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2009 Further Mathematics Trial Exam 1 Solutions

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SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	D	B	A	D	D	C	A	C	C	E	A

SECTION B

Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
E	A	D	A	C	C	E	E	C

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
D	E	D	A	C	C	B	E	D

Module 6: Matrices

1	2	3	4	5	6	7	8	9
C	A	C	D	E	D	A	D	C

SECTION A Core: Data analysis

Q1 The interval 160 -164 has the highest frequency. C

Q2 By inspection 164 cm cannot be the mean of the right side of the back-to-back stemplot. $\therefore$ the right side must be the data of male students. The mean is 157.8 cm. B

Q3 Mean height $= \frac{164\times 42 + 158\times 32}{74} = 161.4$ cm. D

Q4 A back-to-back stemplot is used to display the relationship between a numerical variable (height) and a two-valued categorical variable (gender). B

Q5 25% in the interval 15-23 mm, less than 25% in the interval 35-52 mm. A

Q6 Number of fish greater than 35 mm is 80. There are 4 outliers greater than 35 mm.

Percentage $= \frac{4}{80}\times 100\% = 5\%$. D

Q7 The mean is 250 and the standard deviation is about 50. The interval 200-400 is from $\mu-\sigma$ to $\mu+3\sigma$ approximately.

Percentage $\approx \frac{68\%}{2} + 50\% = 84\%$. D

Q8 105 is about 3σ below μ. $\therefore z \approx -3$. C

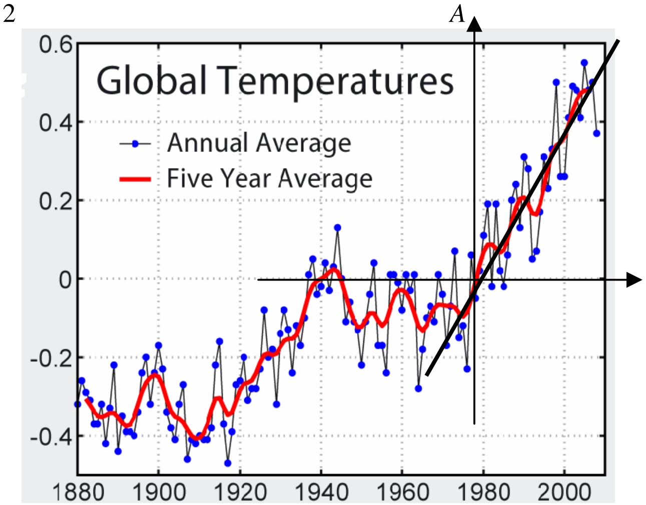
Q9 $\frac{s_Y}{s_X} = \frac{b}{r}$, $r = 0.80$ (given), $b = 1.2$ (gradient).

$\frac{s_{(\log y)}}{s_x} = \frac{1.2}{0.80} = 1.5$. A

Q10 C

Q11 Sum $\approx -0.05 + 0.01 + 0.11 + 0.19 - 0.02 + 0.19 = 0.43$. C

Q12



Gradient of trend line ≈ 0.019, vertical axis intercept ≈ -0.04. E

Q13 A

SECTION B

Module 1: Number patterns and applications

Q1 There is no common ratio in each of the four series. E

Q2 Consider 71.5 and 67 as the first and second terms of the sequence. $\therefore a = 71.5$, $d = 67 - 71.5 = -4.5$.

$S_{108} = \frac{108}{2}\left(2\times 71.5 + (108-1)\times {}^{-}4.5\right) = -18279$.

The sum of the middle 54 terms $= \frac{1}{2}\times {}^{-}18279 = {}^{-}9139.5$. A

Q3 Common difference: $\frac{1}{a} - \frac{1}{2} = \frac{1}{4} - \frac{1}{a}$, $\therefore \frac{2}{a} = \frac{3}{4}$,

$\therefore a = \frac{8}{3}$. D

Q4 By inspection, $t_{n+1} = t_n + 4n$. A

Q5 Given $t_4 = 25$, $t_5 = 25 + 4\times 4 = 41$, ……….., $t_9 = 145$, $t_{10} = 145 + 4\times 9 = 181$. The tenth pattern has 181 ten-cent coins valued at $18.10. C

Note: The choices in the question have the decimal point incorrectly placed. A. 14.50, B. 15.30, C. 18.10, D. 22.10, E. 22.50.

Q6 $11001.10011001...... = 10000 + 1001.10011001......$

$= 10000 + 1001 + 0.1001 + 0.00001001 + 0.000000001001 + ...$

$= 10000 + \frac{1001}{1 - 0.0001}$. $\therefore a = 1001$. C

Q7 Common ratio $= \frac{t_2}{t_1} = \frac{-30}{45} = -\frac{2}{3}$, $\therefore \frac{t_{n+1}}{t_n} = -\frac{2}{3}$,
$\therefore 3t_{n+1} + 2t_n = 0$. E

Q8 Use $u_{n+1} = a + bu_n$ and 20 and 10 to set up equation (1) $10 = a + 20b$. Use $u_{n+1} = a + bu_n$ and 10 and 25 to set up equation (2) $25 = a + 10b$.
Solve the 2 equations simultaneously to obtain $a = 40$ and $b = -1.5$. E

Q9 Third bounce distance $= 98$ cm.
Third fall distance $= 98 + 5 = 103$ cm.
Total distance for the third fall and bounce $= 103 + 98 = 201$ cm.
Total distance for the second fall and bounce $= 108 + 103 = 211$ cm.
Total distance for the first fall and bounce $= 113 + 108 = 221$ cm.
$a = 221$, $d = -10$, $S_{18} = \frac{18}{2}\left(2\times 221 + (18-1)\times{}^{-}10\right) = 2448$. C

Module 5: Networks and decision mathematics

Q1 D

Q2 E

Q3 Euler's formula: $v - e + f = 2$. f is an even number.
$\therefore$ either both v and e are even numbers or both v and e are odd numbers. D

Q4 A

Q5 An Euler path is a path that includes every edge just once. A Hamiltonian path is a path that passes through each vertex just once, and the starting vertex is different from the finishing vertex. C

Q6 A spanning tree is a subgraph and a tree containing all the vertices of the graph. There are 8 vertices in the graph. $\therefore$ exactly 7 edges. C

Q7 Maximum flow from P to Q = minimum cut = 60.
Maximum flow from R to Q $= 20 + 20 + 10 = 50$.
Minimum flow from R to Q $= 20 + 20 + 0 = 40$.
Difference $= 50 - 40 = 10$. B

Q8 E

Q9 One-step dominance matrix:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Two-step dominanace matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \end{bmatrix}$$

Sum of missing entries $= 3 + 0 = 3$. D

Module 6: Matrices

Q1 The order of product $X\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ must be the same as that of $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix}$, i.e. 2×3. $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ has an order of 4×3, $\therefore X$ must have an order of 2×4. C

Q2 $\begin{bmatrix} a & 2 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} -2a+2 & a+2b \\ 0 & \frac{1}{2}+b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
$\therefore -2a + 2 = 1$ and $\frac{1}{2} + b = 1$.
$\therefore a = \frac{1}{2}$ and $b = \frac{1}{2}$. A

Q3 $\begin{bmatrix} -1 & -2 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{3}{5} & -\frac{1}{5} \end{bmatrix}$. $\therefore X = \begin{bmatrix} -6 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{3}{5} & -\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & -1 \end{bmatrix}$. C

Q4 D

Q5 A transition matrix (i) is a square matrix, (ii) has no negative elements, and (iii) the sum of elements in each column must be 1. E

Q6 D

Q7 $\begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 1 \\ 1 & -5 & -3 \\ 1 & 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0.6 & 1.4 \end{bmatrix}$. A

Q8 Apply the transition matrix repeatedly:
$\begin{bmatrix} 3500 \\ 2800 \\ 3800 \\ 2500 \end{bmatrix} \to \begin{bmatrix} \\ 2966 \\ \\ \\ \end{bmatrix} \to \begin{bmatrix} \\ 3095 \\ \\ \\ \end{bmatrix} \to \ldots\ldots$.

% change in the first transition $\frac{2966 - 2800}{2800} \times 100\% = 5.9\%$.
% change in the next transition $\frac{3095 - 2966}{2966} \times 100\% = 4.3\%$.
D

Q9 For large enough n, $\begin{bmatrix} 0.88 & 0.02 & 0.04 & 0.01 \\ 0.05 & 0.92 & 0.05 & 0.01 \\ 0.03 & 0.03 & 0.85 & 0.01 \\ 0.04 & 0.03 & 0.06 & 0.97 \end{bmatrix}^n \begin{bmatrix} 3500 \\ 2800 \\ 3800 \\ 2500 \end{bmatrix} \to \begin{bmatrix} \\ \\ \\ 7193 \end{bmatrix}$.

% of 12600 customers $\frac{7193}{12600} \times 100\% \approx 57\%$. C

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors