 960. All staff members work 8 hours per day.

Let x be the number of junior staff and y be the number of senior staff.

Some of the constraints on x and y are as follows:

$$x \geq 0$$
$$y \geq 0$$
$$5x + 10y \geq 120$$

a. There is one additional constraint based on the information given above.

Name this constraint.

_____

_____

1 mark

The cost of employing each junior staff member is $20 per hour while a senior staff member is to be paid $25 per hour.

b. Give an expression for C, the hourly cost of employing the required staff.

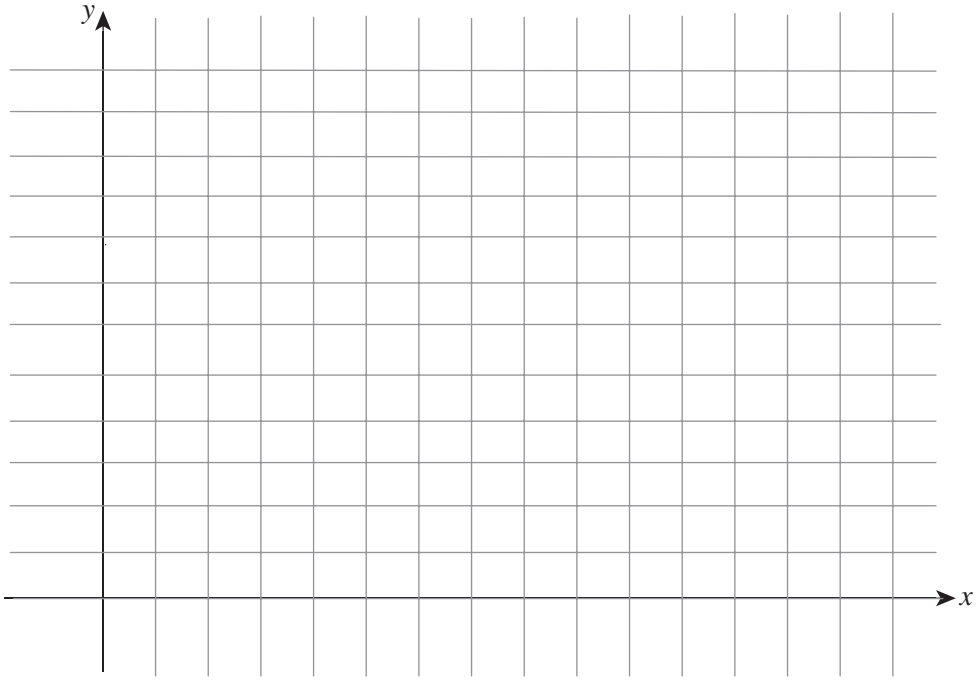
_____

_____

_____

1 mark

c. Sketch a graph showing all the constraints.



2 marks

d. Determine the staffing that results in the minimum cost and state this cost.

_____

_____

_____

_____

_____

_____

4 marks

e. An additional condition is placed on the number of senior and junior staff. To maintain standards and experience, it is required that at least $\frac{1}{3}$ of **all** staff are senior staff.

Write down an inequation to describe this condition.

_____

_____

_____

2 marks
Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

Mel and Paul caanot decide whether to keep renting for the next five years or to buy their first home. They currently pay $350 a week in rent but over the next three years the rent will be adjusted annually in line with inflation, which is expected to average 4.5% per annum.

a. Calculate the total rent paid in the next 12 months.

_____

_____

1 mark

b. To the nearest dollar, calculate the total rent to be paid over the next three years, allowing for the annual rent adjustment.

_____

_____

_____

2 marks

They have $35 000 in savings and a combined income of around $3000 per fortnight. The house Mel and Paul are thinking of buying is priced at $310 000 and the Credit union is offering a loan of $275 000 over 20 years at a fixed rate of 6.9% per annum compounding monthly.

c. To the nearest dollar, calculate the monthly repayments over the 20 year period.

_____

_____

1 mark

d. Assuming a constant interest rate, how much money will be paid in loan repayments cost over the first three years?

_____

_____

1 mark

e. Assuming the value of the house also increases with the rate of inflation, what will its value be at the end of three years, to the nearest dollar?

_____

_____

1 mark

f. If they sold the house for the value calculated in part **e** after three years, repay the outstanding debt on the loan and then deposit the profit, how much money will they have in their account?

_____

_____

_____

2 marks

Question 2

After making the big housing decision Mel and Paul decide to expand their nail factory and double the number of nails they manufacture. They cannot afford the $67 000 to buy a new nail making machine outright so they take it on hire purchase under the following conditions: 10% deposit and the remainder in monthly repayments of $1 533 over five years.

a. Calculate

i. the deposit, to the nearest dollar

_____

ii. the flat rate of interest to one decimal place

_____

_____

_____

_____

_____

1 + 2 = 3 marks

b. The nail machine depreciates at 12% in the first year and then an average rate of 8% over the following four years.

To the nearest dollar, calculate the book value of the machine after five years.

_____

_____

_____

2 marks

Question 3

Mel and Paul want to save for a six month trip through Europe in five years' time. They estimate they will need a total of $60 000 to cover the trip. They open an account and contribute $700 a month.

a. If interest is calculated at 7.6% p.a. compounded monthly, how far short of their goal will they be?

_____

_____

1 mark

b. To reach their goal, what interest rate, to one decimal place, will they need to earn on their savings if it is calculated compounding monthly?

_____

_____

1 mark

Total 15 marks

END OF MODULE 4

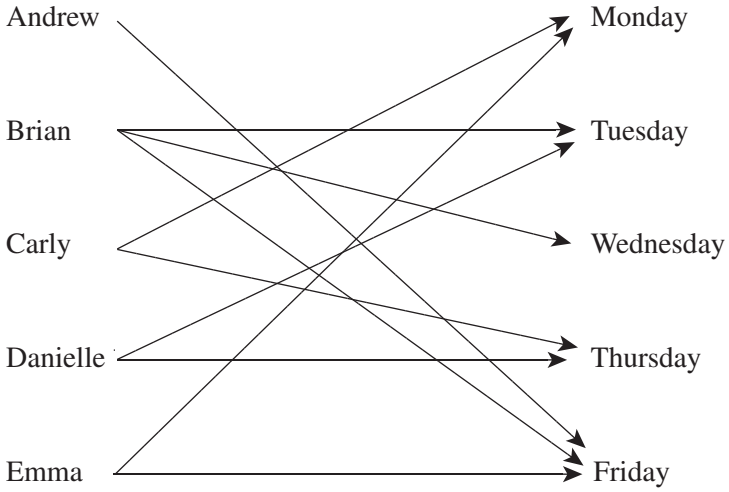
Module 5: Networks and decision mathematics

Question 1

Andrew, Brian, Carly, Danielle and Emma are the directors of a new company.

Each director must visit the tax office once on a weekday next week, and a visit must occur each weekday.

The following bipartite graph represents the day(s) that each person is available to visit the tax office.



a. Which director must visit the tax office on Wednesday?

_____

1 mark

b. Complete the table below to show who must visit the tax office each day.

Day	Person
Monday	
Tuesday	
Thursday	
Friday	

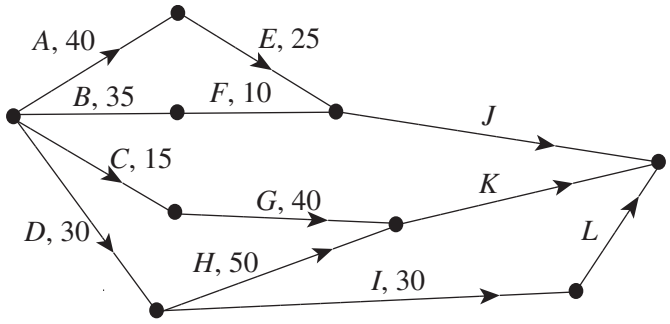
2 marks

Question 2

The five directors have decided to design a business plan. This will require the completion of twelve activities.

Some of the activities, with their immediate predecessor, duration, earliest start time and latest finish time are shown in the table below.

Activity	Immediate predecessor	Duration (minutes)	Earliest start time	Latest finish time
A	–	40	0	40
B	–	35	0	35
C	–	15	0	15
D	–	30	0	30
E			40	65
F	*B*	10	35	
G	*C*	40	15	



a. Use the information in the above network diagram to complete the table above by filling in the shaded cells.

4 marks

It is known that two out of the three activities *J*, *K* and *L* have a duration of 30 minutes and that the remaining activity (out of *J*, *K* and *L*) has a duration of 25 minutes.

It is also known that the minimum project completion time is 105 minutes.

b. Complete the table below to show the duration of activities *J*, *K* and *L*.

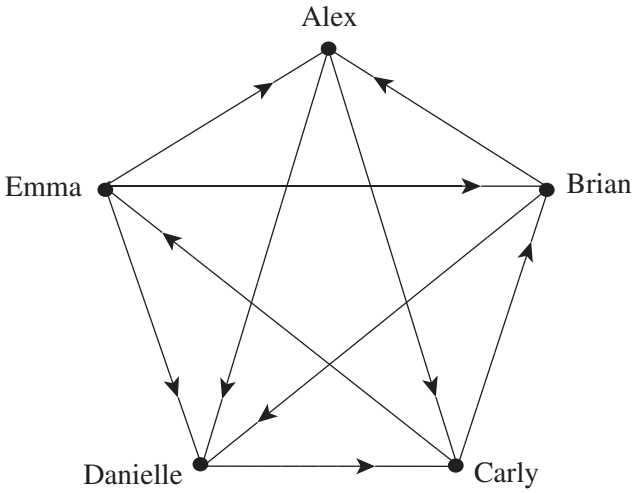
Activity	*J*	*K*	*L*
Duration (minutes)			

2 marks

Question 3

The five directors compete against each other in a business management IQ test that will assist in their selection for a job promotion. Each director competes in a test against each of the other directors until a winner is found.

The results are shown in the directed graph and the dominance matrix below.



	loser A	B	C	D	E	
	0	0	1	1	0	A
	1	0	0	1	0	B
	0	1	0	☐	1	C winner
	0	0	1	0	0	D
	1	1	☐	1	0	E

Two entries in the dominance matrix are missing.

a. Complete the dominance matrix by filling in the shaded boxes.

2 marks

b. Explain the two-step dominance that Alex has over Carly.

_____

_____

1 mark

c. Determine which director was ranked first using one-step dominance.

_____

_____

_____

_____

1 mark

d. Determine which directors were ranked first and second using, both one-step and two-step dominances.

_____

_____

_____

_____

_____

2 marks

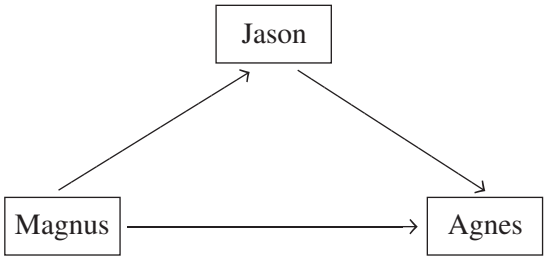
Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

In the Gooragin basin tennis tournament, all the players have played each other once. The results are summarised in the diagram below. An arrow pointing from player A to player B shows that A beat B.



The matrix shown below describes this situation but it is incomplete.

$$\begin{matrix} & J & M & A \\ \begin{matrix} J \\ M \\ A \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & a & c \\ 1 & b & 0 \end{bmatrix} \end{matrix}$$

a. State the values of a, b and c.

_____

_____

2 marks

b. A fourth player, Beatrice, is added to the tournament. She beats Jason but loses to Agnes and Magnus.

Write all the results in a matrix in the form $\begin{matrix} & J & M & A & B \\ \begin{matrix} J \\ M \\ A \\ B \end{matrix} & \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \end{matrix}$ including those in the original matrix.

1 mark

Question 2

Each of the four players has a rating and the points system works like this.

The loser gives 20% of their points to the winner of a match.

Jason is responsible for the system of tallying points. He decides to determine the new points after a round of matches by premultiplying the current points, matrix P, by another matrix A.

In round six, matrix A is $\begin{array}{c} \begin{matrix} J & M & A & B \end{matrix} \\ \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 0.8 \end{bmatrix} \end{array} \begin{matrix} J \\ M \\ A \\ B \end{matrix}$

The matrix P, $\begin{bmatrix} 27 \\ 18 \\ 12 \\ 17 \end{bmatrix}$, is the points after round five.

a. Find the product AP.

1 mark

b. Who were the winners in round six and who did they defeat?

_____

_____

_____

2 marks

c. The points totals before and after round eight were $\begin{bmatrix} 20 \\ 20 \\ 16 \\ 18 \end{bmatrix}$ and $\begin{bmatrix} 16 \\ 16 \\ 20 \\ 22 \end{bmatrix}$

Determine matrix A for this round if the third row is $\begin{bmatrix} 0 & 0.2 & 1 & 0 \end{bmatrix}$.

_____

_____

_____

_____

3 marks

After eight rounds, the singles matches are complete and the doubles begin. A new system is instituted for these. Each player is rated individually regardless of the result.

Jason makes a prediction that in each doubles round the points total will be altered according to the matrix equation $P_{n+1} = T.P_n$, where

$$T = \begin{bmatrix} 0.9 & 0.1 & 0 & 0.1 \\ 0.05 & 0.8 & 0.05 & 0 \\ 0 & 0.1 & 0.80 & 0.10 \\ 0.05 & 0 & 0.15 & 0.80 \end{bmatrix}$$

d. Determine the points after round nine (first doubles round).

1 mark

e. Determine if a steady state exists for this prediction and if so, state it to one decimal place.

2 marks

Question 3

Jason designs a series of equations to predict the proportion of points that each player of the four players will score against opponents from other clubs next weekend.

$$2j + 3m + a = 2.8$$
$$3j - m + 2b = 2.0$$
$$m + a + b = 1.1$$
$$j + 2a + 2b = 2.0$$

a. Write these equations as a single matrix equation in the form

$$\begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \begin{bmatrix} j \\ m \\ a \\ b \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}$$

1 mark

b. It is found that Jason predicted himself to win 60% of matches and Magnus wins 40%.
Write a new matrix equation that incorporates this data.

1 mark

c. What proportion of matches does Jason predict that Beatrice wins?

1 mark
Total 15 marks

END OF QUESTION AND ANSWER BOOKLET