

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1



2010 Trial Examination 1

SOLUTIONS

SECTION A – Multiple-choice questions

Core: Data analysis

Question 1

Answer: B

Explanation:

$$\Pr(-1 < Z < 2) = 0.68 + \frac{1}{2}(0.95 - 0.68) = 0.815.$$

Question 2

Answer: A

Explanation:

$$0.25 - 0.16 = 0.09.$$

Question 3

Answer: A

Explanation:

The mode is the most frequent score, so 2. The median of the 22 data points is mid-way between the 11th and 12th data points so 2.

Question 4

Answer: E

Explanation:

Enter the data, in minutes, into a calculator and request 1-Var Stats to get a mean of 163.6 and standard deviation of 129.9.

Question 5

Answer: B

Explanation:

The data are close to symmetric apart from the outlying value at 10 hours per week.

Question 6

Answer: E

Explanation:

$$\frac{17}{251 + 236 + 17} \times 100\% = 3.4\%$$

Question 7

Answer: B

Explanation:

The absence of the outlier would make the negative correlation even stronger, hence a decrease in Pearson's correlation coefficient.

Question 8

Answer: C

Explanation:

$$\frac{60.5 - 30}{2} = 15.25.$$

Question 9

Answer: D

Explanation:

Recall that residual = actual value – predicted value.

Question 10

Answer: A

Explanation:

Need to bring larger x -values in (i.e. make them smaller) so log transform will do this.

Question 11

Answer: B

Explanation:

The median of the final five data points is 155.

Question 12

Answer: C

Explanation:

The seasonal means are 118.7, 159.3, 243.3 and 149 for Summer, Autumn, Winter and Spring. Hence the seasonal index for Winter is in excess of 1 so C.

Question 13

Answer: A

Explanation:

23 months difference so $23 \times 1.1 = 25.3$.

Module 1: Number Patterns

Question 1

Answer: B

Explanation:

$$20 + 2 \times 9 = 38.$$

Question 2

Answer: A

Explanation:

$$t_{10} = 70 \times 0.7^{10} = 1.977.$$

Question 3

Answer: D

Explanation:

$$15 \times 1.05^7 = 21.107.$$

Question 4

Answer: E

Explanation:

$$131 - 2(n - 1) = 1 \Rightarrow n = 66. \text{ So } S_{66} = \frac{66}{2}(1 + 131) = 4,356.$$

Question 5

Answer: D

Explanation:

7% increase so multiply by 1.07 and kill 1000 so subtract 1000. Initially $K_0=10,000$.

Question 6

Answer: C

Explanation:

Use SEQ mode on calculator to get 2,461.

Question 7

Answer: A

Explanation:

Terms are increasing uniformly so $d > 0$ and it is arithmetic.

Question 8

Answer: A

Explanation:

$$t_{10} = t_9 + t_8 \Rightarrow t_9 = 130.$$

Question 9

Answer: D

Explanation:

$$S_{\infty} = \frac{a}{1-r}$$

$$\text{For } a = 1, S_{\infty} = 2, \frac{1}{1-r} = 2 \Rightarrow r = \frac{1}{2}$$

Module 2: Geometry and Trigonometry

Question 1

Answer: E

Explanation:

$$\sin \theta = \frac{10}{15} \Rightarrow \theta = 42^\circ.$$

Question 2

Answer: B

Explanation:

$$AD = \sqrt{244} \text{ and } AE^2 = 244 + 81 \Rightarrow AE = 18.$$

Question 3

Answer: A

Explanation:

$$5 \times (10000)^2 \text{ cm}^2 \Rightarrow 50000 \text{ m}^2 \text{ or } 5 \text{ hectares.}$$

Question 4

Answer: B

Explanation:

$$\text{Using Heron's formula } A = \sqrt{22.5(4.5)(5.5)(12.5)} = 83.4 \text{ cm}^2.$$

Question 5

Answer: A

Explanation:

$$\frac{CD}{8} = \frac{25}{10} \Rightarrow CD = 20.$$

Question 6*Answer:* E*Explanation:*

$$AB^2 = 10^2 + 12^2 - 2(10)(12)\cos 15^\circ \Rightarrow AB = 3.5.$$

Question 7*Answer:* E*Explanation:*

$$x = \sqrt{500^2 + 100^2} = 510.$$

Question 8*Answer:* A*Explanation:*

$$\tan(\hat{D}\hat{B}\hat{C}) = 0.3 \Rightarrow \hat{D}\hat{B}\hat{C} = 16.7^\circ \text{ and then } \tan 21.7^\circ = \frac{x+3}{10} \Rightarrow x = 0.979.$$

Question 9*Answer:* B*Explanation:*

$$\frac{AC}{\sin 40^\circ} = \frac{12}{\sin 75^\circ}, AC = 7.99 \text{ and } \frac{AD}{\sin 65^\circ} = \frac{12}{\sin 75^\circ}, AD = 11.259 \text{ and}$$

$$BC^2 = 12^2 + 21.259^2 - 2(12)(21.259)\cos 40^\circ, BC = 14.32 \text{ hence } \frac{\sin \theta}{7.99} = \frac{\sin 105^\circ}{14.32} \Rightarrow \theta = 33^\circ.$$

Module 3: Graphs and Relations

Question 1

Answer: C

Explanation:

$$0.5 + 3 \times 0.25 = \$1.25.$$

Question 2

Answer: B

Explanation:

Increase by 500 for each unit increase in t so B.

Question 3

Answer: A

Explanation:

$$\frac{5}{40} = 0.125.$$

Question 4

Answer: C

Explanation:

$$20x + 80y \leq 4000 \Leftrightarrow x + 4y \leq 200 \text{ so C.}$$

Question 5

Answer: A

Explanation:

$y = \frac{3}{x^2}$ can only return positive values and has branches with shape similar to the hyperbola so A.

Question 6

Answer: A

Explanation:

Total score is $a + b$. Difference of scores is $a - b$.

Question 7

Answer: D

Explanation:

$m = \frac{2}{-4} = -0.5$. Line has equation $y - 2 = -0.5(x - 5) \Rightarrow x + 2y = 9$.

Question 8

Answer: A

Explanation:

Gradient is 1 as is y-intercept, so A.

Question 9

Answer: A

Explanation:

Since positive contributions to profit come only from the y values we want to maximise the y contribution. This occurs at (0,8).

Module 4: Business-related mathematics

Question 1

Answer: B

Explanation:

$$\frac{880}{1.1} = 800$$

Question 2

Answer: B

Explanation:

$$640.5 = P + \frac{P(0.09)(9)}{12} \Rightarrow P = \$600.$$

Question 3

Answer: B

Explanation:

Divide 12% by 100 to remove % sign and by m since compounding m times per year. The number of periods is $3m$.

Question 4

Answer: D

Explanation:

$$\frac{5000}{0.1}$$

Question 5

Answer: A

Explanation:

No principal repaid and all interest paid so outstanding balance has not changed in the first five years.

Question 6

Answer: A

Explanation:

$$r_f = \frac{100(2100)}{13500} = 15.56\%.$$

Question 7

Answer: D

Explanation:

$$r_{eff} = \frac{100(2100)}{13500} \times \frac{24}{13} = 28.72\%.$$

Question 8

Answer: B

Explanation:

Anne has $2000(1.05)^8 = \$2,954.91$.

Bill has $2000(1.025)^{16} = \$2,969.01$.

Question 9

Answer: C

Explanation:

$1000(1.05)^2(1.08) = \$1,190.70$.

Module 5: Networks and decision mathematics

Question 1

Answer: E

Explanation:

$$v - e + f = 2.$$

Question 2

Answer: C

Explanation:

$$\frac{12 \times 11}{2} = 66.$$

Question 3

Answer: B

Explanation:

Definition of Hamiltonian path.

Question 4

Answer: D

Explanation:

By observation.

Question 5

Answer: E

Explanation:

Use Prim's algorithm. Choose shortest branch each time so as not to form a loop.

Question 6

Answer: C

Explanation:

Total number of edges at vertex D.

Question 7

Answer: C

Explanation:

Property of Euler paths.

Question 8

Answer: B

Explanation:

Draw a graph with 6 vertices all in a straight line with 5 connecting edges.

Question 9

Answer: C

Explanation:

Two-step dominance when a player beats another player and that defeated player beats a third player, so C.

Module 6: Matrices

Question 1

Answer: A

Explanation:

$$\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 3 & 3 \end{bmatrix}.$$

Question 2

Answer: B

Explanation:

Inverses of X and Y are not defined since neither X nor Y are square matrices so D and E are incorrect. You can't add multiples of X and Y since they are of different dimension so A is incorrect. Also C cannot be correct as you cannot add a matrix and a scalar term. By lining up dimensions only B can be correct.

Question 3

Answer: E

Explanation:

Need determinant of the matrix A to be zero for it to be singular.

Question 4

Answer: D

Explanation:

Multiply the given row vector by the given column vector in D to get the total revenue.

Question 5

Answer: B

Explanation:

$$B = A^{-1} \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}.$$

Question 6

Answer: C

Explanation:

$(XY)^{-1}$ is $n \times n$ and XZ is $n \times 1$.

Question 7

Answer: E

Explanation:

$$T = \text{given matrix. } S_0 = \begin{bmatrix} 0.8 \\ 0.2 \\ 0 \\ 0 \end{bmatrix} \text{ so } T^2 S_0 = \begin{bmatrix} 0.584 \\ 0.203 \\ 0.113 \\ 0.100 \end{bmatrix}.$$

Question 8

Answer: B

Explanation:

$$S_{2008} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}.$$

$$S_{2008} = T^{-1} S_{2009} \Rightarrow \text{B.}$$

Question 9

Answer: B

Explanation:

All companies that hit state IV won't escape so ultimately they will all be in that state.