The Mathematical Association of Victoria

FURTHER MATHEMATICS

Trial Written Examination 2

2011

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of Book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 29 pages, with a detachable sheet of miscellaneous formulas at the back.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Core

Question 1

Table 1 shows the value of the Australian dollar relative to the United States (U.S.) dollar and the Japanese yen for each month from January 1984 to December 1984.

Table 1

Month	Month no.	U.S. (dollar)	Japanese (yen)
January	1	0.906	212.79
February	2	0.936	219.70
March	3	0.952	215.68
April	4	0.925	209.14
May	5	0.908	210.02
June	6	0.886	207.49
July	7	0.837	203.86
August	8	0.847	206.09
September	9	0.833	205.21
October	10	0.837	207.52
November	11	0.861	210.31
December	12	0.844	210.03

For the year 1984 determine

a. the median value of the Australian dollar relative to the U.S. dollar, correct to three decimal places.

___

b. the percentage of months that the Australian dollar was worth more than 90 U.S. cents. Write your answer to the nearest percent.

___

c. the range of the value of the Australian dollar relative to the U.S. dollar, correct to three decimal places.

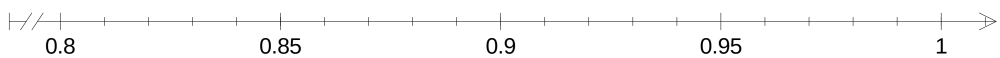
___

1+ 1 + 1 = 3 marks

Question 2

a. **i.** Find the five figure summary statistics for the distribution of the value of the Australian relative to the U.S. dollar for 1984.

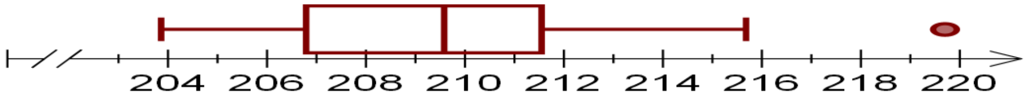
ii. On the axis below, construct a boxplot of the distribution of the value of the Australian dollar relative to the U.S. dollar for 1984.



Value of the Australian dollar relative to the U.S. dollar

1 + 1 = 2 marks

b. The following boxplot shows the distribution of the value of the Australian dollar relative to the Japanese yen for 1984.



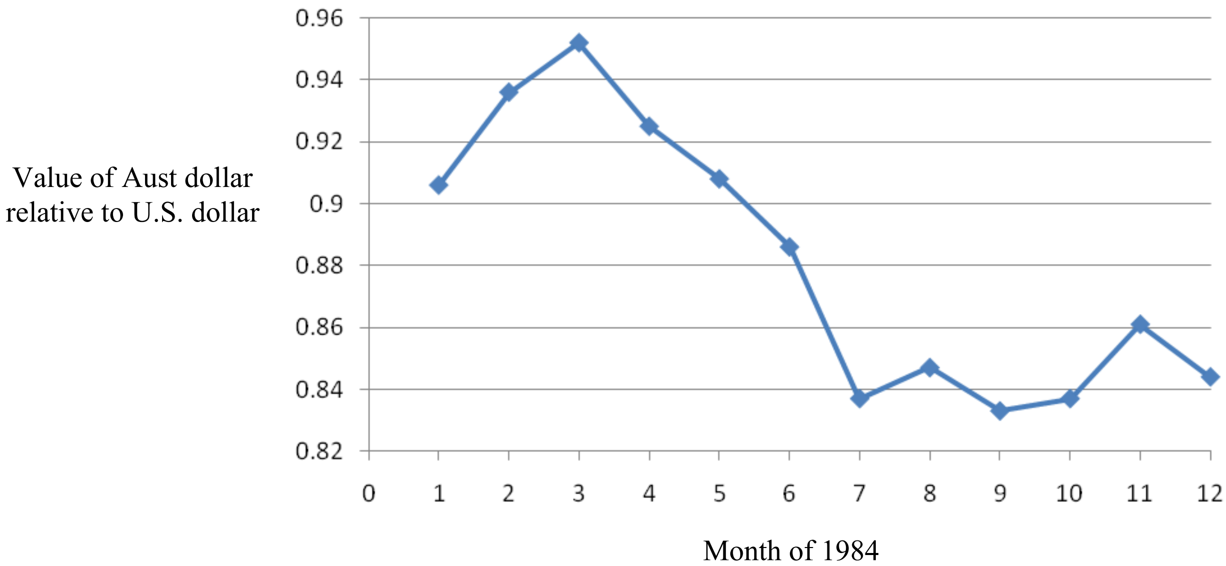
Value of Australian dollar relative to the Japanese yen

Comment on the shape and features of this boxplot.

1 mark

Question 3

The time series plot below, constructed from the data in **Table 1**, shows the value of the Australian dollar relative to the U.S. dollar for the year 1984.



An equation for the least squares regression line for this data set is

Australian dollar value (\$) = 0.946 − 0.01 × *month* where *month* 1 = January 1984.

a. Draw this line on the time series plot above. Show the coordinates of two points on this line correct to three decimal places.

2 marks

b. Calculate, to three decimal places, the residual value for November 1984.

1 mark

c. Use the equation of the least squares regression line to predict the value of the Australian dollar relative to the U.S. dollar for July 1985. Write your answer correct to three decimal places.

2 marks

d. Suggest a reason why this least squares regression line may not be appropriate to predict future values of the Australian dollar relative to the U.S. dollar.

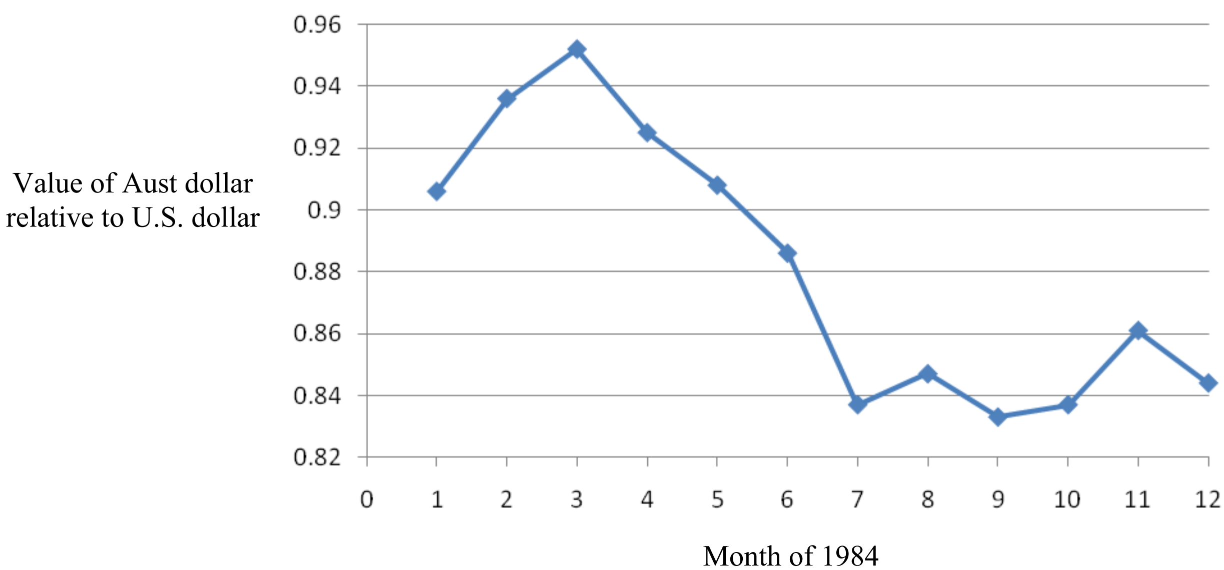
1 mark

e. Pearson's correlation coefficient for this data is – 0.834.

Interpret the **coefficient of determination** for the data in terms of the variables *Value of Australian dollar* and *month of the year*.

1 mark

f. Three median smoothing will be used to smooth the time series plot.
Plot the smoothed time series on the axes below, marking each smoothed point with a cross (x)



2 marks

Total 15 marks

END OF CORE

Module 1: Number patterns

Question 1

In Macca's Olive Grove there are a number of varieties of olive tree. The initial planting of Corregiola olive trees was around a large dam, and the number of trees planted in each row varied.
The first row had 11 trees, the second row 13 trees, the next 15 and so on, following an arithmetic sequence.

a. How many Corregiola olive trees are planted in row 12?

1 mark

b. What is the total number of Corregiola olive trees planted in the first 15 rows?

1 mark

The last row of Corregiola olive trees has 55 trees.

c. **i.** Find how many rows of Corregiola olive trees have been planted.

ii. Determine the total number of Corregiola olive trees that have been planted.

1 + 1 = 2 marks

Question 2

Macca decided to plant a second variety of olive tree on a steep hill. Because the soil contains less moisture the further up the hill the tree is planted, Macca planted the Paragon olive trees progressively further apart. The spacing between the Paragon olive trees followed a geometric sequence, with the spacing between the first and second trees in the row being 2 metres; the spacing between the second and third trees being 2.2 metres and so on.

a. Find the common ratio of this sequence.

1 mark

b. Find the spacing between the 7^{th} and 8^{th} olive trees, in metres, correct to one decimal place.

1 mark

c. Determine how many Paragon olive trees are planted in a row that is 150 metres long.

2 marks

Question 3

At 'first yield', the Corregiola olive trees produce an average yield of 10 kg per tree. The average yield per tree continues to increase each year.

A difference equation that estimates the average yield per tree is given by

$$t_{n+1} = 1.1t_n + 5, \quad t_0 = 10$$

where t_n is the average yield of a Corregiola olive tree, in kilograms, n years after 'first yield'.

a. Use the difference equation to predict the average yield per tree in the 5^{th} year after 'first yield'. Write your answer correct to two decimal places.

1 mark

b. Show that the sequence generated by the difference equation is not geometric.

1 mark

c. How many years after 'first yield' will the average yield per tree exceed 80 kg?

1 mark

d. What was the increase in average yield per tree during the 6^{th} year after 'first yield'? Write your answer in kilograms correct to two decimal places.

1 mark

The average yield per Paragon olive tree is modelled by the difference equation

$$p_{n+1} = 1.3 p_n, \quad p_0 = 5$$

where p_n is the average yield of a Paragon olive tree, in kilograms, n years after 'first yield'.

e. Write an expression in terms of n that can be used to predict the average yield per Paragon olive tree n years after 'first yield'.

1 mark

f. Describe the difference in the yield patterns between the Corregiola and the Paragon olive trees.

1 mark

g. How many years after 'first yield' does the average yield of a Paragon olive tree first exceed the average yield of a Corregiola olive tree?

1 mark

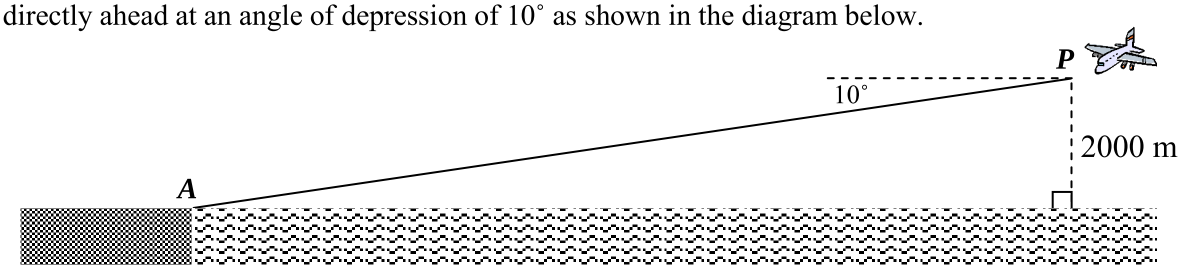
Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

A plane at P is flying horizontally at an altitude of 2000 metres. It is going to land at airstrip, A, located directly ahead at an angle of depression of 10˚ as shown in the diagram below.



a. Calculate the angle of elevation of the plane, P, from the airstrip, A.

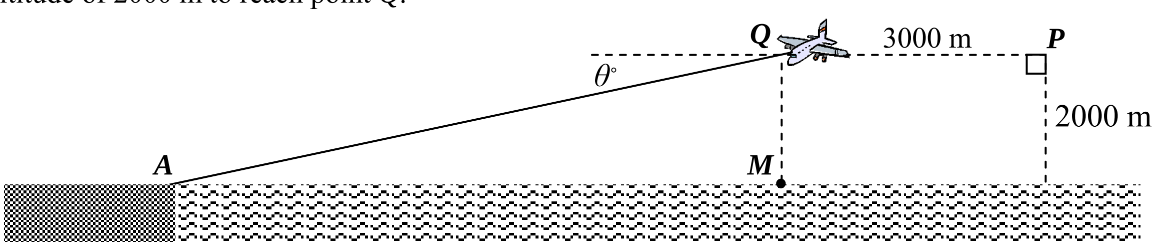
1 mark

b. Find the distance, PA, from the plane to the airstrip.

Write your answer correct to the nearest metre.

1 mark

Before beginning its descent to the airstrip, the plane travels horizontally for a further 3000 m at an altitude of 2000 m to reach point Q.



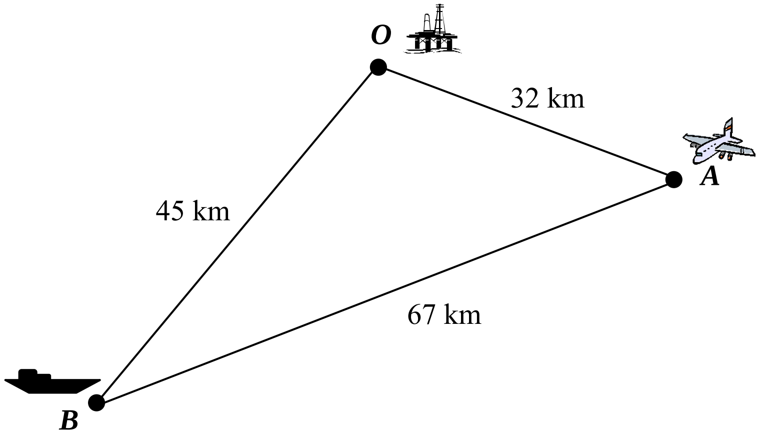
c. **i.** Point M is directly below Q in the same horizontal plane as airstrip A.
Determine the distance, AM correct to the nearest metre.

ii. Calculate the angle of depression, θ, the plane should take from point Q in order to land at A.
Write your answer in degrees correct to one decimal place.

1 + 1 = 2 marks

Question 2

Some people have been lost at sea in the triangular region *AOB* marked out by the airstrip, *A*, an oil rig, *O*, and a boat, *B*. The distances, in kilometres, between each point are shown in the diagram below.



a. **i.** Show that the angle *AOB* is 120°.

____

____

____

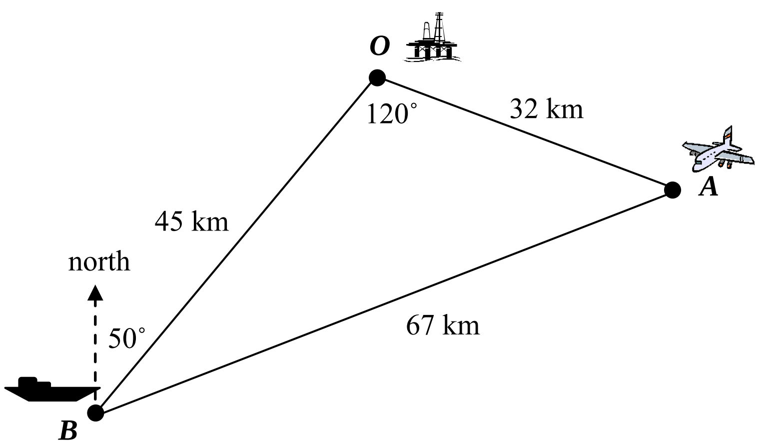
ii. Find the area of the triangle *AOB* in square kilometres correct to one decimal place.

____

____

1 + 1 = 2 marks

The oil rig is located on a bearing of 50˚ from the boat.



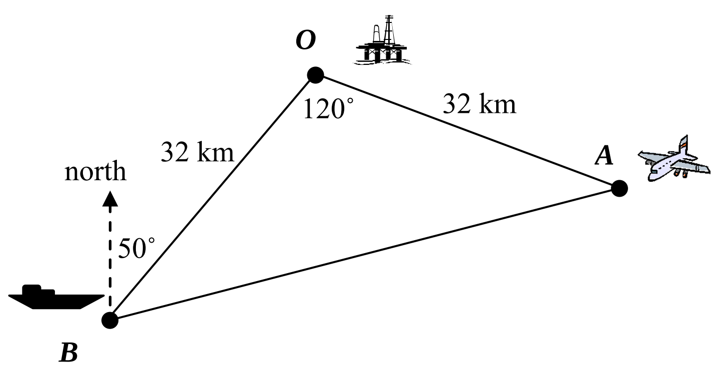
b. Determine the bearing of the boat from the oil rig.

1 mark

c. What is the bearing of the boat from the airstrip? Write your answer correct to one decimal place.

2 marks

The boat travels on a bearing of 50° until it is 32 km from the oil rig.



d. **i.** Calculate the distance from the airstrip to the boat.
Write your answer in kilometres correct to one decimal place.

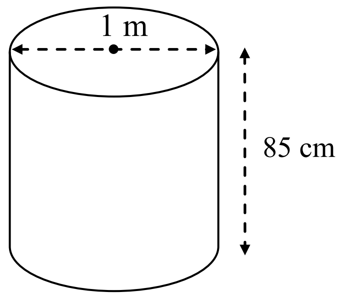
ii. What is the new bearing of the airstrip from the boat?

1 + 1 = 2 mark

Question 3

The boat is carrying containers of varying shapes and sizes.

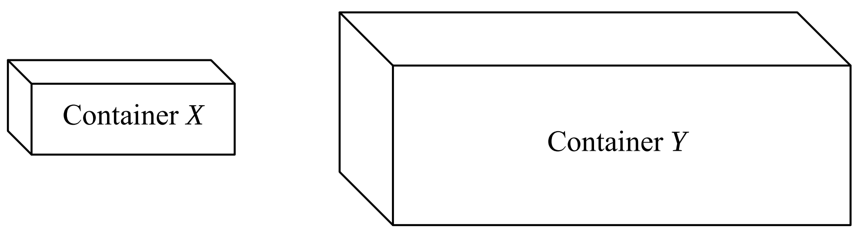
a. A cylindrical oil barrel has a diameter of 1 metre and a height of 85 cm.



How many litres of oil will this barrel contain when it is full?
Write your answer correct to the nearest litre.

2 marks

b. Another two containers, *X* and *Y*, are similar in shape.



The volume of container *Y* is 3.375 times the volume of container *X*.
The total surface area of container *Y* is 13.95 m^2.

Calculate the total surface area of container *X*.

2 marks

Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

A restaurant has both small and large tables in the dining area.
Up to 15 small tables and up to 8 large tables may be used at a special function.

Let x be the number of small tables that are used

and y be the number of large tables that are used.

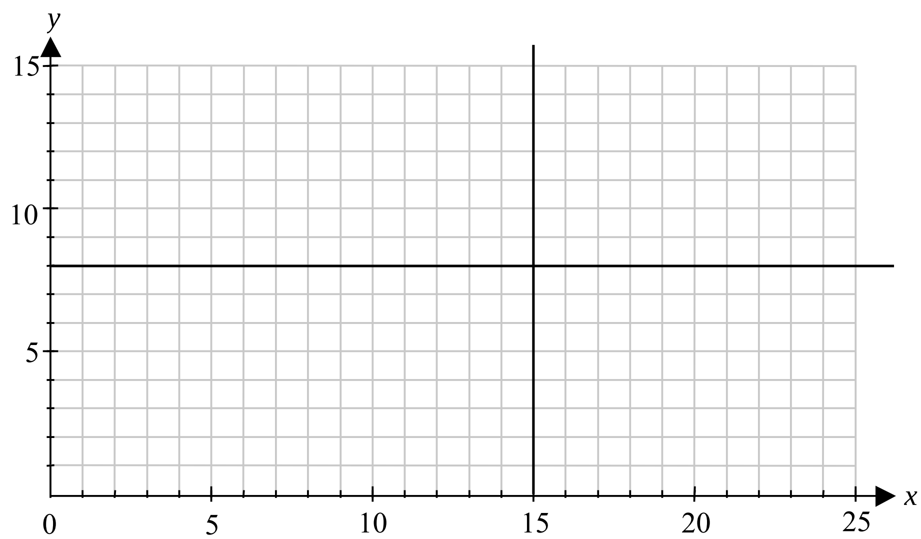
a. Write inequalities involving the variable x and the variable y to describe the number of small and large tables respectively that may be used.

Inequality 1: ____
(Small tables)

Inequality 2: ____
(Large tables)

1 mark

b. Two lines are drawn on the axes below. Label each line with its equation.



1 mark

On average, a waitress spends 5 minutes at a small table and 12 minutes at a large table. Millie is working a two hour shift as a waitress at the special function. The time, in minutes, she spends at tables can be written as Inequality 3.

Inequality 3: $5x + 12y \le 120$

c. Use Inequalities 1, 2, and 3 to construct the feasible region for the time Millie spends at tables. **Shade** this feasible region on the graph in **part b**.

2 marks

d. A small table can seat up to 4 people and a large table can seat up to 10 people.
Use Inequalities 1, 2 and 3 to determine the maximum number of people that Millie can assist at the tables during her two hour shift.

2 marks

Question 2

The manager uses a formula to work out the number of staff, n, that he will need working in the kitchen to prepare the meals in time, t, minutes at a special function.

The formula he uses is

$$n = \frac{k}{t} \qquad \text{where } k \text{ is a constant}$$

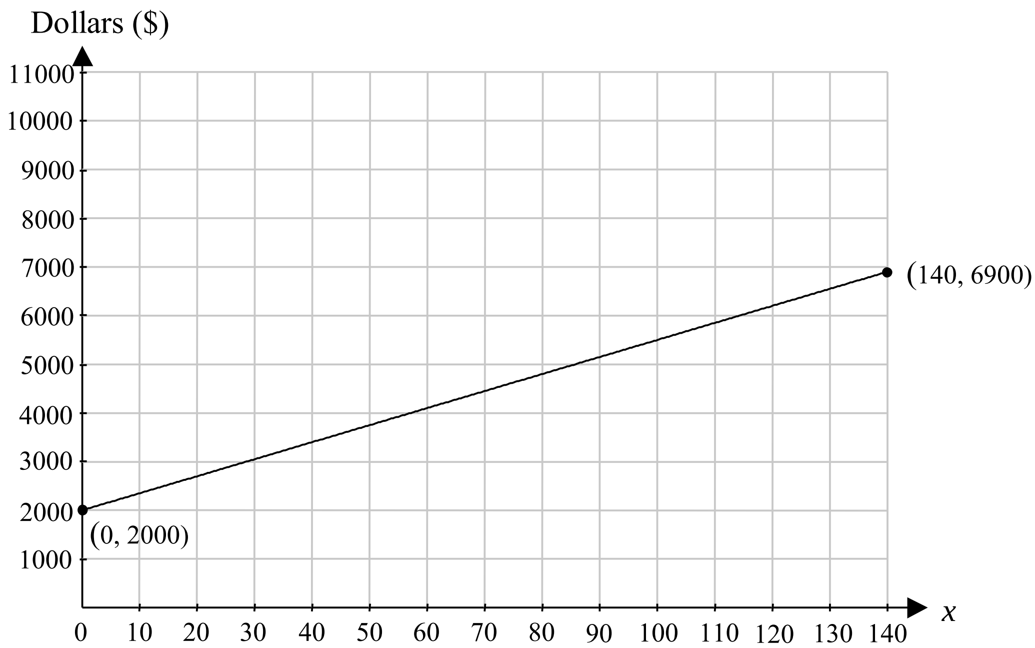
According to this formula, the meals will take 120 minutes to prepare if 5 staff work in the kitchen.

Determine how many staff will need to work in the kitchen if the manager wants the meals for this function to be prepared in 75 minutes.

2 marks

Question 3

The manager has a linear graph of the cost to the restaurant when x people attend a special function. This line is drawn on the set of axes below.



a. Determine the equation of the cost, C, in dollars, to the restaurant when x people attend a special function.

1 mark

b. If 10 more people than expected attend a function how much extra will it cost the restaurant?

1 mark

The restaurant will serve a set menu for this special functions and each person will pay $75 to attend.

c. Calculate the number of people who will need to attend for the restaurant to breakeven.

2 marks

d. If the restaurant is to make a profit of $3480 on this function, how many people will need to attend?

1 mark

e. The manager wants to make a higher profit on this function and decides to increase the price people will pay. If his costs do not change, what is the lowest amount he can charge each person in order to make a profit of at least $4000 when 125 people attend?

2 marks

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

A small hobby farm was purchased as an investment for \$490 000 three years ago. Since then it has increased by 6% of its value each year.

a. Calculate the present value of the farm. Write your answer correct to the nearest hundred dollars.

1 mark

b. Assume the farm will continue to increase in value by 6% per annum.
How many years after the farm was purchased will it be valued at \$700 000?
Write your answer correct to one decimal place.

2 marks

c. The owner anticipates that he will make a profit of \$300 000 when the farm is sold.
He will pay capital gains tax on half of the profit at the rate of 45 cents in the dollar.
Calculate the capital gains tax that will be paid.

1 mark

Question 2

The owner takes out a reducing balance loan of \$35 000 at an interest rate of 10.4% per annum compounding monthly to renovate the house on the farm.

a. Assume the \$35 000 loan will be fully repaid over five years.

i. Calculate the monthly repayment correct to the nearest cent.

ii. Determine the total interest paid over five years correct to the nearest dollar.

1 + 1 = 2 marks

b. Suppose, instead, the owner pays \$800 each month for the first two years of the loan and then he reduces his payments to \$600 per month for the following three years.
Find the principal outstanding on the loan at the end of the five year period.
Write your answer correct to the nearest dollar.

2 marks

Question 3

The owner will claim depreciation on the improvements on the farm in his income tax. He wants to know the differences between the flat rate method of depreciation and the reducing balance method of depreciation.

Explain the differences to him.

2 marks

Question 4

The flat rate depreciation method will be used to depreciate a new tractor that will be used on the farm. Initially the tractor is valued at \$20 000. It will be depreciated at 15% per annum over six years. At the end of six years, the tractor will be considered as written off.

a. Calculate the annual flat rate depreciation in dollars.

1 mark

b. By how much has the tractor depreciated after three years?

1 mark

c. Determine the value of the tractor when it is written off.

1 mark

Question 5

The owner's bank account pays interest on the minimum monthly balance at the simple interest rate of 3.75% per annum at the end of each month. A copy of his bank statement for October is given below. Due to a computer malfunction the balance after each transaction is shown as $xxxx.xx.

Date	**Transaction**	**Debit**	**Credit**	**Balance**
01 October	OPENING BALANCE			$xxxx.xx
07 October	Cheque		$520.00	$xxxx.xx
15 October	EFT		$1359.05	$xxxx.xx
24 October	Automatic payment	$1506.93		$xxxx.xx
31 October	CLOSING BALANCE			**$ 2049.88**

It is known that the closing balance of the account for October is $ 2049.88
Determine the interest earned on this account during October correct to the nearest cent.

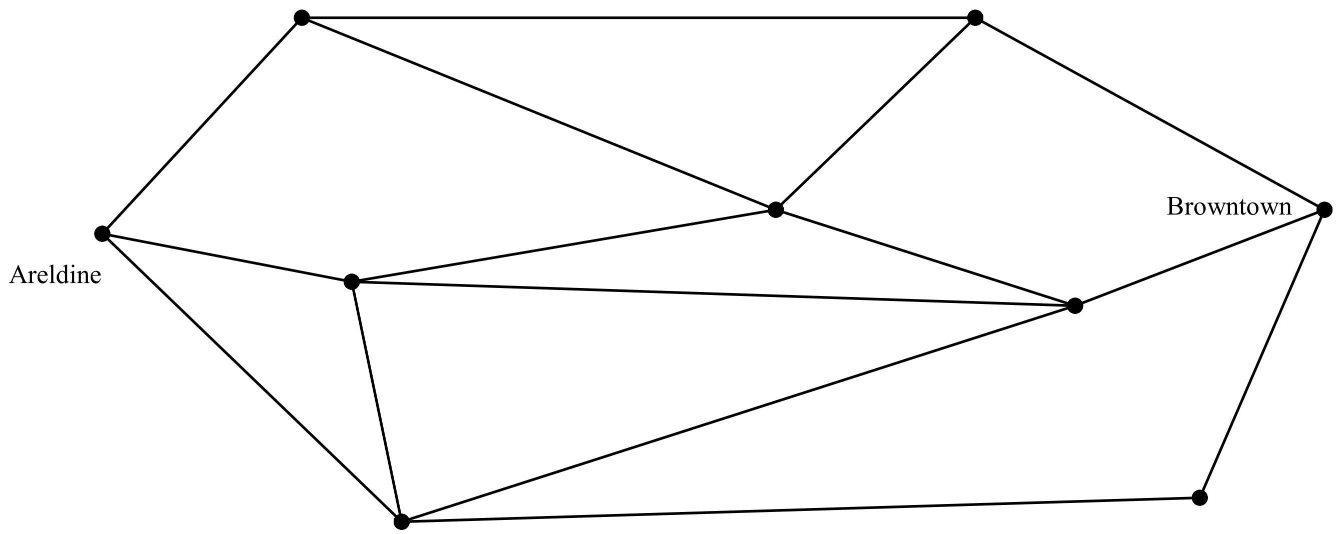
2 marks

Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

A region in central Victoria has roads connecting the towns of Areldine and Browntown and surrounding farms with the intersections (road corners) indicated by the vertices in the network diagram below.



Question 1

Matt the mailman has to travel down all of the roads in the map shown above.
He cannot travel down any roads twice.

a. If Matt started at Areldine and travelled down each road exactly once finishing at Browntown, would this be a circuit or a path?

____

1 mark

b. What is the name of such a path or circuit?

____

1 mark

c. Explain why such a path or circuit described in question **1. a**. is impossible.

____

____

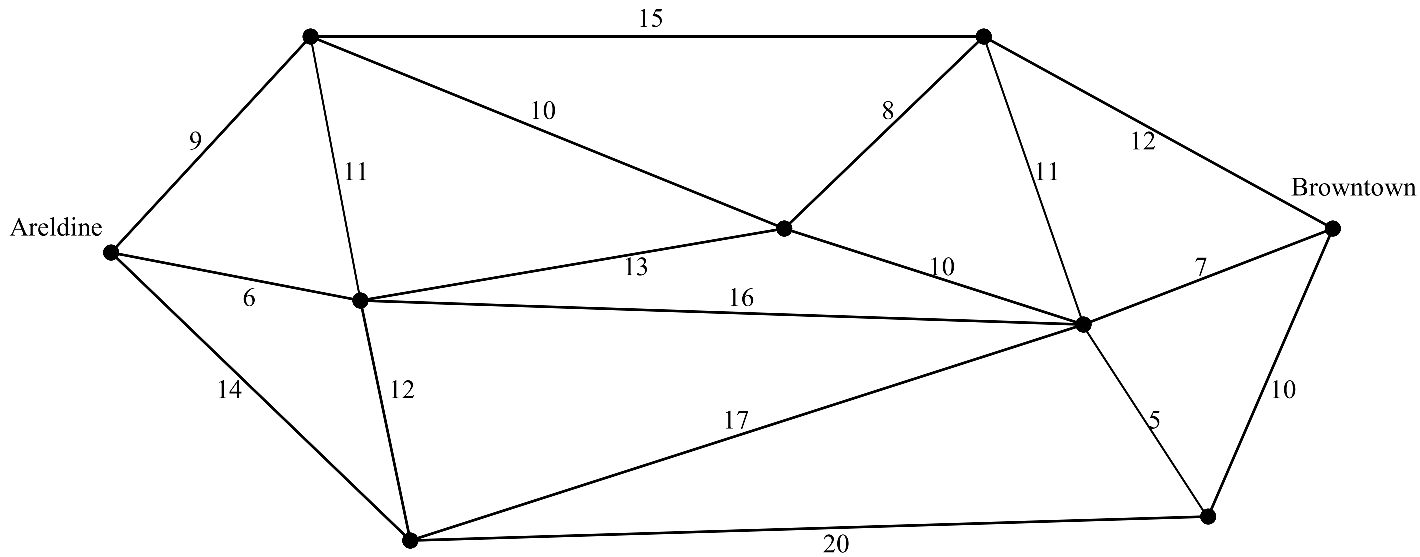
1 mark

d. Mark the road that Matt would have to travel down twice (while travelling down each other road exactly once) by writing *TWICE* on the appropriate road on the diagram above.

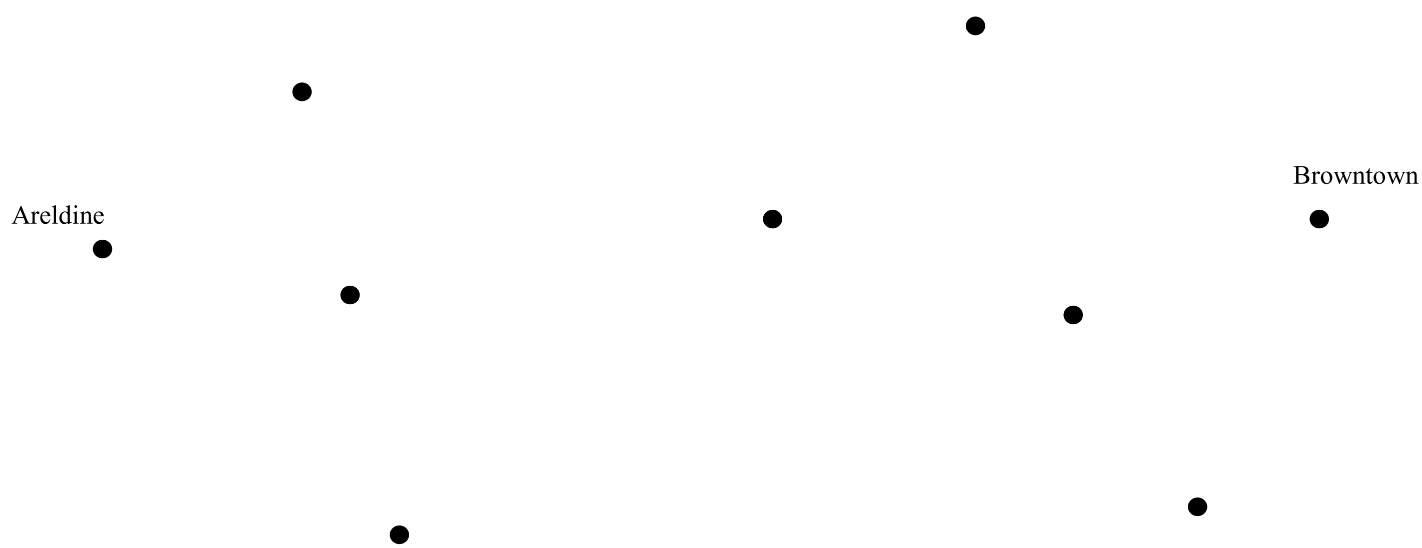
1 mark

Question 2

The local shire council intends to install a fiber-optic network joining each intersection. They want to do this at a minimum cost. The cost in thousands of dollars to connect each intersection is shown on the edges in the network diagram below.



a. Join the vertices in the diagram below to form the minimum spanning tree for this network.



1 mark

b. What is the minimum cost (in thousands of dollars) to connect all the intersections?

____

1 mark

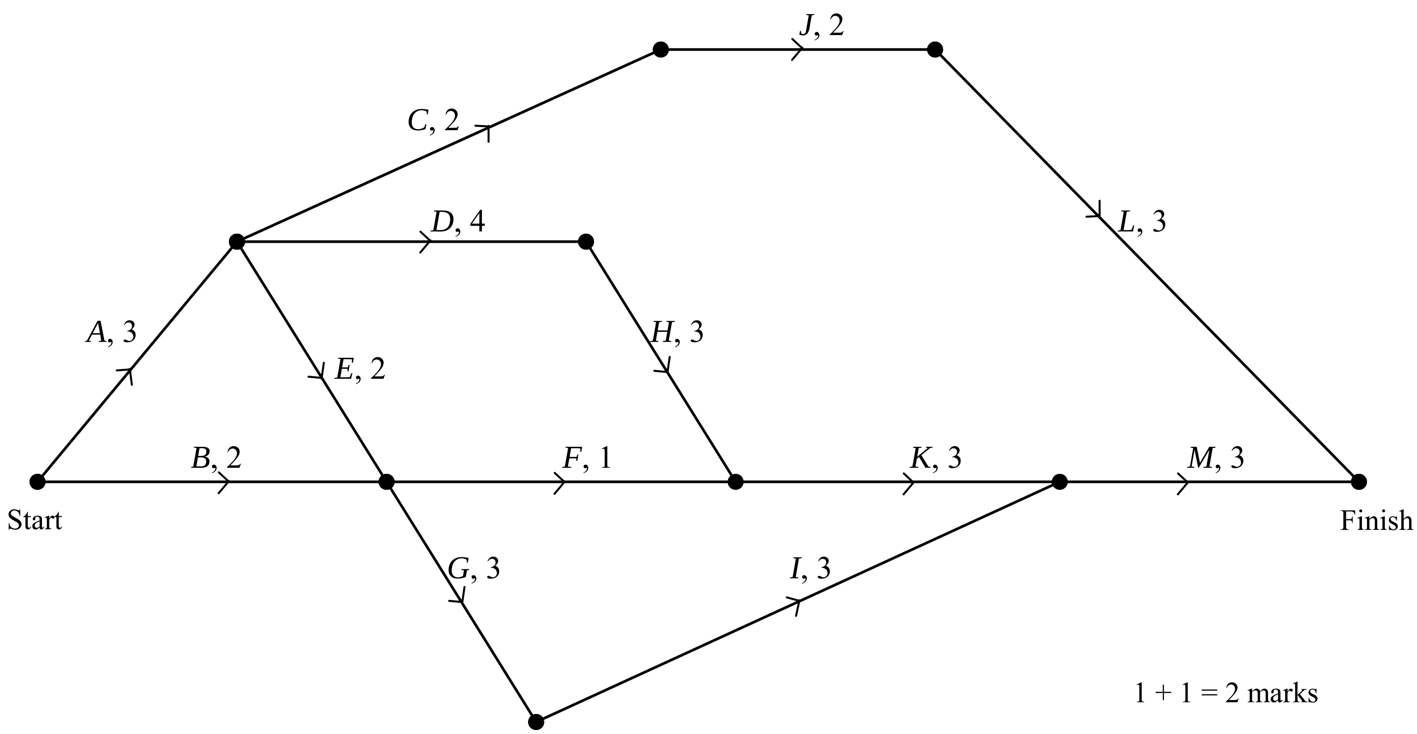
Question 3

The town of Areldine is in the process of building a new central business complex. Patricia, the Town Planner, has been given the task of planning the construction of this new business complex. She has a made list of the tasks that need to be completed and has partially drawn a directed network diagram to show the flow of tasks.

Task	Time to complete Task(days)	Predecessors
A	3	-
B	2	-
C	2	*A*
D	4	*A*
E	2	
F	1	*B, E*
G	3	*B, E*
H	3	*D*
I	3	*G*
J	2	*C*, ***X***
K	3	*F, H*
L	3	*J*
M	3	*I, K*
X	1	*D*

a. **i.** Enter the predecessor(s) for Task *E* in the shaded box in the table above.

ii. Complete the directed graph below by correctly placing and labelling task ***X*** on the network diagram, showing its direction.



1 + 1 = 2 marks

b. Complete the missing information in the shaded boxes in the table below

Task	**Time to complete Task (days)**	**Earliest Start Time (days)**	**Latest Start Time (days)**
A	3	0	0
B	2	0	5
C	2	3	9
D	4	3	3
E	2	3	5
F	1	5	9
G	3		
H	3	7	7
I	3	8	10
J	2	8	11
K	3	10	10
L	3	10	
M	3	13	
X	1		

2 marks

c. Write down the critical path for this project.

1 mark

d. What is the earliest completion time, in days, for this project?

1 mark

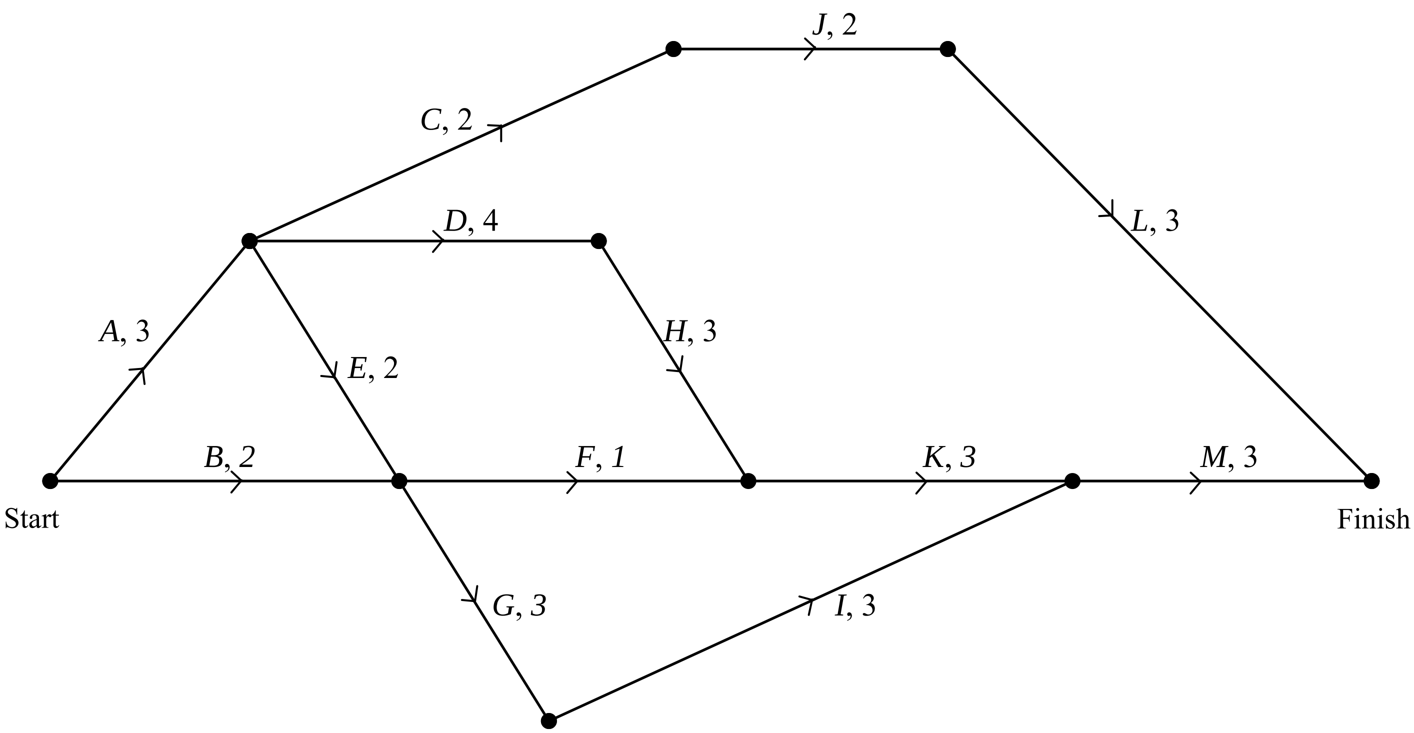
e. Find the float time, in days, for activity *F*.

1 mark

Question 4

Tasks *H*, *J* and *K* can have their completion time reduced by hiring more workers.
The cost to do this is shown in the table below.

Task	Possible reduction in time (days)	Cost per day
H	1	1000
J	1	2000
K	2	2000



Assume that more workers are hired and a new earliest completion time for this project is achieved.

a. What is the minimum cost of achieving the new earliest completion time?

1 mark

b. Specify the number of days that each task will be reduced in order to achieve the new earliest completion time at the minimum cost.

1 mark
Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

A nursery sells budget, *B*, regular, *R*, and premium, *P*, potting mix.
The cost of a bag of each type of potting mix is \$3.95, \$6.50 and \$10 respectively.

a. Complete the column matrix below to show the cost of a bag of each type of potting mix.

$$\begin{array}{c c} \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} & \begin{array}{c} B \\ R \\ P \end{array} \end{array}$$

1 mark

b. A gardener purchased three bags of budget potting mix, one bag of regular potting mix and five bags of premium potting mix. Display this potting mix purchase in row matrix.

1 mark

c. i. Write down a matrix product that, when evaluated, will contain the total cost of potting mix that was purchased.

ii. Evaluate this matrix product.

1 + 1 = 2 marks

Question 2

The nursery also sells punnets of seedlings.
Carol, Doris and Joan, each bought bean, *b*, tomato, *t*, and lettuce, *l*, seedlings to plant in their gardens.

The matrix equation below shows the number of punnets of each type of seedling the gardeners purchased and the total cost, in dollars, they paid.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 0 \\ 3 & 0 & 4 \end{bmatrix}\begin{bmatrix} b \\ t \\ l \end{bmatrix} = \begin{array}{cl} \text{Cost (\$)} & \\ \begin{bmatrix} 16.95 \\ 37.85 \\ 36.45 \end{bmatrix} & \begin{array}{l} \textit{Carol} \\ \textit{Doris} \\ \textit{Joan} \end{array} \end{array}$$

a. Which gardener bought the greatest number of punnets of seedlings, in total? How many punnets of seedlings did she buy?

1 mark

b. Interpret the meaning of the element in the second row and third column of the 3×3 matrix in the equation shown above.

1 mark

In order to find the cost per punnet of each type of seedling, the following matrix equation is used. It has a missing element.

$$\begin{bmatrix} b \\ t \\ l \end{bmatrix} = \begin{bmatrix} \boxed{} & -4 & -5 \\ -4 & 1 & 1 \\ -15 & 3 & 4 \end{bmatrix}\begin{bmatrix} 16.95 \\ 37.85 \\ 36.45 \end{bmatrix}$$

c. **i.** What is the value of the missing element in the shaded box in the matrix equation above?

ii. Another gardener, Kate, bought three punnets of bean seedlings, two punnets of tomato seedlings and one punnet of lettuce seedlings at the same nursery.
Use matrices to determine the total cost of Kate's seedlings.

1 + 2 = 3 marks

Question 3

A gardener's preference for fertilizer (organic, *O*, inorganic, *I*, no fertilizer, *N*) changes from year to year according to the transition matrix, *T*.

$$\begin{array}{c} \text{this year} \\ \begin{array}{ccc} O & I & N \end{array} \end{array}$$

$$T = \begin{bmatrix} 0 \cdot 3 & 0 \cdot 4 & 0 \cdot 2 \\ 0 \cdot 4 & 0 \cdot 5 & 0 \cdot 7 \\ 0 \cdot 3 & 0 \cdot 1 & 0.1 \end{bmatrix} \begin{array}{l} O \\ I \\ N \end{array} \quad \text{next year}$$

a. What percentage of gardeners who use no fertilizer this year will use organic fertilizer next year?

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1 mark

The fertilizer preference of 1000 gardeners who use no fertilizer this year will be examined over time.

b. Use the transition matrix, T, to determine how many of these 1000 gardeners will prefer to use organic fertilizer in three years time.

1 mark

c. Find the steady state matrix for these gardeners' fertilizer preference, showing justification.

1 mark

d. In the long run, what proportion of these 1000 gardeners will prefer to use inorganic fertilizer?

1 mark

Question 4

The nursery manager wants to increase the number of customers to the nursery. He decides to offer demonstrations on horticulture, *H*, and landscaping, *L*, each weekend.

On the first weekend, 10 people attended the horticulture demonstration and 20 people attended the landscaping demonstration. This information is shown in matrix, D_1.

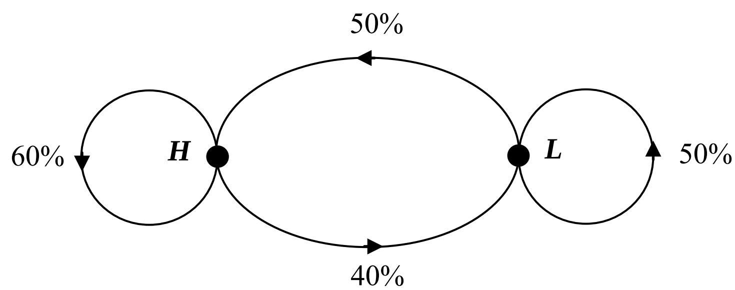
$$D_1 = \begin{array}{c} \begin{bmatrix} 10 \\ 20 \end{bmatrix} \end{array} \begin{array}{l} H \\ L \end{array}$$

The manager believes that the number of people attending the demonstrations will increase each weekend according to the matrix equation

$$D_{n+1} = T D_n + \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

where D_n contains the number of people who attend the horticulture and landscaping demonstrations on the n^{th} weekend

and T is a transition matrix, found from the transition diagram below.



Determine how many people will attend the horticulture demonstration on the third weekend.

2 marks
Total 15 marks

END OF MODULE 6

Further Mathematics Formulae Sheet

Core: Data Analysis

Standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

Least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

Residual value: residual value = actual value – predicted value

Seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number Patterns

arithmetic series: $a + (a+d) + \ldots\ldots + (a + (n-1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}\left(a + l\right)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}$, $r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots\ldots = \dfrac{a}{1-r}$, $|r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a+b+c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base $\times$ height

volume of a pyramid: $\dfrac{1}{3}$ area of base $\times$ height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{Prt}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$