VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2011

FURTHER MATHEMATICS
Written examination 1

Friday 4 November 2011

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 36 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A

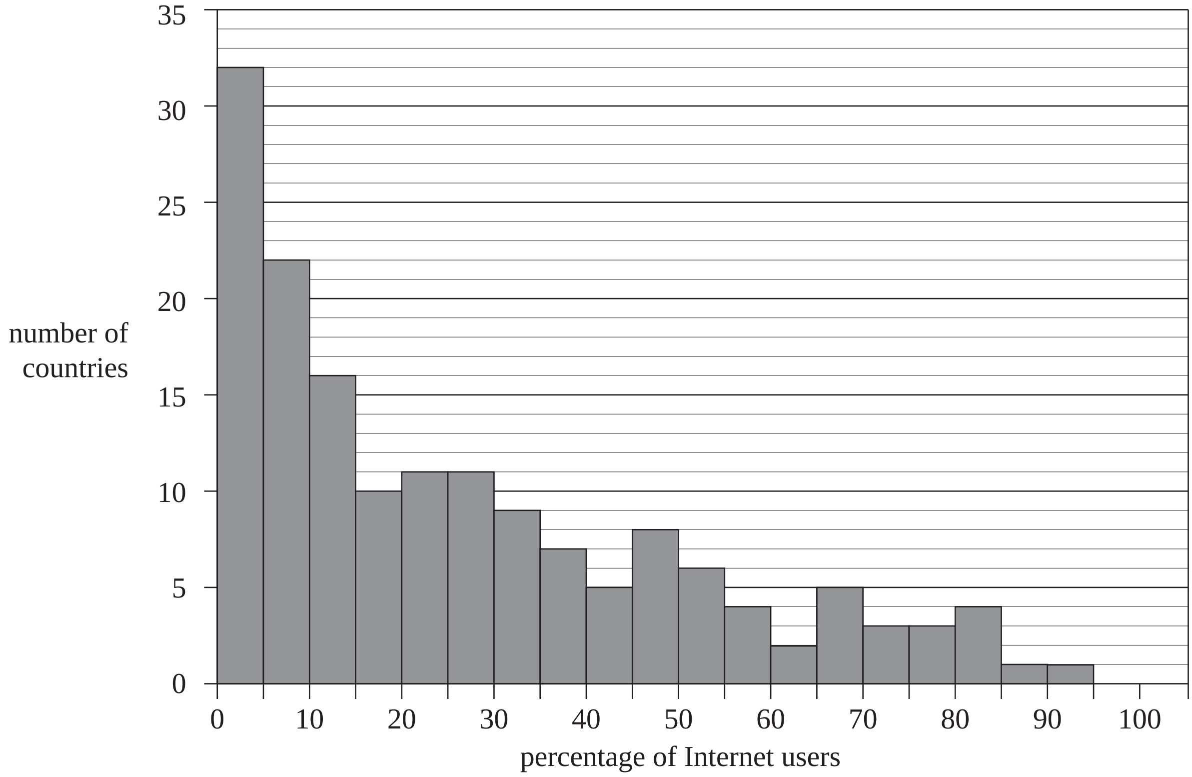
Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Core: Data analysis

Use the following information to answer Questions 1, 2 and 3.

The histogram below displays the distribution of the percentage of Internet users in 160 countries in 2007.



Based on data obtained from: www.data.un.org

Question 1

The shape of the histogram is best described as

A. approximately symmetric.
B. bell shaped.
C. positively skewed.
D. negatively skewed.
E. bi-modal.

Question 2

The number of countries in which less than 10% of people are Internet users is closest to

A. 10
B. 16
C. 22
D. 32
E. 54

Question 3

From the histogram, the median percentage of Internet users is closest to

A. 10%
B. 15%
C. 20%
D. 30%
E. 40%

Question 4

The variables

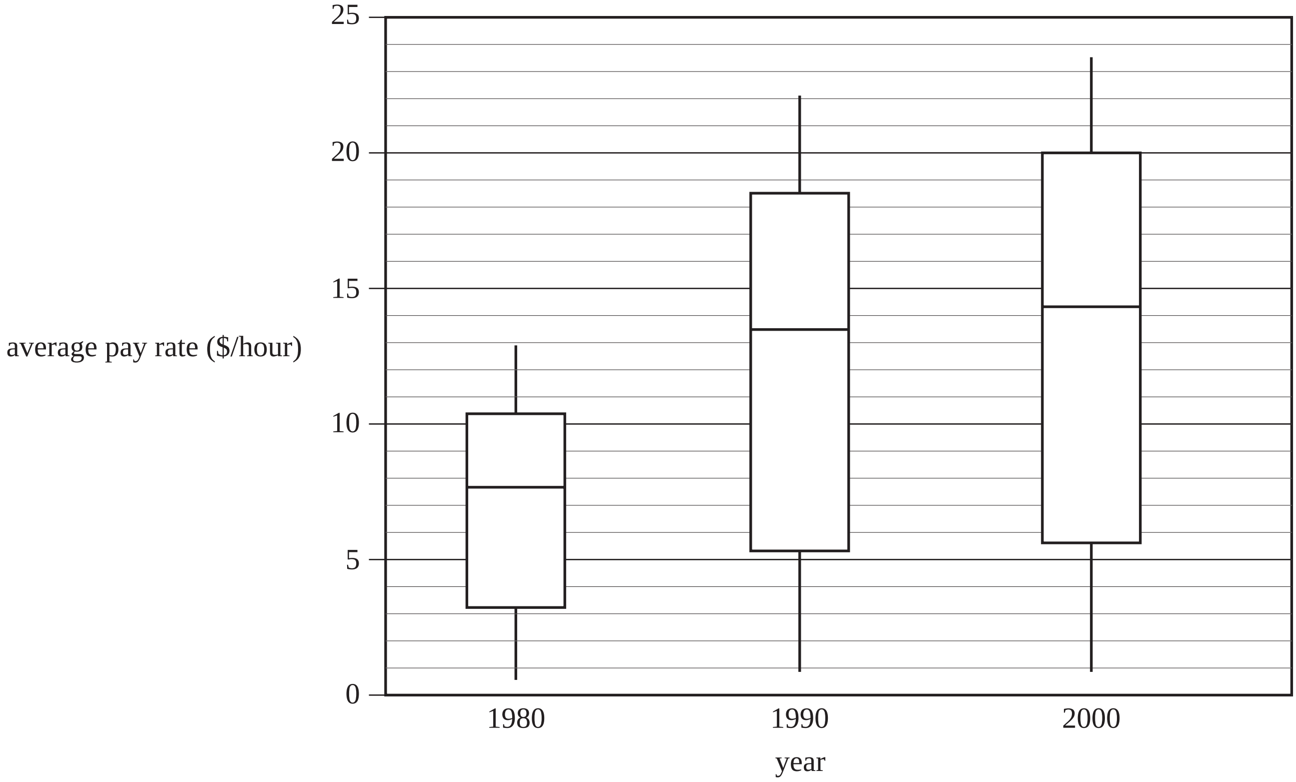
region (city, urban, rural)

population density (number of people per square kilometre)

A. are both categorical.
B. are both numerical.
C. are categorical and numerical respectively.
D. are numerical and categorical respectively.
E. are neither categorical nor numerical.

Question 5

The boxplots below display the distribution of average pay rates, in dollars per hour, earned by workers in 35 countries for the years 1980, 1990 and 2000.



Based on data obtained from: www.data.un.org

Based on the information contained in the boxplots, which one of the following statements is **not** true?

A. In 1980, over 50% of the countries had an average pay rate less than $8.00 per hour.

B. In 1990, over 75% of the countries had an average pay rate greater than $5.00 per hour.

C. In 1990, the average pay rate in the top 50% of the countries was higher than the average pay rate for any of the countries in 1980.

D. In 1990, over 50% of the countries had an average pay rate less than the median average pay rate in 2000.

E. In 2000, over 75% of the countries had an average pay rate greater than the median average pay rate in 1980.

Use the following information to answer Questions 6, 7 and 8.

When blood pressure is measured, both the systolic (or maximum) pressure and the diastolic (or minimum) pressure are recorded.

Table 1 displays the blood pressure readings, in mmHg, that result from fifteen successive measurements of the same person's blood pressure.

Table 1

	Blood pressure	
Reading number	**systolic**	**diastolic**
1	121	73
2	126	75
3	141	73
4	125	73
5	122	67
6	126	74
7	129	70
8	130	72
9	125	69
10	121	65
11	118	66
12	134	77
13	125	70
14	127	64
15	119	69

Question 6

Correct to one decimal place, the mean and standard deviation of this person's **systolic** blood pressure measurements are respectively

A. 124.9 and 4.4

B. 125.0 and 5.8

C. 125.0 and 6.0

D. 125.9 and 5.8

E. 125.9 and 6.0

Question 7

Using systolic blood pressure (*systolic*) as the dependent variable, and diastolic blood pressure (*diastolic*) as the independent variable, a least squares regression line is fitted to the data in Table 1.

The equation of the least squares regression line is closest to

A. *systolic* = 70.3 + 0.790 × *diastolic*

B. *diastolic* = 70.3 + 0.790 × *systolic*

C. *systolic* = 29.3 + 0.330 × *diastolic*

D. *diastolic* = 0.330 + 29.3 × *systolic*

E. *systolic* = 0.790 + 70.3 × *diastolic*

Question 8

From the fifteen blood pressure measurements for this person, it can be concluded that the percentage of the variation in systolic blood pressure that is explained by the variation in diastolic blood pressure is closest to

A. 25.8%

B. 50.8%

C. 55.4%

D. 71.9%

E. 79.0%

Use the following information to answer Questions 9 and 10.

The length of a type of ant is approximately normally distributed with a mean of 4.8 mm and a standard deviation of 1.2 mm.

Question 9

From this information it can be concluded that around 95% of the lengths of these ants should lie between

A. 2.4 mm and 6.0 mm

B. 2.4 mm and 7.2 mm

C. 3.6 mm and 6.0 mm

D. 3.6 mm and 7.2 mm

E. 4.8 mm and 7.2 mm

Question 10

A standardised ant length of $z = -0.5$ corresponds to an actual ant length of

A. 2.4 mm

B. 3.6 mm

C. 4.2 mm

D. 5.4 mm

E. 7.0 mm

Question 11

For a group of 15-year-old students who regularly played computer games, the correlation between the time spent playing computer games and fitness level was found to be $r = -0.56$.

On the basis of this information it can be concluded that

A. 56% of these students were not very fit.

B. these students would become fitter if they spent less time playing computer games.

C. these students would become fitter if they spent more time playing computer games.

D. the students in the group who spent a short amount of time playing computer games tended to be fitter.

E. the students in the group who spent a large amount of time playing computer games tended to be fitter.

Question 12

The seasonal index for headache tablet sales in summer is 0.80.

To correct for seasonality, the headache tablet sales figures for summer should be

A. reduced by 80%

B. reduced by 25%

C. reduced by 20%

D. increased by 20%

E. increased by 25%

Question 13

The table below shows the number of broadband users in Australia for each of the years from 2004 to 2008.

Year	2004	2005	2006	2007	2008
Number	1 012 000	2 016 000	3 900 000	4 830 000	5 140 000

Based on data obtained from: www.data.worldbank.org

A two-point moving mean, with centring, is used to smooth the time series.

The smoothed value for the number of broadband users in Australia in 2006 is

A. 2 958 000

B. 3 379 600

C. 3 455 500

D. 3 661 500

E. 3 900 000

END OF SECTION A

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Stefan swam laps of his pool each day last week.

The number of laps he swam each day followed a geometric sequence.

He swam 1 lap on Monday, 2 laps on Tuesday and 4 laps on Wednesday.

The number of laps that he swam on Thursday was

A. 5

B. 6

C. 8

D. 12

E. 16

Question 2

The first three terms of an arithmetic sequence are –3, –7, –11 . . .

An expression for the *n*th term of this sequence, t_n , is

A. $t_n = 1 - 4n$

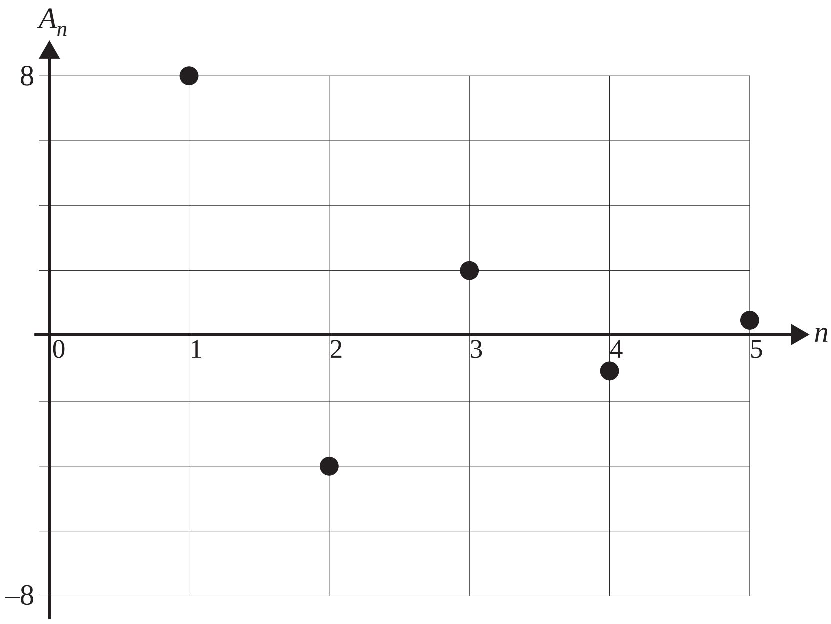
B. $t_n = 1 - 8n$

C. $t_n = -3 - 4n$

D. $t_n = -3 + 4n$

E. $t_n = -7 + 4n$

Question 3



The graph above shows the first five terms of a sequence.
Let A_n be the nth term of the sequence.
A difference equation that generates the terms of this sequence is

A. $A_{n+1} = 2A_n - 2$ where $A_1 = 8$

B. $A_{n+1} = 3A_n$ where $A_1 = 8$

C. $A_{n+1} = -2A_n$ where $A_1 = 8$

D. $A_{n+1} = -\frac{1}{2}A_n$ where $A_1 = 8$

E. $A_{n+1} = -A_n - 1$ where $A_1 = 8$

Question 4

The number of bees in a colony was recorded for three months and the results are displayed in the table below.

Month	1	2	3
Number of bees	10	30	90

If this pattern of increase continues, which one of the following statements is **not** true?

A. There will be nine times as many bees in the colony in month 5 than in month 3.
B. In month 4, the number of bees will equal 270.
C. In month 6, the number of bees will equal 7290.
D. In month 8, the number of bees will exceed 20 000.
E. In month 10, the number of bees will be under 200 000.

Question 5

The difference equation

$$t_{n+2} = t_{n+1} + t_n \quad \text{where} \quad t_1 = a \quad \text{and} \quad t_2 = 7$$

generates a sequence with $t_5 = 27$.

The value of a is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 6

For which one of the following geometric sequences is an infinite sum **not** able to be determined?

A. $4, 2, 1, \frac{1}{2} \ldots$

B. $1, 2, 4, 8 \ldots$

C. $-4, 2, -1, \frac{1}{2} \ldots$

D. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$

E. $-1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8} \ldots$

Question 7

Let P_{2011} be the number of pairs of shoes that Sienna owns at the end of 2011.

At the beginning of 2012, Sienna plans to throw out the oldest 10% of pairs of shoes that she owned in 2011. During 2012 she plans to buy 15 new pairs of shoes to add to her collection.

Let P_{2012} be the number of pairs of shoes that Sienna owns at the end of 2012.

A rule that enables P_{2012} to be determined from P_{2011} is

A. $P_{2012} = 1.1\, P_{2011} + 15$
B. $P_{2012} = 1.1\, (P_{2011} + 15)$
C. $P_{2012} = 0.1\, P_{2011} + 15$
D. $P_{2012} = 0.9\, (P_{2011} + 15)$
E. $P_{2012} = 0.9\, P_{2011} + 15$

Question 8

The first three terms of an arithmetic sequence are 1, 3, 5 . . .

The sum of the first n terms of this sequence, S_n , is

A. $S_n = n^2$
B. $S_n = n^2 - n$
C. $S_n = 2n$
D. $S_n = 2n - 1$
E. $S_n = 2n + 1$

Question 9

A toy train track consists of a number of pieces of track which join together.

The shortest piece of the track is 15 centimetres long and each piece of track after the shortest is 2 centimetres longer than the previous piece.

The total length of the complete track is 7.35 metres.

The length of the longest piece of track, in centimetres, is

A. 21

B. 47

C. 49

D. 55

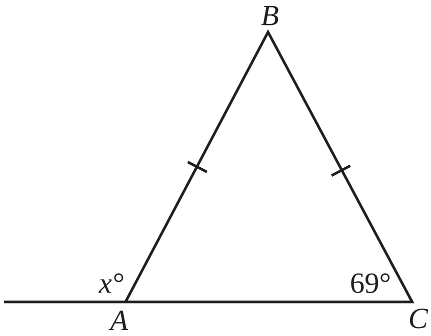
E. 57

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

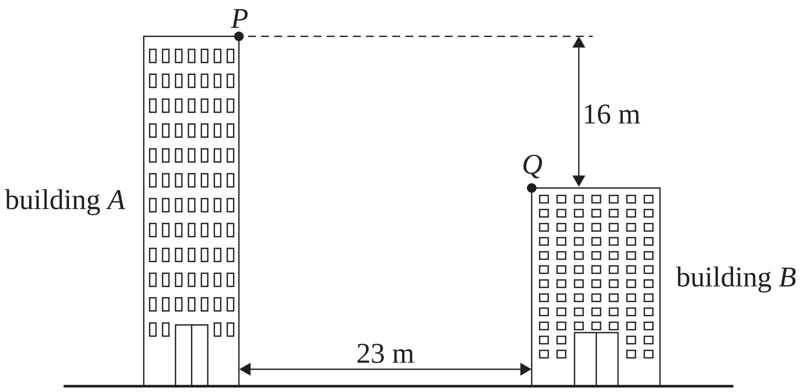
In triangle *ABC*, $\angle BCA = 69°$ and $AB = BC$.



The value of *x* is

A. 42
B. 55.5
C. 84
D. 111
E. 138

Question 2



The point *Q* on building *B* is visible from the point *P* on building *A*, as shown in the diagram above.
Building *A* is 16 metres taller than building *B*.
The horizontal distance between point *P* and point *Q* is 23 metres.
The angle of depression of point *Q* from point *P* is closest to

A. 35°
B. 41°
C. 44°
D. 46°
E. 55°

Question 3

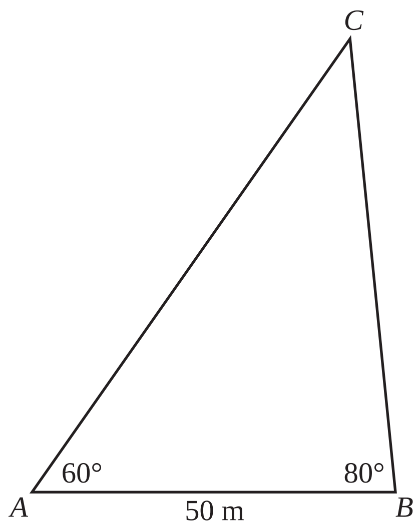
The radius of a circle is 6.5 centimetres.
A square has the same area as this circle.

The length of each side of the square, in centimetres, is closest to

A. 6.4
B. 10.2
C. 11.5
D. 23.0
E. 33.2

Question 4

In triangle *ACB*, $\angle CAB = 60°$ and $\angle ABC = 80°$
The length of side *AB* = 50 m.

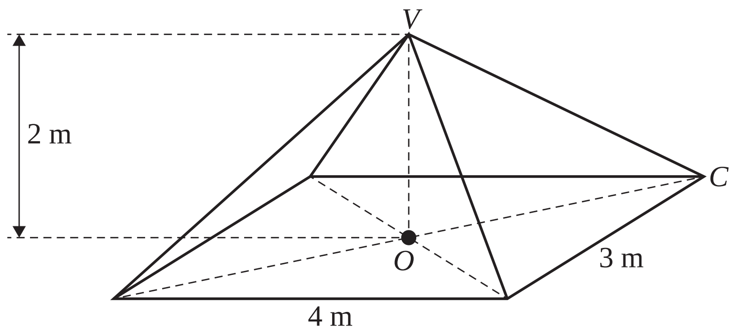


The length of side *AC* is closest to

A. 57 m
B. 67 m
C. 77 m
D. 81 m
E. 100 m

Question 5

A right pyramid, shown below, has a rectangular base with length 4 m and width 3 m.
The height of the pyramid is 2 m.

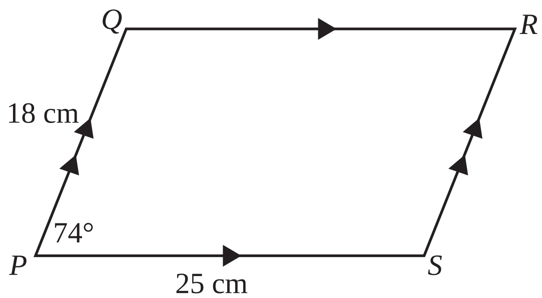


The angle *VCO* that the sloping edge *VC* makes with the base of the pyramid, to the nearest degree, is

A. 22°
B. 27°
C. 34°
D. 39°
E. 45°

Question 6

In parallelogram *PQRS*, ∠*QPS* = 74°.
In this parallelogram, *PQ* = 18 cm and *PS* = 25 cm.



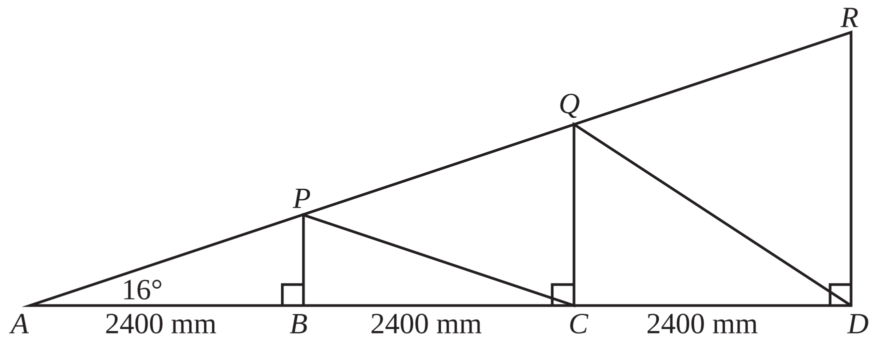
The length of the longer diagonal of this parallelogram is closest to

A. 26.5 cm
B. 30.1 cm
C. 30.8 cm
D. 34.6 cm
E. 39.9 cm

Question 7

The structure of a roof frame is shown in the diagram below.

In this diagram, $AB = BC = CD = 2400$ mm and $\angle PAB = 16°$.



The length of QD, in mm, is closest to

A. 2741

B. 2767

C. 2830

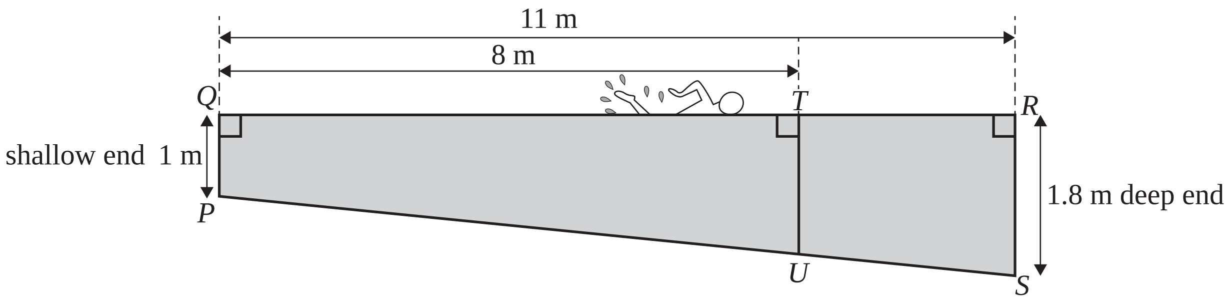
D. 3394

E. 5201

Question 8

The diagram below shows a cross-section, *PQRS*, of a swimming pool.

The swimming pool is 11 metres long and the depth increases uniformly from 1 metre at the shallow end to 1.8 metres at the deep end.



The depth of the water at a point 8 metres from the shallow end, represented by TU on the diagram, is closest to

A. 1.25 metres

B. 1.31 metres

C. 1.34 metres

D. 1.58 metres

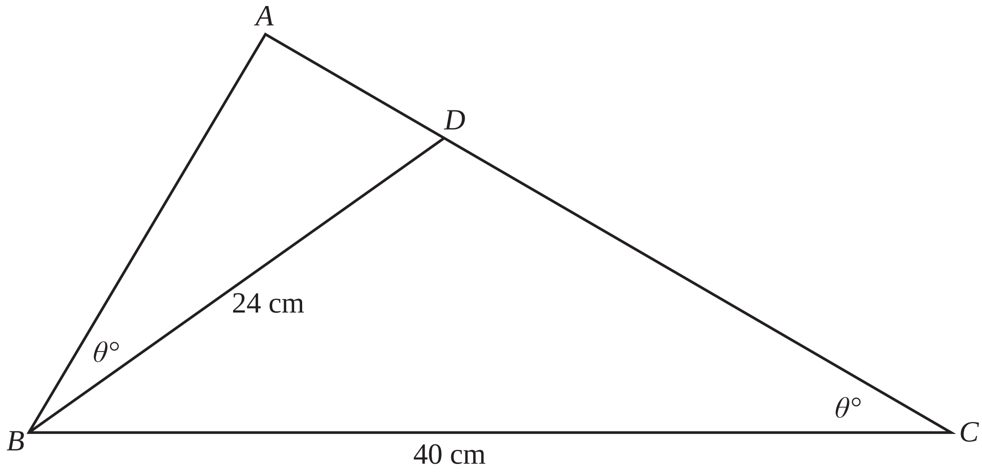
E. 1.62 metres

Question 9

In the diagram below, $\angle ABD = \angle ACB = \theta°$.

$BD = 24$ cm and $BC = 40$ cm.

The area of triangle *ABD* is 100 cm^2.



The area of triangle *ABC*, in cm^2, is closest to

A. 167

B. 178

C. 267

D. 278

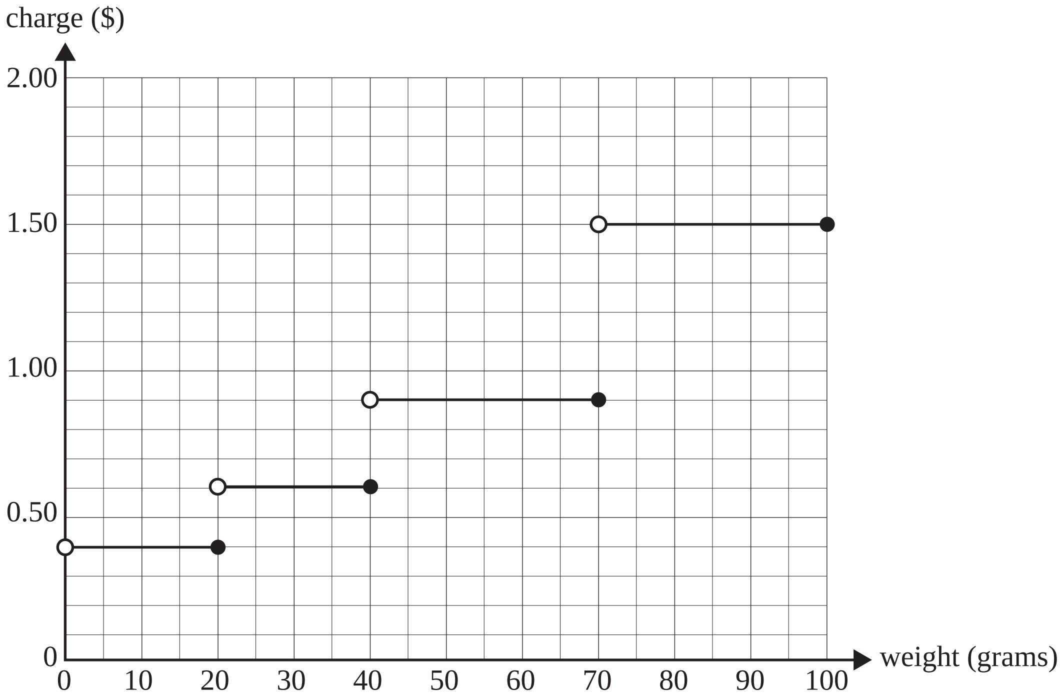
E. 378

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Use the following information to answer Questions 1 and 2.

The charges for posting letters that weigh 100 g or less are shown in the graph below.



Question 1

The charge for posting a 35 g letter is

A. $0.40

B. $0.60

C. $0.90

D. $1.50

E. $2.00

Question 2

Two letters are posted.

The total postage charge **cannot** be

A. $0.80

B. $1.20

C. $1.40

D. $2.10

E. $3.00

Question 3

Two lines intersect at point A on a graph.
The equation of one of the lines is

$$3x + 4y = 26$$

The coordinates of point A could be

A. (2, 5)
B. (3, 4)
C. (4, 3)
D. (5, 2)
E. (7, 22)

Question 4

The fare, $\$F$, to travel a distance of n kilometres in a taxi is given by the rule

$$F = a + bn$$

To travel a distance of 20 kilometres, the taxi fare is \$18.20
To travel a distance of 30 kilometres, the taxi fare is \$25.70

The charge per kilometre, b, is

A. \$0.75
B. \$0.88
C. \$0.91
D. \$1.33
E. \$3.20

Question 5

The cost, $\$C$, of making x kilograms of chocolate fudge is given by $C = 60 + 5x$.
The revenue, $\$R$, from selling x kilograms of chocolate fudge is given by $R = 15x$.
A particular quantity of chocolate fudge was made and sold. It resulted in a loss of \$20.

The number of kilograms of chocolate fudge made and sold was

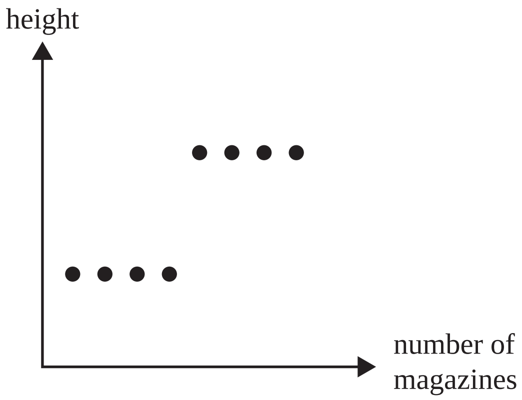
A. 2
B. 4
C. 8
D. 12
E. 16

Question 6

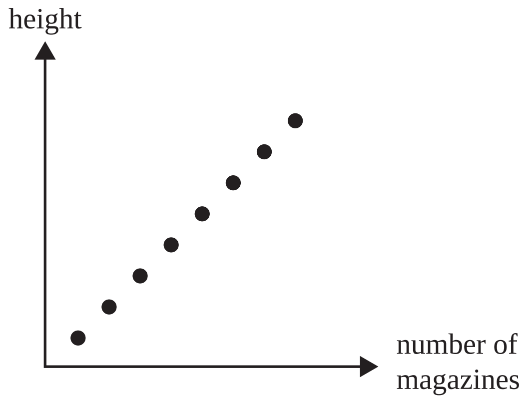
A newsagent has four thick magazines and four thin magazines.
The magazines are stacked one by one into a pile.
The thick magazines are all placed in the bottom part of the pile, and the thin magazines are all placed in the top part of the pile.

A graph that indicates the changing height of the pile of magazines as each magazine is added to the pile could be

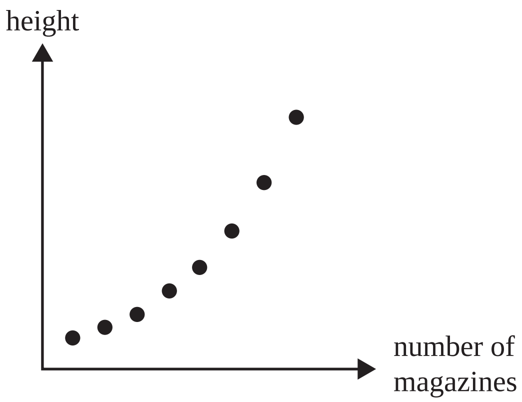
A.



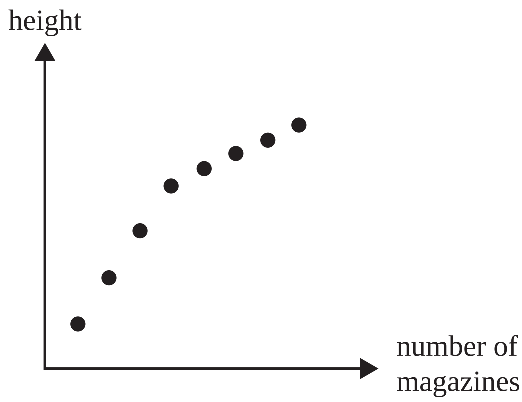
B.



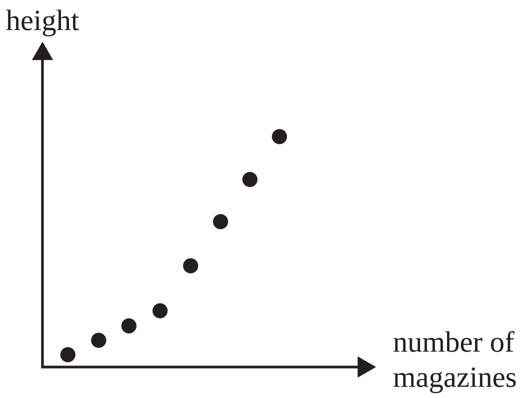
C.



D.



E.

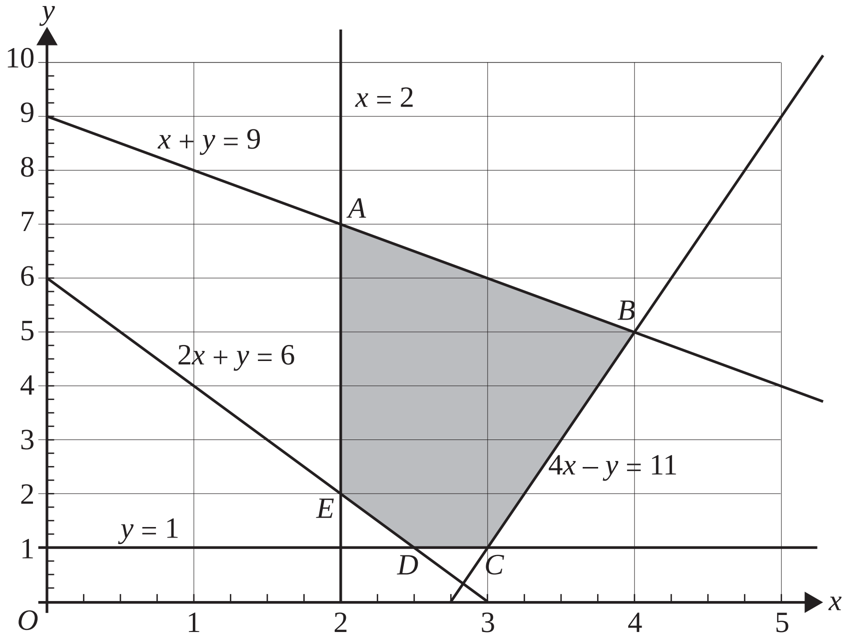


Use the following information to answer Questions 7, 8 and 9.

Craig plays sport and computer games every Saturday.

Let x be the number of hours that he spends playing sport.
Let y be the number of hours that he spends playing computer games.

Craig has placed some constraints on the amount of time that he spends playing sport and computer games. These constraints define the feasible region shown shaded in the graph below. The equations of the lines that define the boundaries of the feasible region are also shown.



Question 7

One of the constraints that defines the feasible region is

A. $y \le 1$

B. $x \le 2$

C. $x + y \ge 9$

D. $2x + y \le 6$

E. $4x - y \le 11$

Question 8

By spending Saturday playing sport and computer games, Craig believes he can improve his health.
Let W be the health rating Craig achieves by spending a day playing sport and computer games.
The value of W is determined by using the rule $W = 5x - 2y$.

For the feasible region shown in the graph above, the maximum value of W occurs at

A. point A

B. point B

C. point C

D. point D

E. point E

Question 9

By spending Saturday playing sport and computer games, Craig believes that he can improve his mental alertness.

Let M be the mental alertness rating that Craig achieves by spending a day playing sport and computer games.

For the feasible region shown in the graph on page 22, the maximum value of M occurs at any point that lies on the line that joins points A and B in the feasible region.

The rule for M could be

A. $M = 2x - 5y$

B. $M = 5x - 2y$

C. $M = 5x - 5y$

D. $M = 5x + 2y$

E. $M = 5x + 5y$

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

An electrician charges $68 per hour to complete a job.
A Goods and Services Tax (GST) of 10% is added to the charge.

Including GST, the cost of a job that takes three hours is

A. $6.80

B. $20.40

C. $204.00

D. $210.80

E. $224.40

Question 2

An amount of $22 000 is invested for three years at an interest rate of 3.5% per annum, compounding annually.

The value of the investment at the end of three years is closest to

A. $2 310

B. $9 433

C. $24 040

D. $24 392

E. $31 433

Question 3

A van is purchased for $56 000.
Its value depreciates at a rate of 42 cents for each kilometre that it travels.

The value of the van after it has travelled 32 000 km is

A. $13 440

B. $26 880

C. $29 120

D. $32 480

E. $42 560

Question 4

Nathan bought a $2 500 bedroom suite on a contract that involves no deposit and an interest-free loan for a period of 48 months.
He has to pay an initial set-up fee of $25.
In addition, he pays an administration fee of $3.95 per month.

The total amount that Nathan will have to pay in fees for the entire 48 months, as a percentage of the original price of $2 500, is closest to

A. 1.6%

B. 4.0%

C. 7.6%

D. 8.5%

E. 8.6%

Question 5

Jane invests in an ordinary perpetuity to provide her with a weekly payment of $500.
The interest rate for the investment is 5.9% per annum.

Assuming there are 52 weeks per year, the amount that Jane needs to invest in the perpetuity is closest to

A. $26 000

B. $102 000

C. $154 000

D. $221 000

E. $441 000

Question 6

Shaun decides to buy a new sound system on a time-payment (hire-purchase) plan.
The sound system is priced at $3 500.
Shaun pays a deposit of $500 and repayments of $80 per month for five years.

The flat rate of interest charged per annum, correct to one decimal place, is

A. 6.0%

B. 8.7%

C. 10.3%

D. 12.0%

E. 15.3%

Question 7

Anthony invested $15 000 in an account. It earned r% interest per annum, compounding monthly.

The amount of interest that is earned in the third year of the investment is given by

A. $15000\left(1+\frac{r}{1200}\right)^{3} - 15000\left(1+\frac{r}{1200}\right)^{2}$

B. $15000\left(1+\frac{r}{1200}\right)^{36} - 15000\left(1+\frac{r}{1200}\right)^{24}$

C. $15000\left(1+\frac{r}{100}\right)^{3} - 15000\left(1+\frac{r}{100}\right)^{2}$

D. $15000\left(1+\frac{r}{100}\right)^{36} - 15000\left(1+\frac{r}{100}\right)^{24}$

E. $15000\left(1+\frac{r}{1200}\right)^{4} - 15000\left(1+\frac{r}{1200}\right)^{3}$

Question 8

Teresa borrowed \$120 000 at an interest rate of 7.67% per annum, compounding monthly.
The loan is to be repaid with equal monthly payments.
She decides to repay the loan by making monthly payments of \$1 430.

Which of the following statements is **true**?

A. She will pay out the loan fully in less than ten years.

B. The amount of interest that she pays on the loan will increase each year.

C. After four years the amount that she owes on the loan will be less than \$80 000.

D. Every monthly payment that she makes reduces the amount that she owes on the loan by the same amount.

E. Monthly payments of \$1 560 (instead of \$1 430) will enable her to repay this loan in less than nine years.

Question 9

Xavier borrows \$45 000 from the bank to buy a car.
He is offered a reducing balance loan for three years with an interest rate of 9.75% per annum, compounding monthly.
He can repay this loan by making 36 equal monthly payments.
Instead, Xavier decides to repay the loan in 18 equal monthly payments.
If there are no penalties for repaying the loan early, the amount he will save is closest to

A. \$2 697

B. \$3 530

C. \$3 553

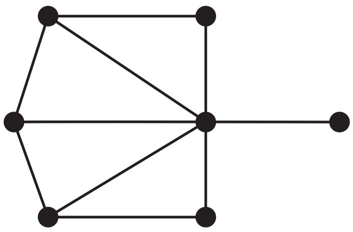
D. \$6 581

E. \$7 083

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1



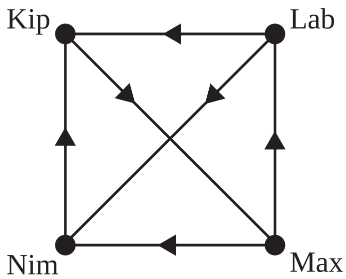
In the network shown, the number of vertices of even degree is

A. 2
B. 3
C. 4
D. 5
E. 6

Question 2

The graph below shows the one-step dominances between four farm dogs, Kip, Lab, Max and Nim.

In this graph, an arrow from Lab to Kip indicates that Lab has a one-step dominance over Kip.

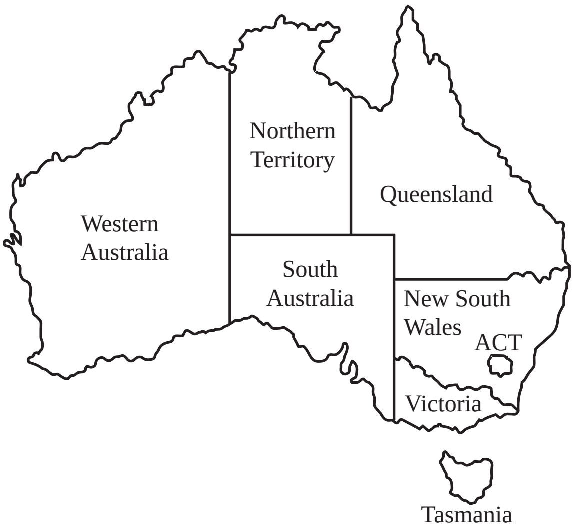


From this graph, it can be concluded that Kip has a two-step dominance over

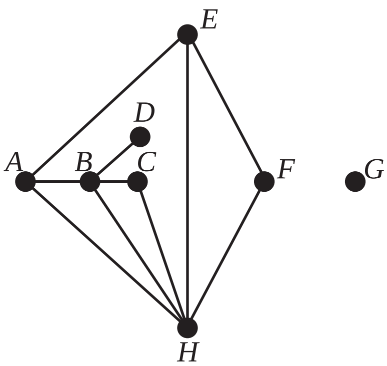
A. Max only.
B. Nim only.
C. Lab and Nim only.
D. all of the other three dogs.
E. none of the other three dogs.

Use the following information to answer Questions 3 and 4.

The map of Australia shows the six states, the Northern Territory and the Australian Capital Territory (ACT).



In the network diagram below, each of the vertices A to H represents one of the states or territories shown on the map of Australia. The edges represent a border shared between two states or between a state and a territory.



Question 3

In the network diagram, the order of the vertex that represents the Australian Capital Territory (ACT) is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 4

In the network diagram, Queensland is represented by

A. vertex *A*.

B. vertex *B*.

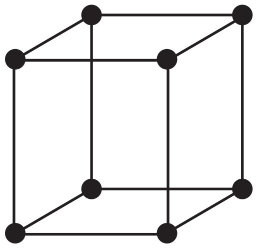
C. vertex *C*.

D. vertex *D*.

E. vertex *E*.

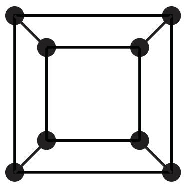
Question 5

A network is represented by the following graph.

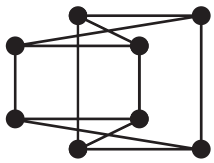


Which one of the following graphs could **not** be used to represent the same network?

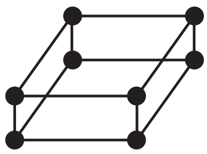
A.



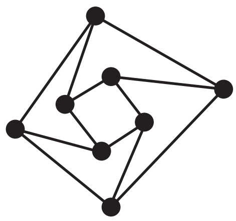
B.



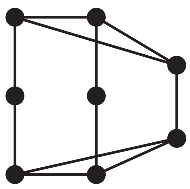
C.



D.



E.



Question 6

A store manager is directly in charge of five department managers.
Each department manager is directly in charge of six sales people in their department.

This staffing structure could be represented graphically by

A. a tree.
B. a circuit.
C. an Euler path.
D. a Hamiltonian path.
E. a complete graph.

Question 7

Andy, Brian and Caleb must complete three activities in total (*K*, *L* and *M*).

The table shows the person selected to complete each activity, the time it will take to complete the activity in minutes and the immediate predecessor for each activity.

Person	**Activity**	**Duration**	**Immediate predecessor**
Andy	*K*	13	–
Brian	*L*	5	*K*
Caleb	*M*	16	*L*

All three activities must be completed in a total of 40 minutes.
The instant that Andy starts his activity, Caleb gets a telephone call.

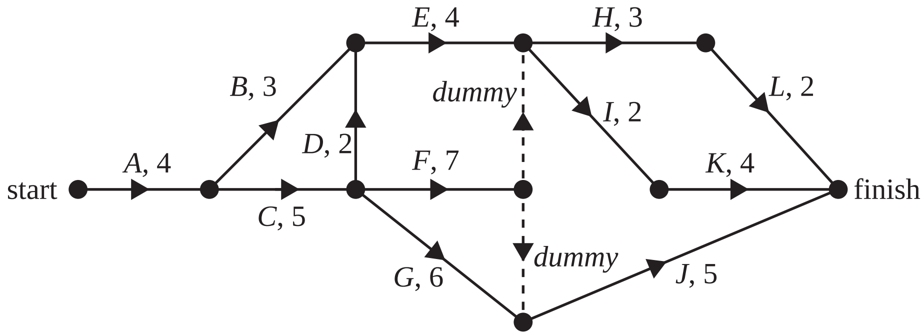
The maximum time, in minutes, that Caleb can speak on the telephone before he must start his allocated activity is

A. 5
B. 13
C. 18
D. 24
E. 34

Question 8

The diagram shows the tasks that must be completed in a project.

Also shown are the completion times, in minutes, for each task.



The critical path for this project includes activities

A. *B* and *I*.

B. *C* and *H*.

C. *D* and *E*.

D. *F* and *K*.

E. *G* and *J*.

Question 9

An Euler path through a network commences at vertex *P* and ends at vertex *Q*.
Consider the following five statements about this Euler path and network.

- In the network, there could be three vertices with degree equal to one.
- The path could have passed through an isolated vertex.
- The path could have included vertex *Q* more than once.
- The sum of the degrees of vertices *P* and *Q* could equal seven.
- The sum of the degrees of all vertices in the network could equal seven.

How many of these statements are true?

A. 0

B. 1

C. 2

D. 3

E. 4

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The matrix below shows the airfares (in dollars) that are charged by Zeniff Airlines to fly between Adelaide (A), Melbourne (M) and Sydney (S).

$$\begin{array}{c} \textit{from} \\ \begin{array}{ccc} A & M & S \end{array} \end{array}$$

$$\begin{bmatrix} 0 & 85 & 89 \\ 85 & 0 & 99 \\ 97 & 101 & 0 \end{bmatrix} \begin{array}{l} A \\ M \\ S \end{array} \quad \textit{to}$$

The cost to fly from Melbourne to Sydney with Zeniff Airlines is

A. $85

B. $89

C. $97

D. $99

E. $101

Question 2

If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then $AB + 2C$ equals

A. $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

B. $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Question 3

Each of the following four matrix equations represents a system of simultaneous linear equations.

$$\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

How many of these systems of simultaneous linear equations have a unique solution?

A. 0
B. 1
C. 2
D. 3
E. 4

Question 4

Matrix A is a 3×4 matrix.
Matrix B is a 3×3 matrix.
Which one of the following matrix expressions is defined?

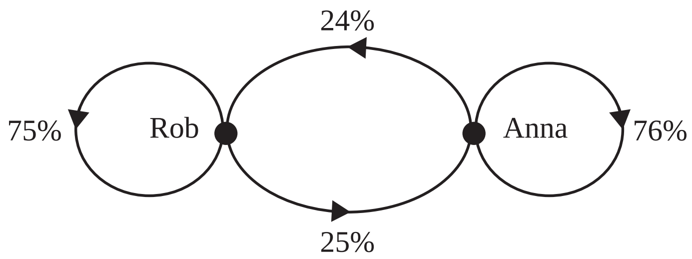
A. BA^2
B. $BA - 2A$
C. $A + 2B$
D. $B^2 - AB$
E. A^{-1}

Use the following information to answer Questions 5 and 6.

Two politicians, Rob and Anna, are the only candidates for a forthcoming election. At the beginning of the election campaign, people were asked for whom they planned to vote. The numbers were as follows.

Candidate	**Number of people who plan to vote for the candidate**
Rob	5692
Anna	3450

During the election campaign, it is expected that people may change the candidate that they plan to vote for each week according to the following transition diagram.



Question 5

The total number of people who are expected to change the candidate that they plan to vote for one week after the election campaign begins is

A. 828
B. 1423
C. 2251
D. 4269
E. 6891

Question 6

The election campaign will run for ten weeks.

If people continue to follow this pattern of changing the candidate they plan to vote for, the expected winner after ten weeks will be

A. Rob by about 50 votes.
B. Rob by about 100 votes.
C. Rob by fewer than 10 votes.
D. Anna by about 100 votes.
E. Anna by about 200 votes.

Question 7

Each night, a large group of mountain goats sleep at one of two locations, *A* or *B*.
On the first night, equal numbers of goats are observed to be sleeping at each location.
From night to night, goats change their sleeping locations according to a transition matrix *T*.
It is expected that, in the long term, more goats will sleep at location *A* than at location *B*.

Assuming the total number of goats remains constant, a transition matrix *T* that would predict this outcome is

A.

$$\begin{matrix} & \textit{this night} & \\ & A \quad B & \\ T = & \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} & \begin{matrix} A \\ B \end{matrix} \; \textit{next night} \end{matrix}$$

B.

$$\begin{matrix} & \textit{this night} & \\ & A \quad B & \\ T = & \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} & \begin{matrix} A \\ B \end{matrix} \; \textit{next night} \end{matrix}$$

C.

$$\begin{matrix} & \textit{this night} & \\ & A \quad B & \\ T = & \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} & \begin{matrix} A \\ B \end{matrix} \; \textit{next night} \end{matrix}$$

D.

$$\begin{matrix} & \textit{this night} & \\ & A \quad B & \\ T = & \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} & \begin{matrix} A \\ B \end{matrix} \; \textit{next night} \end{matrix}$$

E.

$$\begin{matrix} & \textit{this night} & \\ & A \quad B & \\ T = & \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} & \begin{matrix} A \\ B \end{matrix} \; \textit{next night} \end{matrix}$$

Question 8

Consider the following matrix *A*.

$$A = \begin{bmatrix} 3 & k \\ -4 & -3 \end{bmatrix}$$

A is equal to its inverse A^{-1} for a particular value of *k*.

This value of *k* is

A. –4

B. –2

C. 0

D. 2

E. 4

Question 9

Matrix *A* is a 3 × 3 matrix. Seven of the elements in matrix *A* are zero.
Matrix *B* contains six elements, none of which are zero.

Assuming the matrix product *AB* is defined, the minimum number of zero elements in the product matrix *AB* is

A. 0

B. 1

C. 2

D. 4

E. 6

END OF MULTIPLE-CHOICE QUESTION BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}$, $r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}$, $|r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET