THE HEFFERNAN GROUP
P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
Fax 03 9836 5025
info@theheffernangroup.com.au
www.theheffernangroup.com.au

Student Name…………………………………

FURTHER MATHEMATICS

TRIAL EXAMINATION 2

2009

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains a set of extended answer questions from the core, 'Data Analysis'.
Section A is compulsory and is worth 15 marks.
Section B begins on page 7 and consists of 6 modules. You should choose 3 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 15 marks.
Section B is worth 45 marks.
There is a total of 60 marks available for this exam.
The marks allocated to each of the questions are indicated throughout.
Students may bring one bound reference into the exam.
An approved graphics or CAS calculator may be used in the exam.
Formula sheets can be found on pages 35 and 36 of this exam.
Unless otherwise stated the diagrams in this exam are not drawn to scale.

Section A

Core

This section is compulsory.

Question 1

A persons body fat percentage is the weight of their body fat expressed as a percentage of their total body weight.

The body fat percentage (BFP) of a sample of 15 teenage boys is given below.

			%				
9	15	16	23	16	10	24	
16	15	21	12	17	13	17	19

Teenage boys with a BFP of 9% - 15% (inclusive) are considered healthy.

a. What is the percentage of teenage boys in this sample who are considered healthy?

1 mark

b. For this set of data, what is the

i. mode

ii. mean

$1+1=2$ marks

The standard deviation of the BFP for this sample is 4.3 (to 1 decimal place).
One of the boys in the sample has a BFP of 19.

c. Calculate the standardized BFP (*z* score) of this boy relative to the sample. Express your answer correct to one decimal place.

1 mark

Question 2

The stemplot below shows the body fat percentage (BFP) for a sample of teenage girls.

0	9						
1	2	3	3	4	4	4	4
1	6	7	7	8			
2	0	0	1				
2	6						
3	1						
3							
4	0						

a. What is the median?

1 mark

b. Give two reasons why the median is a better measure of the centre of this distribution than the mean.

2 marks

Question 3

The body fat percentage (BFP) was calculated for a sample of 52 women and 43 men. These men and women were then classified as having an acceptable or unacceptable BFP. A total of 12 women and 19 men in this sample were found to have an unacceptable (BFP).

a. Complete the two-way frequency table below using this information.

	Gender	
BFP	female	male
acceptable		
unacceptable		
total		

2 marks

The researchers conducting this investigation had an hypothesis that body fat percentage is related to gender.

b. State whether or not the data in the two-way frequency table in part **a.** supports this hypothesis. Give appropriate percentages to support your answer.

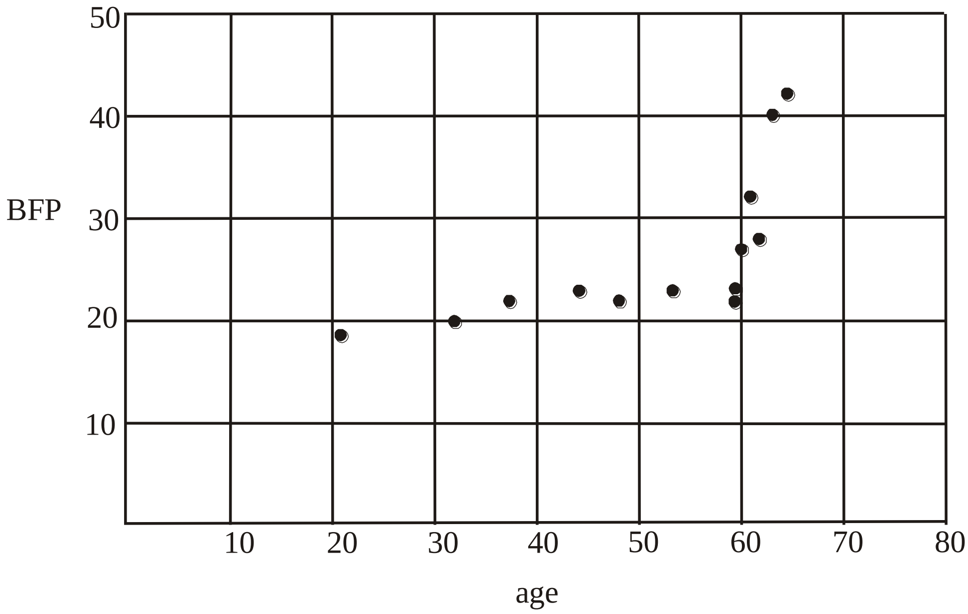
2 marks

Question 4

The age in years and the body fat percentage (BFP) for a group of women is given in the table below.

Age (years)	Body fat percentage (%)
21	18
59	23
32	20
59	22
37	22
44	23
48	22
53	23
60	27
63	40
62	28
61	32
64	42

The results are displayed on the scatterplot below.



a. Explain why it is not appropriate to determine an equation for a least squares regression line for the variables age and BFP.

1 mark

A transformation is to be applied to the data so that a least squares regression line can be fitted.

The variable “BFP” is to be replaced with the variable “log(BFP)”.

b. What other type of transformation could have been used?

1 mark

c. Find the equation of the least squares regression line linking the variables “age” and “log (BFP)”. Express coefficients correct to 4 decimal places.

2 marks

Total 15 marks

SECTION B

Module 1: Number patterns

If you choose this module all questions must be answered.

Question 1

On a large timber plantation, a block of 96 hectares of trees is about to be logged.
Six hectares are to be logged each week.

a. How many hectares of trees will be left on this block 3 weeks after logging commences?

1 mark

b. How many weeks will it take to completely log all the trees on this block?

1 mark

c. The number of hectares of trees left on this block 10 weeks after logging commences is given by T_{10} where

$$T_{10} = 96 + 10 \times m$$

What is the value of *m*?

1 mark

A difference equation that gives us the number of hectares of trees; T_n, left on the block *n* weeks after logging commences is given by

$$T_n = T_{n-1} - c, \qquad T_0 = 96$$

d. What is the value of *c*?

1 mark

Question 2

Logging also begins on a different block on the south side of the plantation. On this south block, 80 hectares of trees remained at the end of the first week of logging, 76 hectares remained at the end of the second week and 72.2 hectares remained at the end of the third week.
The number of hectares of trees that remained on this south block at the end of each week during logging follows a geometric progression.

a. Show that the common ratio, r, is equal to 0.95.

1 mark

b. How many hectares of trees remained at the end of the sixth week? Express your answer correct to two decimal places.

1 mark

c. How many more hectares of trees will be left at the end of the tenth week than at the end of the eleventh week after logging commences? Express your answer correct to two decimal places.

1 mark

d. How many weeks of logging will it take for the number of hectares of trees remaining on this south block to first drop below 10 hectares?

1 mark

e. Write an expression for the number of hectares; S_n, of trees remaining on this south block n weeks after logging begins.

1 mark

Question 3

In an experiment on a block on the west side of the plantation, logging and replanting of trees is begun during the same week.
The number of hectares, W, of trees (old and new) that remain at the end of the *n*th week after the logging/replanting process begins is given by the difference equation

$$W_n = 0 \cdot 97 W_{n-1} + 2 \cdot 5, \qquad W_0 = 120$$

a. How many hectares of trees (both old and new) are there on this west block 3 weeks after the logging/replanting process begins? Express your answer correct to two decimal places.

1 mark

b. How many hectares of trees are being replanted each week?

1 mark

c. How many hectares of trees were logged in the third week after the logging/replanting process started? Express your answer correct to two decimal places.

1 mark

Question 4

On a block on the east side of the plantation, the number of hectares of trees remaining *n* weeks after logging begins is given by the difference equation

$$E_n = 0 \cdot 94E_{n-1}, \quad E_0 = 285$$

a. What type of sequence is defined by this difference equation?

1 mark

Assume that the logging/replanting process described in **Question 3** on the west side starts the same week as the logging described in this question on the east side.

b. After how many weeks will the number of hectares of trees on the east side block and the west side block be equal (to one decimal place). State the number of hectares on each side at this time.

2 marks

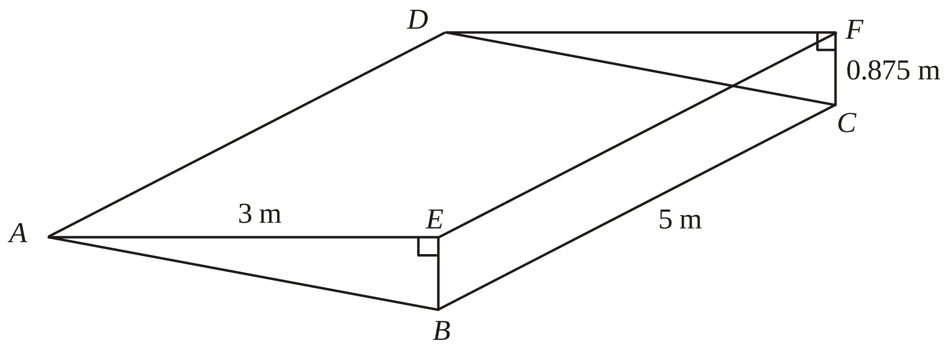
Total 15 marks

Module 2: Geometry and trigonometry

If you choose this module all questions must be answered.

Question 1

A reinforced concrete base is constructed on sloping land. The base is a right triangular prism as shown in the diagram below.



The rectangle *ABCD* is in contact with the ground and the rectangle *AEFD* is a horizontal surface.
The distance *AE* is 3m, *BC* is 5m and *CF* is 0.875m.

a. Show that the length of *AB* is 3.125m.

1 mark

b. Find the angle *BAE*. Express your answer in degrees correct to 1 decimal place.

1 mark

A piece of reinforcing steel runs from point *A* to *C*.

c. Find the length of this piece of steel. Express your answer in metres correct to 4 decimal places.

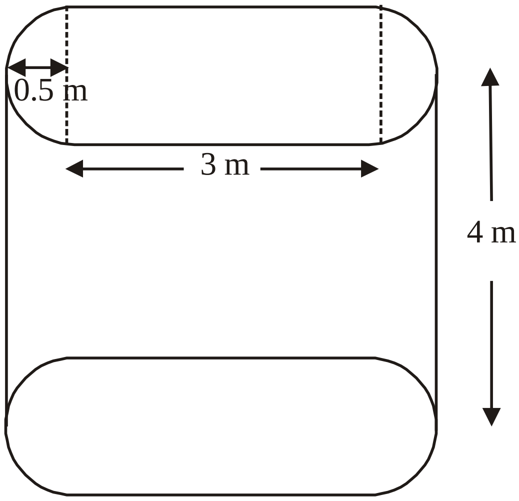
1 mark

d. Find the angle *CAF*. Express your answer in degrees correct to 2 decimal places.

1 mark

Question 2

One of the items to sit on the base is a water tank.
The tank is a prism with a cross-sectional area made up of a rectangle with a semi-circle at each end.
The tank is shown below.



The height of the water tank is 4m and the length is 4m. The radius of each semi-circular cross-section is 0.5m and the length of the rectangular cross-section is 3m as indicated in the diagram.

a. Find the surface area of the top of the tank. Express your answer in square metres correct to 4 decimal places.

1 mark

b. What is the volume of the tank? Express your answer in cubic metres correct to 2 decimal places.

1 mark

c. One cubic metre can hold 1000 litres of water. Use your answer to part **b.** to find out how many litres of water this tank can hold.

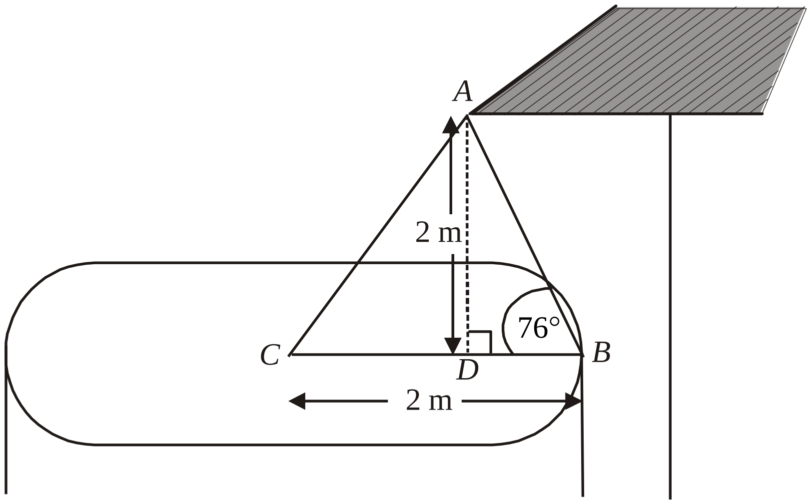
1 mark

d. Find the total surface area (including the base) of the outside of the water tank. Express your answer in square metres correct to 2 decimal places.

2 marks

Question 3

Water enters the tank from a pipe at the edge of an eave of a nearby building.
The pipe *AC*, together with the tank and the building are shown in the diagram below.



The top of the tank has its centre at *C*. The point *B* lies on the edge of the tank and the distance *BC* is 2 metres. The point *D* lies on *BC*, 2 metres vertically below *A*. The angle, *ABC* is 76°.

a. Find the length *AB*. Express your answer in metres correct to 4 decimal places.

1 mark

b. Use the cosine rule to find the length of the pipe from *A* to *C*. Express your answer in metres correct to 1 decimal place.

1 mark

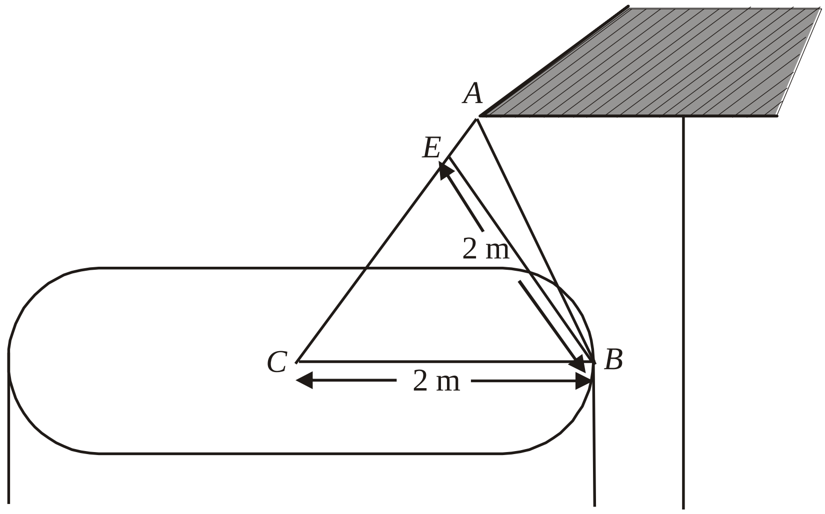
c. Find the angle *ACB*. Express your answer correct to the nearest degree.

1 mark

d. Find the area of triangle *ABC* using Heron's formula. Express your answer to the nearest square metre.

1 mark

A straight metal strut of length 2 metres is attached to the tank at *B* and to the pipe at *E* to give the pipe some extra support. The diagram below shows the metal strut indicated by *BE*.



e. Find the distance *CE*. Express your answer in metres correct to one decimal place.

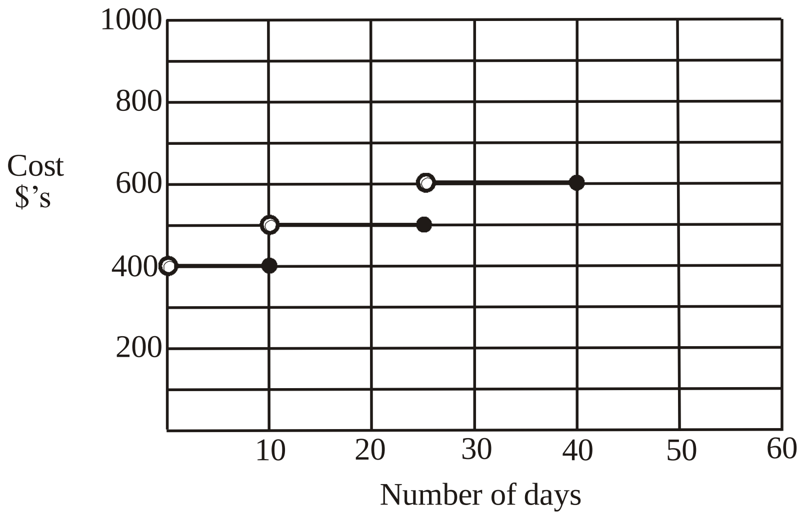
2 marks
Total 15 marks

Module 3: Graphs and relations

If you choose this module all questions must be answered.

Question 1

Kate runs a furniture hire business. She hires out furniture to people who are selling their home and wish to improve its look. She offers different combinations of furniture and decorative items.
The costs of one of her most popular combinations for periods up to 40 days are shown on the graph below.



a. What is the cost of hiring this combination for 20 days?

1 mark

The cost of hiring this combination for more than 40 days up to and including 60 days is $700.

b. On the graph above, draw this information.

1 mark

Kate has noticed that a very high percentage of furniture is hired for 10, 25 or 40 days.

c. Explain why people might do this.

1 mark

Question 2

Kate estimates that across her business, in the coming year, the average revenue she will receive per client will be $1200.

a. Write an equation that gives the total revenue, R, in dollars, forecast for the coming year when Kate has x clients.

1 mark

Kate also estimates that in the coming year the cost of running her business will be $80 000 plus an average of $400 per client.

b. Write an equation that gives the total cost, C, in dollars, of running the business in the coming year in terms of the number of clients x.

1 mark

c. How many clients will Kate need in the coming year for her business to break even?

1 mark

d. If Kate has 60 clients in the coming year state whether her business makes a profit or a loss and find that profit or loss.

1 mark

Question 3

Kate offers two very popular furniture hiring packages; the luxury package and the standard package.
Let x be the number of luxury packages Kate hires out in a month.
Let y be the number of standard packages Kate hires out in a month.

There are constraints on the number of these packages she can hire out in a month due to furniture availability, staff availability and time availability.
The inequalities below define these constraints.

Constraint 1: $x \geq 0$
Constraint 2: $y \geq 0$
Constraint 3: $2x + 5y \leq 75$ (staff availability)
Constraint 4: $x + y \leq 21$ (time availability)
Constraint 5: $x \leq 15$ (furniture availability)

a. Explain whether or not there is any constraint on the amount of furniture available for standard packages.

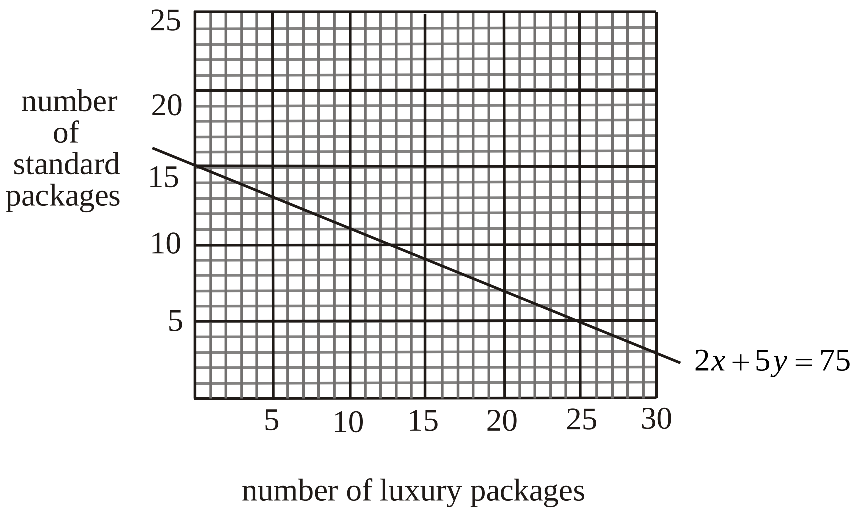
____

____

____

1 mark

The line $2x + 5y = 75$ is drawn on the graph below.



b. On the graph above, sketch and label the line $x + y = 21$.

1 mark

c. On the graph above, sketch and label the line $x = 15$.

1 mark

d. On the graph above, shade the feasible region described by the five constraints.

1 mark

A profit of $800 is made on a luxury package and a profit of $500 is made on a standard package.

e. Write an objective function for the profit P in terms of x and y.

1 mark

f. Find the maximum profit that can be made on these two packages in a month.

1 mark

The constraint associated with the furniture availability is changed to $x \leq 10$.

g. Which one of the six constraints originally given now has no effect on the feasible region? Explain your answer.

1 mark

h. What is the maximum profit now possible on these packages for the month?

1 mark
Total 15 marks

Module 4: Business-related mathematics

If you choose this module all questions must be answered.

Question 1

John runs a green grocery business. The bank statement for one of his business accounts for April is shown below.

Date	Description of Transaction	Withdrawals	Deposits	Balance
01 April	Opening Balance			15 246.28
03 April	Transfer from acc 239782		20 000	35 246.28
15 April	Withdrawal – direct debit			34 618.28
24 April	Deposit – cheque		14 382.61	49 000.89
30 April	Closing Balance			49 000.89

a. How much money was withdrawn from the account on 15 April?

1 mark

For this business account, interest is paid on the minimum monthly balance at the rate of 2.4% per annum.

b. How much interest is paid to this account for April? Express your answer to the nearest cent.

2 marks

The $20 000 that John transferred on 3rd April had been previously in a term deposit account earning annual interest of 4% per annum compounding quarterly.

c. How much interest would John have earned on this investment over a three year period?

2 marks

Question 2

John purchases a fork lift for his business for $20 000.

a. A Goods and Services Tax (GST) of 10% was included in this purchase price. How much GST did John pay?

1 mark

b. John paid cash for his fork lift and therefore paid a discounted price. The original price was $22 000. What was the percentage discount that John received for having paid cash? Express your answer as a percentage correct to 2 decimal places.

1 mark

Question 3

The fork lift that was purchased for $20 000 is to be depreciated over time for taxation purposes.
Three methods of depreciation can be used.

Unit Cost Depreciation

The fork lift is depreciated at the rate of 5 cents per lift.

a. How many lifts will the fork lift have performed before the book value is reduced to $12 000?

1 mark

Flat Rate Depreciation

The fork lift is depreciated by 7% of its purchase price each year.

b. What is the book value of the fork lift after 5 years?

1 mark

Reducing Balance Depreciation

The value of the fork lift depreciates at the rate of 8% per annum.

c. By how much will the fork lift depreciate during its fourth year of operation?

2 marks

Question 4

John needs to increase the size of his cool room. He takes out a reducing balance loan of \$80 000 with interest calculated quarterly at the rate of 6% per annum.
John wants to fully repay the loan in 3 years.

a. What will John's quarterly repayments be?

1 mark

b. How much will John pay in interest over the course of this loan?

1 mark

Two years after the loan was taken out John has to change his quarterly repayments to \$5000.
The interest rate remains at 6% per annum.

c. How much longer does it take John to repay the loan? Express your answer to the nearest quarter.

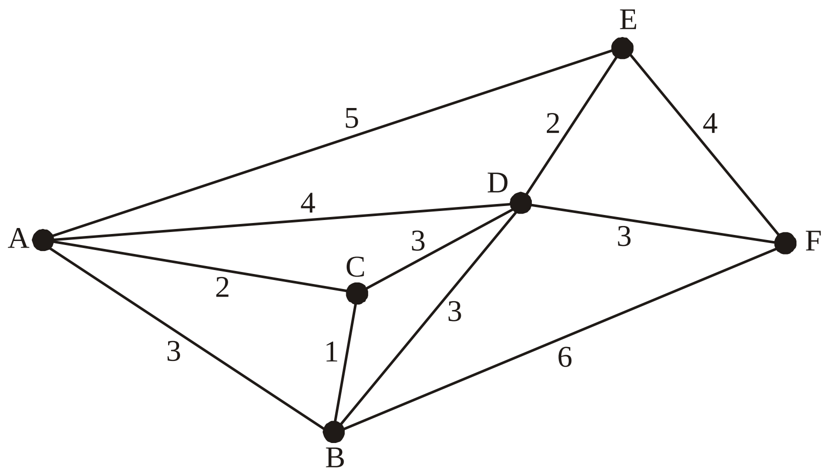
2 marks
Total 15 marks

Module 5: Networks and decision mathematics

If you choose this module all questions must be answered.

Question 1

A group of six islands *A* – *F*, together with the distances in kilometres between them, are shown on the graph below.



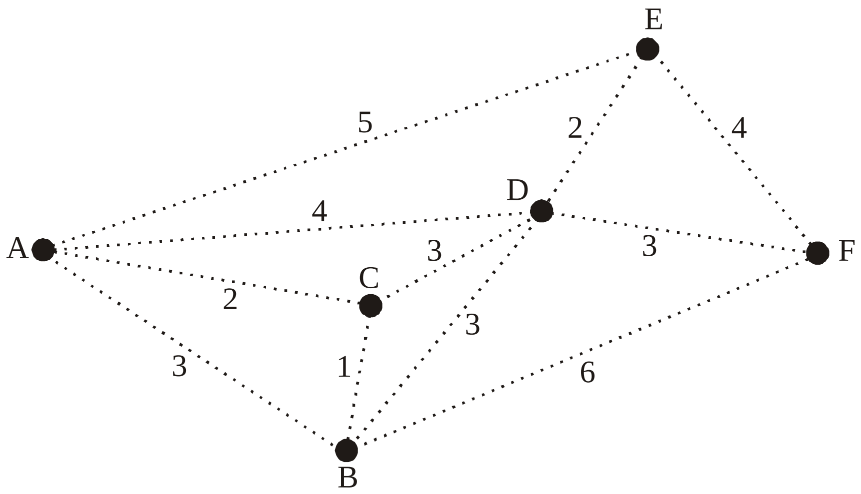
a. Find the shortest distance from *A* to *F*.

1 mark

b. Explain why the path *E A B F E D C* does not form a tree.

1 mark

c. **i.** On the graph below representing the group of islands, draw a minimal spanning tree.



1 mark

ii. How many different minimal spanning trees exist for this graph?

____

1 mark

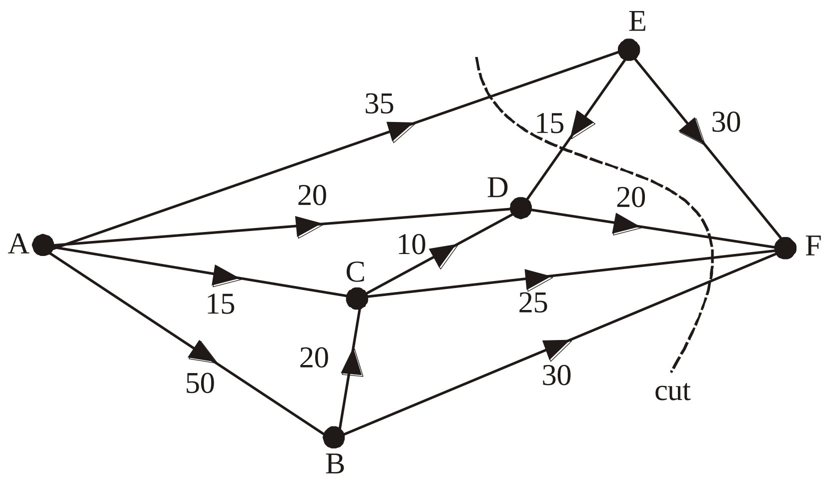
iii. Communication cable is to be laid between the islands. What is the minimum length of cable that has to be laid so that all the islands can be connected?

____

1 mark

Question 2

During the week water taxis provide transport for people travelling between the islands. The number of people who can be transported in a day from one island to another in a particular direction is indicated on the edges of the directed graph below.



A cut is made through the network.

a. What is the capacity of this cut?

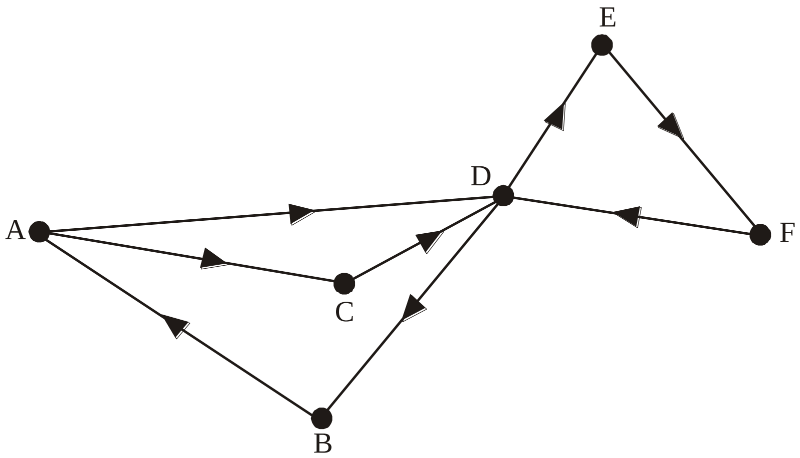
1 mark

b. What is the maximum number of people who can be transported by water taxi from island *A* to island *F* on a weekday?

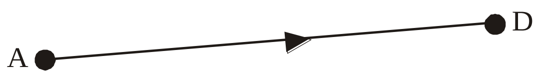
1 mark

Question 3

The directed graph below shows the reachability of the islands by commercial ferries on a Sunday.



A directed line such as



means that island *D* can be reached from island *A* by commercial ferry on a Sunday.

a. Which island is considered most reachable when considering just one-step reachability?

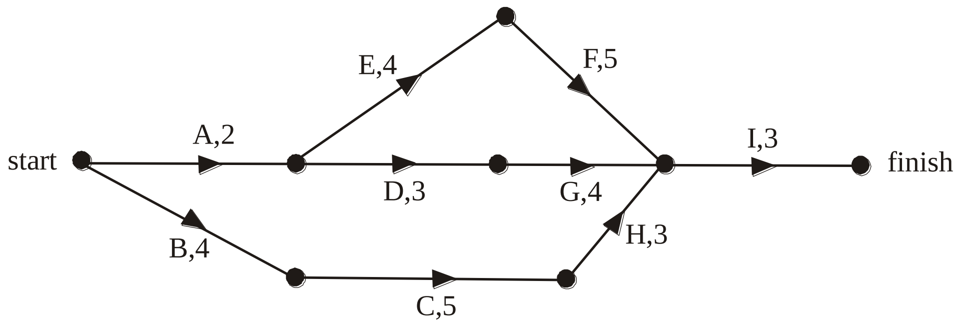
1 mark

b. By adding together the one-step and two-step reachability for each island, rank the top three islands in order of reachability.

2 marks

Question 4

An upgrade is being carried out on the wharf on island *E*. There are 9 activities that need to be completed. The directed network below shows these 9 activities, *A* – *I*, and the time, in days, required to complete each one.



a. What is the shortest time in which these improvements can be completed?

1 mark

b. Which two activities have the greatest slack time?

1 mark

c. What is the latest start time for activity *E*?

1 mark

Activity *C* is to be reduced by 3 days.

d. Which 3 activities have now become critical to the upgrade being completed in the shortest possible time?

1 mark

e. Explain whether it was worth reducing activity *C* by 3 days given that it cost money each day to do so. Use completion times for the project to justify your opinion.

1 mark
Total 15 marks

Module 6: Matrices

If you choose this module all questions must be answered.

Question 1

At a basketball club, junior players can play in the Under 8, 10, 12 and 14 age groups. The matrix *J* below shows the number of players at each of these age groups.

$$J = \begin{bmatrix} 100 \\ 140 \\ 120 \\ 80 \end{bmatrix} \begin{matrix} \text{U/8} \\ \text{U/10} \\ \text{U/12} \\ \text{U/14} \end{matrix}$$

a. What is the order of matrix *J* ?

1 mark

The proportion of junior players playing in division 1 competition and division 2 competition is given by the matrix *D* where

$$\begin{matrix} & \text{division 1} & \text{division 2} \\ D = & [0 \cdot 55 & 0 \cdot 45] \end{matrix}$$

b. Evaluate the matrix *JD*.

1 mark

c. If $X = JD$, then explain what the element $X_{3,2}$ describes.

1 mark

Question 2

The club runs a coaching program for U/16, U/18 and U/20 players. The cost in dollars to a player to participate in the programme for a season is *b* for beginner level, *d* for development level and *r* for representative level.

The number of players at the three different age groups involved in the three different coaching levels is given by the matrix *A*.

$$\begin{matrix} & \text{beginner} & \text{development} & \text{rep} & \\ A = & \left[\begin{matrix} 25 \\ 15 \\ 12 \end{matrix}\right. & \begin{matrix} 20 \\ 19 \\ 17 \end{matrix} & \left.\begin{matrix} 20 \\ 23 \\ 21 \end{matrix}\right] & \begin{matrix} \text{U/16} \\ \text{U/18} \\ \text{U/20} \end{matrix} \end{matrix}$$

a. Find the determinant of matrix *A*.

1 mark

b. Explain why matrix *A* has an inverse.

1 mark

c. Find matrix A^{-1}, the inverse of *A*.

1 mark

d. Use your answer to part **c.** to find the values of *b*, *d* and *r* in the matrix equation below.

$$AX = B \qquad \text{where } X = \begin{bmatrix} b \\ d \\ r \end{bmatrix} \text{ and } B = \begin{bmatrix} 5625 \\ 5255 \\ 4660 \end{bmatrix}$$

1 mark

Question 3

The profit, in dollars, made by the club on the sale of each new and each secondhand uniform sold in 2009 is given by the matrix

$$P_{2009} = \begin{bmatrix} 15 \\ 10 \end{bmatrix} \begin{matrix} \text{new} \\ \text{secondhand} \end{matrix}$$

The uniform manager believes that the profit in 2010 can be predicted using the matrix equation

$$P_{2010} = QP_{2009} + R \qquad \text{where } Q = \begin{bmatrix} 0 \cdot 92 & 0 \\ 0 & 0 \cdot 85 \end{bmatrix} \text{and } R = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

a. Find the profit predicted to be made by the club on the sale of a new uniform and of a secondhand uniform in 2010.

1 mark

b. If the profit equation is generalized to

$$P_{n+1} = QP_n + R,$$

find the profit predicted to be made by the club on the sale of a new and of a secondhand uniform in 2012.

2 marks

Question 4

In order to attract more coaches to junior teams, the club offers payment to U/16 – U/18 representative (rep.) players to coach junior teams.
At the end of each season these rep. players make a decision to coach, to not coach or to coach jointly with another rep. player.
The club recorded the coaching pattern of these rep. players over time and developed the following transition matrix *T*.

$$\begin{array}{c} \text{this season} \\ \begin{array}{ccc} \text{coach} & \begin{array}{c}\text{didn't}\\\text{coach}\end{array} & \begin{array}{c}\text{joint}\\\text{coach}\end{array} \end{array} \end{array}$$

$$T = \begin{bmatrix} 0\cdot 82 & 0\cdot 79 & 0\cdot 84 \\ 0\cdot 06 & 0\cdot 03 & 0\cdot 01 \\ 0\cdot 12 & 0\cdot 18 & 0\cdot 15 \end{bmatrix} \begin{array}{l} \text{coach} \\ \text{didn't coach} \quad \text{next season} \\ \text{joint coach} \end{array}$$

a. What proportion of rep. players who didn't coach one season decide to jointly coach in the following season?

1 mark

In 2008 there were 64 rep. players eligible to coach and the club permanently maintains this number. Of these, 46 coached, 6 didn't coach and 12 coached jointly.

b. Find the number of rep. players who won't be coaching in the 2010 season.

2 marks

The club decided that if they could maintain more than 75% of rep. players coaching (on their own as opposed to coaching jointly with another rep. player), then their payment scheme to rep. players would be considered a success.

c. Find a steady state matrix and hence state whether the club believes their payment scheme has been a success.

2 marks
Total 15 marks

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a+d) + ... + (a + (n-1)d) = \dfrac{n}{2}[2a + (n-1)d] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + ... + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}$, $r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + ... = \dfrac{a}{1-r}$, $|r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a+b+c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base $\times$ height

volume of a pyramid: $\dfrac{1}{3}$ area of base $\times$ height

Pythagoras' theorem $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

THE HEFFERNAN GROUP
P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
Fax 03 9836 5025
info@theheffernangroup.com.au
www.theheffernangroup.com.au

FURTHER MATHEMATICS TRIAL EXAMINATION 2 SOLUTIONS 2009

SECTION A

Core - solutions

Question 1

a. 6 boys are in the range 9% - 15%.

$\left(\frac{6}{15} \times \frac{100}{1}\right)\% = 40\%$

So 40% of boys in the sample are considered healthy.

(1 mark)

b. **i.** The mode is the most popular (frequently occurring) piece of data.
The mode is 16.

(1 mark)

ii. The mean is 16.2.

(1 mark)

c. $\text{Mean} = 16 \cdot 2$
$\text{standard deviation} = 4 \cdot 3$

$$z = \frac{19 - 16 \cdot 2}{4 \cdot 3}$$
$$= 0 \cdot 7 \text{ (correct to 1 decimal place)}$$

(1 mark)

Question 2

a. The median is the middle score.
It is in the $\left(\frac{n+1}{2}\right)$th position.
Since $n = 18$ it is in the 9.5^{th} position i.e. between the 9^{th} and 10^{th} scores. So the median is between 16 and 17.
The median is 16.5.

(1 mark)

b. The median is a better measure of the centre of the distribution than the mean because

1. the distribution is positively skewed.

(1 mark)

2. there are two outliers (the scores 31 and 40).

(1 mark)

Question 3

a.

BFP	Gender	
	female	male
acceptable	40	24
unacceptable	12	19
total	52	43

(2 marks)

b. Yes the data supports the hypothesis because $\left(\frac{19}{43}\times 100\right)\% = 44.2\%$ of men have an unacceptable BFP compared with $\left(\frac{12}{52}\times 100\right)\% = 23.1\%$ of women. If there was no relationship between BFP and gender we would expect those percentages to be similar.

(1 mark) quoting correct %'s
(1 mark) quoting % in correct context

Question 4

a. It is not appropriate because from the scatterplot we see that the relationship between age and BFP is not linear.

(1 mark)

b. There are two possibilities.
The variable "age" could be replaced with "(age) ^{2}" or the variable "BFP" could be replaced with "$\frac{1}{BFP}$".

(1 mark)

c. $\log(BFP) = 0\cdot 0061 \times \text{age} + 1\cdot 0947$

(1 mark) correct coefficient of age
(1 mark) correct equation including 1.0947
Total 15 marks

SECTION B

Module 1: Number patterns

Question 1

a. Three weeks after logging commences there will be $96-3\times 6=78$ hectares of trees left.

(1 mark)

b. $96\div 6=16$
It will take 16 weeks.

(1 mark)

c. $m=-6$
Check.
$T_{10}=96+10\times -6$
$=36$
There are 36 hectares left after 10 weeks since 60 hectares have been logged during that time.

(1 mark)

d. $c=6$ since 6 hectares are subtracted from last week's total T_{n-1} to find this week's total T_n that is left.

(1 mark)

Question 2

a. $r=\frac{76}{80}=\frac{72\cdot 2}{76}$
$=0\cdot 95$

(1 mark)

b. The geometric sequence is 80, 76,72.2, 68.59, 65.1605, 61.902475,…
At the end of the sixth week there were 61.90 hectares of timber left.

(1 mark)

c. At the end of the 10th week there will be 50.41995 hectares left and at the end of the 11th week there will be 47.89895 hectares left. So there are $50\cdot 41995-47\cdot 89895=2\cdot 52$ (correct to 2 decimal places) left.

(1 mark)

Use your calculator to generate the sequence up to the 11th term.

d. Again use your calculator to generate the sequence. It takes 42 weeks of logging until the number of hectares of timber remaining first drops below 10.

(1 mark)

e. $S_n=80\times(0\cdot 95)^{n-1}$

(1 mark)

Question 3

a. $W_n = 0\cdot 97W_{n-1} + 2\cdot 5, \quad W_0 = 120$

$W_1 = 118\cdot 9$

$W_2 = 117\cdot 833$

$W_3 = 116\cdot 79801$

So there are 116.80 hectares of timber (old and new) on the block 3 weeks after the logging/replanting process begins.

(1 mark)

b. From the difference equation

$W_n = 0\cdot 97W_{n-1} + 2\cdot 5, \quad W_0 = 120$

we see that 2.5 hectares of trees are being replanted each week.

(1 mark)

c. $W_0 = 120$

$W_1 = 118\cdot 9$

$W_2 = 117\cdot 833$

At the end of the second week there were 117.833 hectares of trees (old and new) remaining. In the third week, the number of hectares of trees that remained before any more were planted was $0\cdot 97 \times 117\cdot 833 = 114\cdot 29801$.

So $117\cdot 833 - 114\cdot 29801 = 3\cdot 53499$ or $3\cdot 53$ hectares (correct to 2 decimal places) of trees were logged in the third week.

(1 mark)

Question 4

a. $E_n = 0\cdot 94E_{n-1}, E_0 = 285$

This difference equation describes a geometric sequence because each term is obtained by multiplying the previous term by the common ratio 0.94.

(1 mark)

b. $E_n = 0\cdot 94E_{n-1}, \quad E_0 = 285$

$W_n = 0\cdot 97W_{n-1} + 2\cdot 5, \quad W_0 = 120$

Use a calculator to generate the sequences defined by these difference equations. We see from the table of terms of each of the sequences that at the end of the 16th week of logging the number of hectares of trees remaining is equal (correct to 1 decimal place). There are 105.9 hectares remaining.

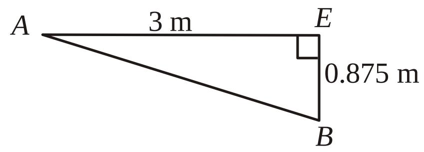
(1 mark) – for 16th week
(1 mark) – for 105.9 hectares
Total 15 marks

Module 2: Geometry and trigonometry

Question 1

a. In ΔABE,

$$(AB)^2 = 3^2 + 0\cdot875^2$$
$$AB = \sqrt{3^2 + 0.875^2}$$
$$= 3\cdot125\text{m}$$



(1 mark)

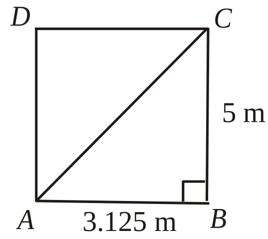
b. Again in ΔABE,

$$\tan(\angle BAE) = \frac{\text{opp}}{\text{adj}}$$
$$= \frac{0\cdot875}{3}$$
$$\angle BAE = \tan^{-1}\left(\frac{0\cdot875}{3}\right)$$
$$= 16\cdot3^\circ \quad \text{(correct to 1 decimal place}$$

(1 mark)

c. In ΔABC, we have

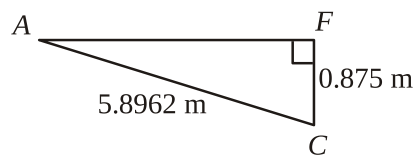
$$(AC)^2 = 3\cdot125^2 + 5^2$$
$$AC = \sqrt{3.125^2 + 5^2}$$
$$= 5\cdot8962\text{m} \quad \text{(correct to 4 decimal places)}$$



(1 mark)

d. In ΔACF

$$\sin(\angle CAF) = \frac{\text{opp}}{\text{hyp}}$$
$$= \frac{0\cdot875}{5\cdot8962}$$
$$\angle CAF = \sin^{-1}\left(\frac{0\cdot875}{5\cdot8962}\right)$$
$$= 8\cdot53^\circ \quad \text{(correct to 2 decimal place}$$



(1 mark)

Question 2

a.

$$\begin{aligned}\text{S.A.} &= \text{area of circle} + \text{area of rectangle}\\ &= \pi r^2 + 3\times 1\\ &= \pi \times 0\cdot 5^2 + 3\\ &= 3\cdot 7854\text{m}^2\end{aligned}$$

(Note that the width of the rectangle is equal to $0\cdot 5+0\cdot 5=1$)

(1 mark)

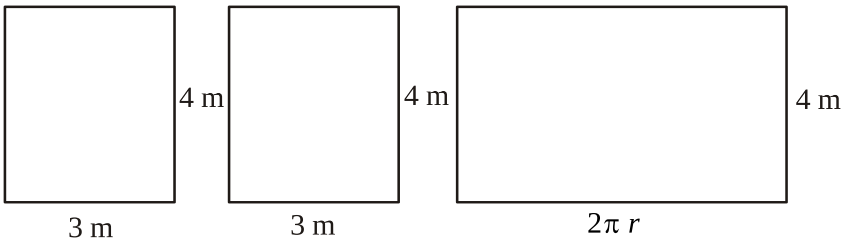
b. $V = \text{area of cross-section} \times \text{height}$
$= 3\cdot 7854\times 4$ (from part **a.**)
$= 15\cdot 14\text{m}^3$ (correct to 2 decimal places)

(1 mark)

c. From part **b.**, $V = 15\cdot 14\text{m}^3$ so the tank can hold $15\cdot 14\times 1000 = 15140$ litres.

(1 mark)

d. From part **a.**, the surface area of the top (and therefore the base) of the tank is $3\cdot 7854\text{m}^2$ (to 4 decimal places).
The surface area of the sides can be broken up into 2 rectangles and the sides of a cylinder with radius $0\cdot 5$m.



$$\begin{aligned}\text{S.A. of sides} &= 4\times 3 + 4\times 3 + 2\pi r\times 4\\ &= 24 + 8\pi \times 0\cdot 5\\ &= 24 + 4\pi\end{aligned}$$

(1 mark)

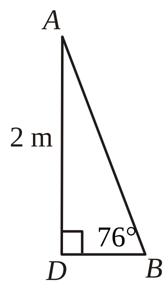
$$\begin{aligned}\text{Total SA} &= 24 + 4\pi + 2\times 3\cdot 7854 \text{(top and base)}\\ &= 44\cdot 14\text{m}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

(1 mark)

Question 3

a. In ΔABD

$$\sin(76°) = \frac{2}{AB}$$



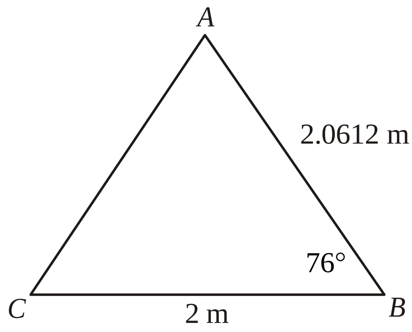
$$AB = \frac{2}{\sin(76°)}$$
$$= 2.0612\text{m (correct to 4 decimal places)}$$

(1 mark)

b. In ΔABC,

$$(AC)^2 = 2^2 + 2\cdot0612^2 - 2\times2\times2\cdot0612\times\cos(76^\circ)$$

$$AC = 2\cdot5007...$$
$$= 2.5\text{m} \quad \text{(correct to 1 decimal place)}$$



(1 mark)

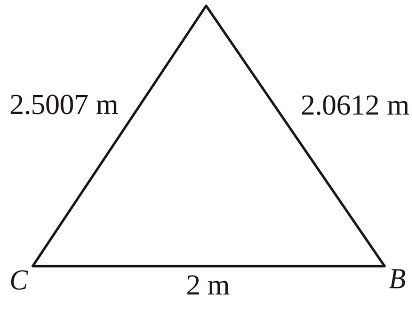
c. Again in ΔABC,

$$\frac{\sin(\angle ACB)}{2\cdot0612} = \frac{\sin(76^\circ)}{2\cdot5007}$$
$$\angle ACB = 53\cdot1077...^\circ$$
$$= 53^\circ \quad \text{(to the nearest degree)}$$

(1 mark)

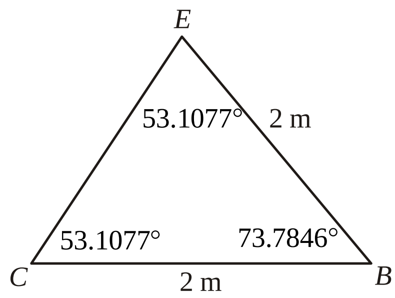
d. Heron's formula.

$$s = \frac{1}{2}(2\cdot0612 + 2\cdot5007 + 2)$$
$$= 3\cdot28095$$
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= 1.999945..$$
$$= 2\text{m}^2$$
(to the nearest square metre)



(1 mark)

e. ΔBCE is an isosceles triangle since $BC = BE = 2\text{ m}$.
So, $\angle BEC = \angle BCE = 53.1077°$ from part **c.**



(1 mark)

<u>Method 1</u> – using the sine rule

$$\frac{CE}{\sin(73.7846°)} = \frac{2}{\sin(53.1077°)}$$

$$CE = \frac{2}{\sin(53.1077°)} \times \sin(73.7846°)$$

$$CE = 2.40125...$$

$$= 2.4 \text{ metres (correct to 1 decimal place)}$$

(1 mark)

<u>Method 2</u> – using the cosine rule

$$(CE)^2 = 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos(73.7846°)$$

$$CE = 2 \cdot 40125...$$

$$= 2.4 \text{ metres (correct to 1 decimal place)}$$

(1 mark)

Total 15 marks

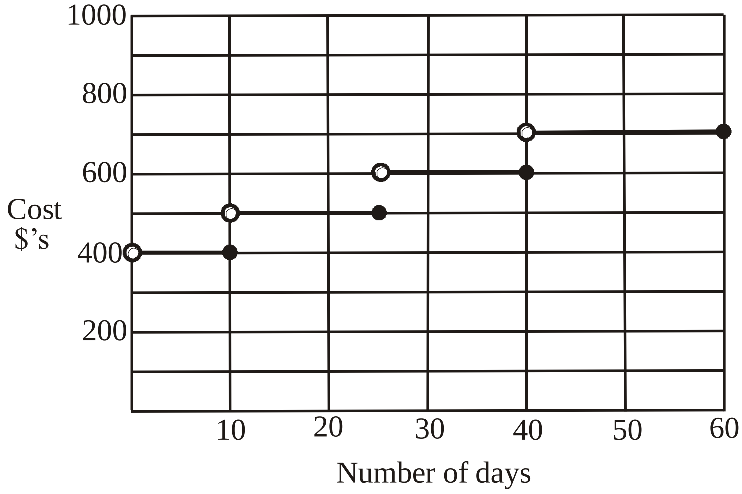
Module 3: Graphs and relations.

Question 1

a. From the graph, it costs \$500.

(1 mark)

b.



(1 mark)

c. People would hire for exactly 10, 25 or 40 days because on the 11th, 26th and 41st day of hiring prices go up. People are getting the most number of days for the money they have paid.

(1 mark)

Question 2

a. $R = 1\,200x$

(1 mark)

b. $C = 80\,000 + 400x$

(1 mark)

c. At break even point, $R = C$

$$1\,200x = 80\,000 + 400x$$
$$800x = 80\,000$$
$$x = 100$$

Kate needs 100 clients to break even.

(1 mark)

d. If she has 60 clients she will make a loss.

$$\text{If } x = 60, \quad R = 1\,200 \times 60 \quad C = 80\,000 + 400 \times 60$$
$$= 72\,000 \qquad = 104\,000$$

$$\text{Loss} = 104\,000 - 72\,000$$
$$= \$32\,000$$

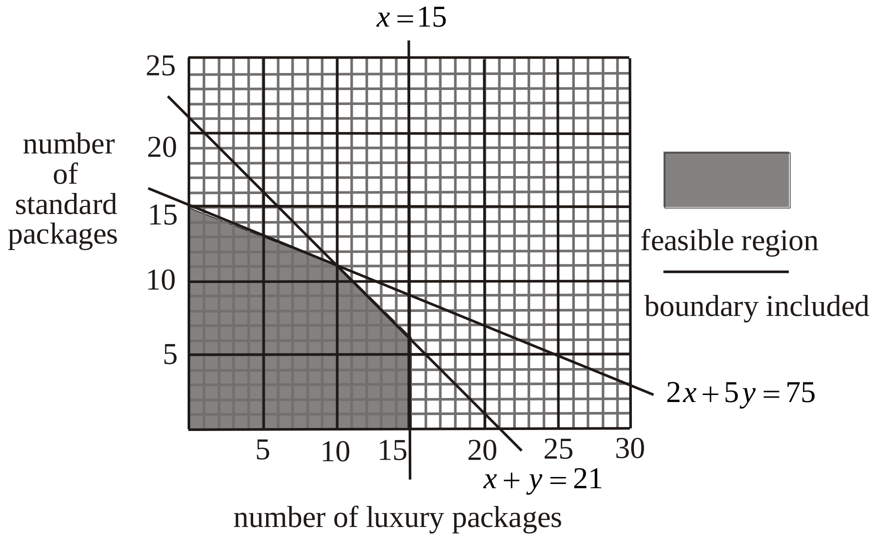
(1 mark)

Question 3

a. Constraint 5 is concerned with furniture availability.
It contains only the variable *x* which represents the number of luxury packages which are limited to 15.
So there are no constraints on the amount of furniture available for standard packages.

(1 mark)

b. c. and **d.**



(1 mark) for the line $x + y = 21$
(1 mark) for the line $x = 15$
(1 mark) for the feasible region

e. $P = 800x + 500y$

(1 mark)

f. A maximum will occur at one of the corner points of the feasible region. From the graph the corner points are $(0,0),(0,15),(10,11),(15,6),(15,0)$.

At $(0,0)$ $\quad P = 0$

At $(0,15)$ $\quad P = 800\times 0 + 500\times 15$
$= 7\,500$

At $(10,11)$ $\quad P = 800\times 10 + 500\times 11$
$= 13\,500$

At $(15,6)$ $\quad P = 800\times 15 + 500\times 6$
$= 15\,000$

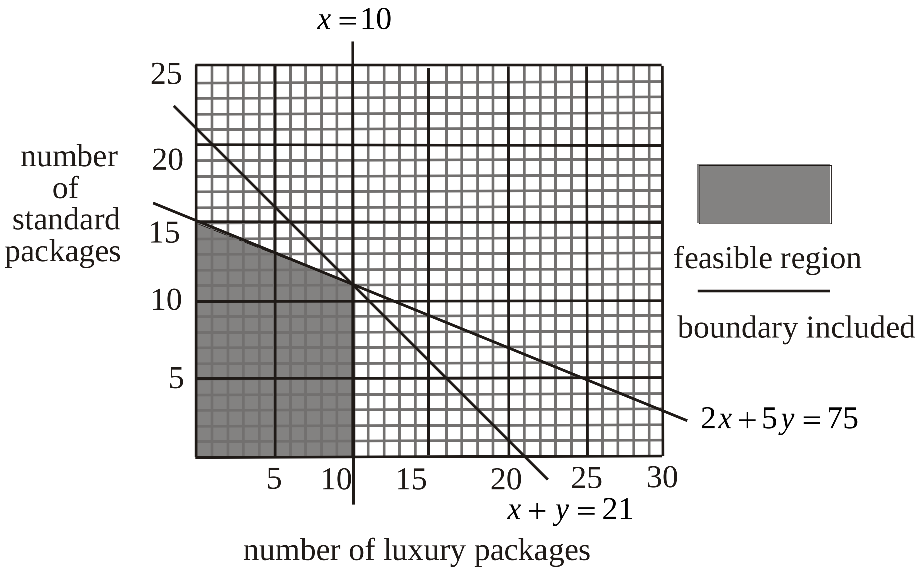
At $(15,0)$ $\quad P = 800\times 15 + 500\times 0$
$= 12\,000$

Maximum profit is $15 000.

(1 mark)

g. The line $x = 15$ moves left to $x = 10$ so the constraint $x + y \leq 21$ now has no effect on the feasible region since only the point $(10,11)$ on the line $x + y = 21$ lies in the feasible region and it is the corner point created by the intersection of the lines $2x + 5y = 75$ and $x = 10$.

(1 mark)



h. The corner points that now remain are $(0,0), (0,15), (10,11)$ and $(10,0)$.

At $(0,0)$ $P = 0$

At $(0,15)$ $P = 7500$ from part **f.**

At $(10,11)$ $P = 13500$ from part **f.**

At $(10,0)$ $P = 800 \times 10 + 0$ from part **f.**

$= 8000$

The maximum profit is now $13 500.

(1 mark)
Total 15 marks

Module 4: Business-related mathematics

Question 1

a.

$$\begin{aligned}\text{amount withdrawn} &= \$35\,246\cdot 28 - \$34\,618\cdot 28\\ &= \$628\end{aligned}$$

(1 mark)

b.

$$\begin{aligned}\text{Interest for April} &= \frac{PrT}{100} \quad \text{(simple interest)}\\ &= \frac{15246\cdot 28 \times 2\cdot 4 \times \frac{1}{12}}{100}\\ &= \$30\cdot 49\end{aligned}$$

(1 mark) correct balance
(1 mark) correct answer

c.

$$A = PR^n \qquad R = 1 + \frac{r}{100} = 1 + \frac{1}{100} = 0.01$$

$$\begin{aligned} &= 20000 \times (1\cdot 01)^{12}\\ &= \$22536\cdot 50\end{aligned}$$

(1 mark)

So John earned $\$22\,536\cdot 50 - \$20\,000 = \$2\,536\cdot 50$ in interest.

(1 mark)

Question 2

a. To find the GST included in a price, divide by 11.
So $\$20\,000 \div 11 = \$1\,818\cdot 18$
John paid $\$1\,818\cdot 18$ in GST.

(1 mark)

b.

$$\begin{aligned}\%\ \text{discount} &= \left(\frac{22\,000 - 20\,000}{22\,000} \times \frac{100}{1}\right)\%\\ &= \left(\frac{2\,000}{22\,000} \times \frac{100}{1}\right)\%\\ &= 9\cdot 09\%\end{aligned}$$

(1 mark)

Question 3

a. $\$20\,000 - \$12\,000 = \$8\,000$

$\$8\,000 \div \$0 \cdot 05 = 160000$

So 160 000 lifts will have been performed.

(1 mark)

b. 7% of \$20 000

$= \$1\,400$

Book value after 5 years

$= \$20\,000 - 5 \times \$1\,400$

$= \$13\,000$

(1 mark)

c. At the end of the third year of operation the value of the fork lift is given by

$$V = 20\,000 \times \left(1 - \frac{8}{100}\right)^3$$

$$= 15\,573 \cdot 76$$

(1 mark)

The value at the end of the fourth year is given by

$$V = 20\,000 \times \left(1 - \frac{8}{100}\right)^4$$

$$= 14\,327 \cdot 8592$$

The fork lift depreciates by $\$15\,573 \cdot 76 - \$14\,327 \cdot 86 = \$1\,245 \cdot 90$ during its fourth year of operation.

(1 mark)

Question 4

a. Using *TVM* solver

$$N = 12$$
$$I\% = 6$$
$$PV = 80\,000 \quad \text{(positive because John receives this from the bank)}$$
$$PMT = ?$$
$$FV = 0$$
$$P/Y = 4$$
$$C/Y = 4$$
$$PMT = -7\,334\cdot 3994... \quad \text{(negative because John has to pay the bank)}$$

John's quarterly repayments will be $7 334.40.

(1 mark)

b. John will pay in total

$$12 \times \$7\,334\cdot 40$$
$$= \$88\,012\cdot 80$$
$$\text{Interest} = \$88\,012\cdot 80 - \$80\,000$$
$$= \$8\,012\cdot 80$$

(1 mark)

c. Use *TVM* solver to find how much John still owes two years into the loan.

$$N = 8$$
$$I\% = 6$$
$$PV = 80\,000$$
$$PMT = -7\,334\cdot 3994$$
$$FV = ?$$
$$P/Y = 4$$
$$C/Y = 4$$

So John owes $28 269.59657 two years into the loan.

(1 mark)

Again using *TVM* solver.

$$N = ?$$
$$I\% = 6$$
$$PV = 28\,269.59657$$
$$PMT = -5000$$
$$FV = 0$$
$$P/Y = 4$$
$$C/Y = 4$$
$$N = 5\cdot 9524$$

It will take John $5\cdot 9524 - 4 = 1\cdot 9524$ extra quarters or 2 quarters correct to the nearest quarter.

(1 mark)

Total 15 marks

Module 5: Networks and business mathematics

Question 1

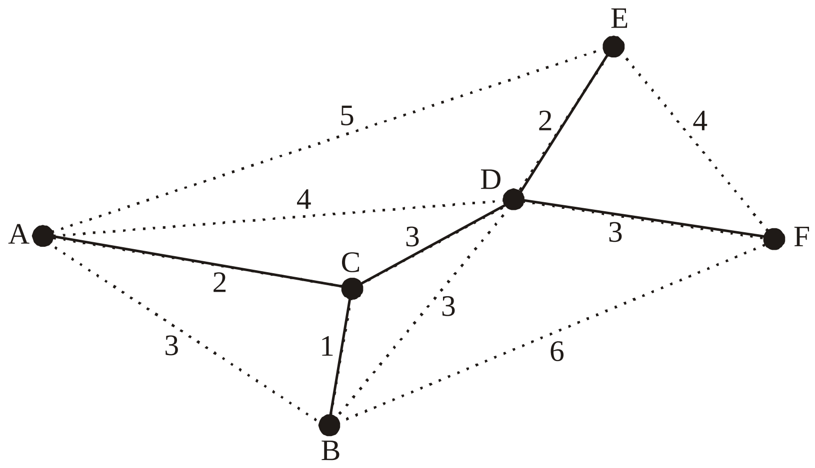
a. By inspection of the graph the shortest distance is along the path *ADF* and is 7km.

(1 mark)

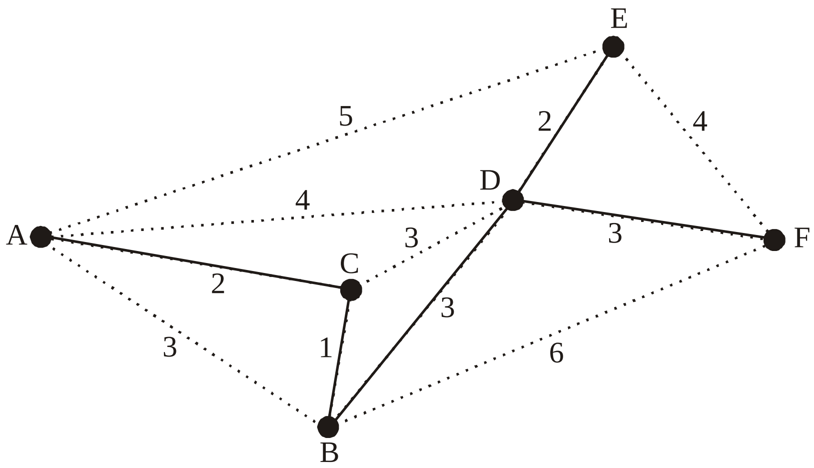
b. A tree is a connected graph with no circuits. Since *E A B F E* is a circuit *E A B F E D C* cannot be a tree.

(1 mark)

c. **i.**



OR



(1 mark) for one of these

ii. There are 2 minimal spanning trees that exist.

(1 mark)

iii. The minimum length of cable required is the sum of the distances on the minimum spanning tree which is $1+2+3+3+2=11$km.

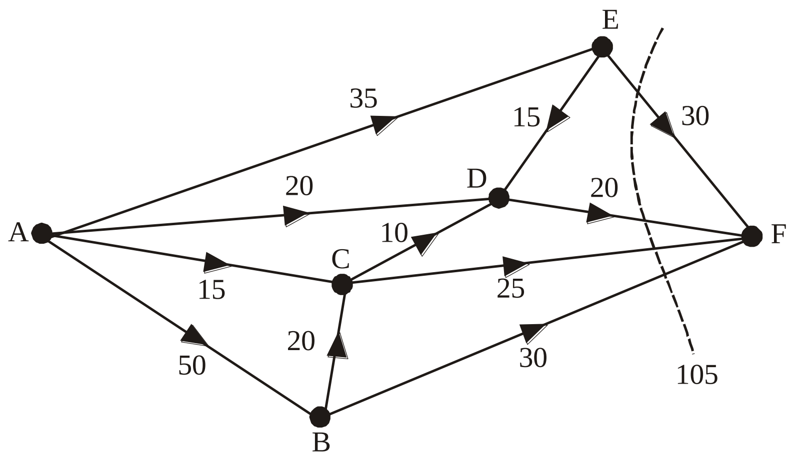
(1 mark)

Question 2

a. The capacity of the cut is $35+20+25+30=110$.
Note that since the edge with a capacity of 15 crosses from right to left and not left to right ($A-F$) it is not counted.

(1 mark)

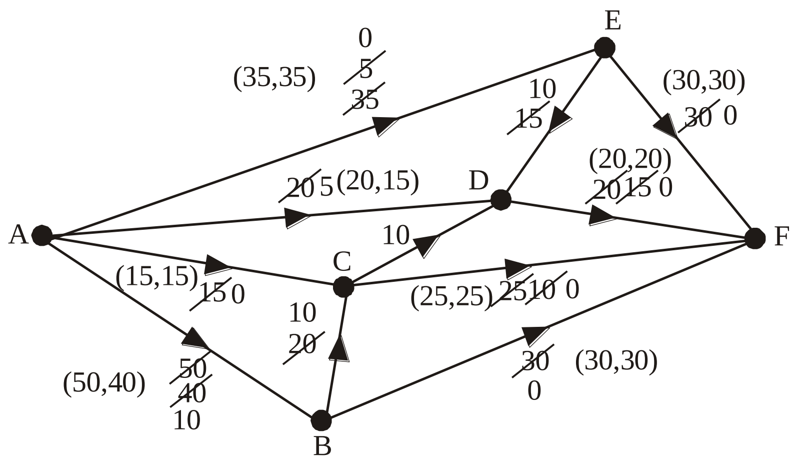
b. Method 1 – trial cuts



The minimum cut is 105 so the maximum number of people is 105.

(1 mark)

Method 2



Each bracket contains (initial capacity, final flow)

$$\text{Number of passengers from } A = 35+15+15+40$$
$$=105$$
$$\text{Number of passengers into } F = 30+20+25+30$$
$$=105$$

The maximum number of people who can travel from A to F is 105.

(1 mark)

Question 3

a. For one-step reachability we have

$A \to D$
$A \to C$
$B \to A$
$C \to D$
$D \to B$
$D \to E$
$E \to F$
$F \to D$

Looking at the destination islands, we see for example that island *A* is reached once.
The one-step reachability score for each island is

A – 1
B – 1
C – 1
D – 3
E – 1
F – 1

So *D* is the most reachable island.

(1 mark)

b. For two-step reachability we have

$A \to B$
$A \to D$
$A \to E$
$B \to C$
$B \to D$
$C \to B$
$C \to E$
$D \to A$
$D \to F$
$E \to D$
$F \to B$
$F \to E$

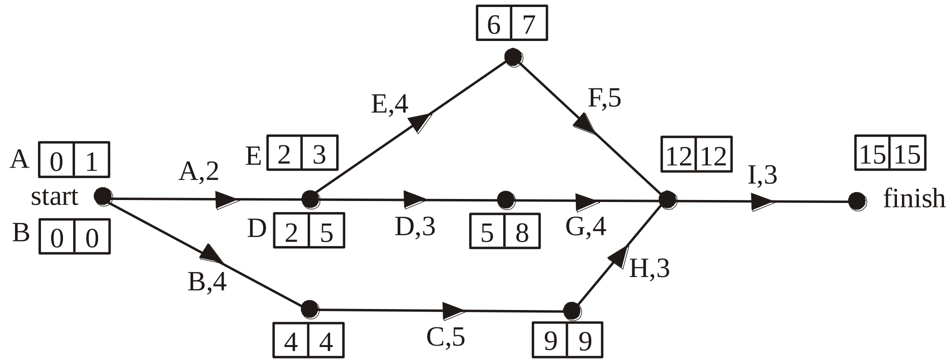
The two-step reachability score for each island is

A – 1
B – 3
C – 1
D – 3
E – 3
F – 1 **(1 mark)**

Adding one-step and two-step we have *A* – 2, *B* – 4, *C* – 2, *D* – 6, *E* – 4, *F* – 2.
So D is the most reachable island and then *B* and E are the next most reachable and are equally reachable.

(1 mark)

Question 4



a. Using our earliest start time and latest start time boxes we see that the shortest time in which the upgrade can be completed is 15 days.

(1 mark)

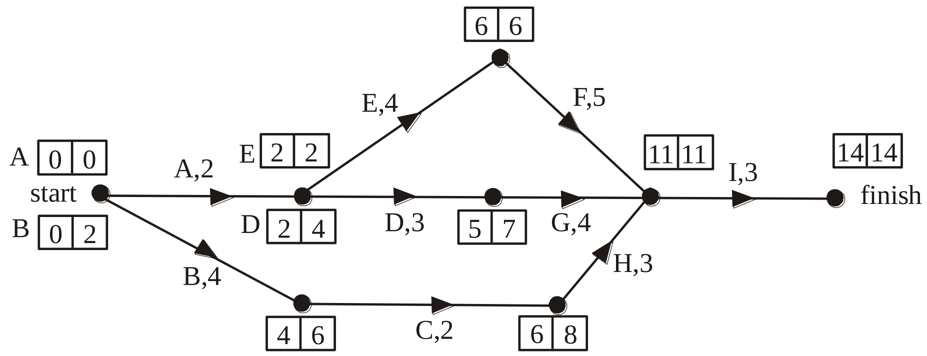
b. Again using our boxes we see that activities *D* and *G* have the greatest slack time (3 days).

(1 mark)

c. Latest start time for activity *E* is day 3.

(1 mark)

d.



The three activities that now lie on the critical path (and didn't previously) are *A*, *E* and *F*.

(1 mark)

e. By reducing activity *C* by 3 days a new critical path *A, E, F, I* has emerged and the new completion time is 14 days. It would only have been necessary to reduce activity *C* by 1 day to achieve this. So two days of money spent have been wasted. It was therefore not worth reducing activity *C* by 3 days.

(1 mark)
Total 15 marks

Module 6: Matrices

Question 1

a. The order of J is 4×1.
(4 rows and 1 column)

(1 mark)

b.
$$JD = \begin{bmatrix} 100 \\ 140 \\ 120 \\ 80 \end{bmatrix} \begin{bmatrix} 0 \cdot 55 & 0 \cdot 45 \end{bmatrix}$$
$$= \begin{bmatrix} 55 & 45 \\ 77 & 63 \\ 66 & 54 \\ 44 & 36 \end{bmatrix}$$

(1 mark)

c. $X = JD$
The element $X_{3,2}$ is 54 and it describes the number of players playing in the U/12 division 2 competition.

(1 mark)

Question 2

a. Enter the matrix into your calculator.
The determinant is – 40.

(1 mark)

b. Matrix A has an inverse because det (A) does not equal zero.

(1 mark)

c. Using your calculator,
$$A^{-1} = \begin{bmatrix} -0 \cdot 2 & 2 & -2 \\ 0 \cdot 975 & -7 \cdot 125 & 6 \cdot 875 \\ -0 \cdot 675 & 4 \cdot 625 & -4 \cdot 375 \end{bmatrix}$$

(1 mark)

d.
$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$
Enter matrix B on your calculator.
Multiply $A^{-1} \times B$ to get X.
$$X = \begin{bmatrix} 65 \\ 80 \\ 120 \end{bmatrix}$$
So $b = 65$, $d = 80$ and $r = 120$.

(1 mark)

Question 3

a.

$$P_{2010} = QP_{2009} + R$$
$$= \begin{bmatrix} 0\cdot 92 & 0 \\ 0 & 0\cdot 85 \end{bmatrix}\begin{bmatrix} 15 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 13\cdot 8 \\ 8\cdot 50 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 15\cdot 80 \\ 10\cdot 50 \end{bmatrix}$$

The profit predicted to be made on the sale of a new uniform is \$15.80 in 2010.
On a secondhand uniform it is \$10.50.

(1 mark)

b.

$$P_{n+1} = QP_n + R$$

$$P_{2011} = QP_{2010} + R$$
$$= \begin{bmatrix} 0\cdot 92 & 0 \\ 0 & 0\cdot 85 \end{bmatrix}\begin{bmatrix} 15\cdot 80 \\ 10\cdot 50 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad . \quad \text{from part } \mathbf{a.}$$
$$= \begin{bmatrix} 14.536 \\ 8\cdot 925 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 16.536 \\ 10\cdot 925 \end{bmatrix}$$

(1 mark)

$$P_{2012} = QP_{2011} + R$$
$$= \begin{bmatrix} 0\cdot 92 & 0 \\ 0 & 0\cdot 85 \end{bmatrix}\begin{bmatrix} 16\cdot 536 \\ 10\cdot 925 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 15\cdot 21312 \\ 9.28625 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 17\cdot 21312 \\ 11.28625 \end{bmatrix}$$

The profit made on a new uniform is predicted to be \$17.21 and on a secondhand uniform it is predicted to be \$11.29.

(1 mark)

Question 4

a. The proportion is 0.18.

(1 mark)

b.

Let $S_{2008} = \begin{bmatrix} 46 \\ 6 \\ 12 \end{bmatrix} \begin{matrix} \text{coach} \\ \text{didn't coach} \\ \text{joint} \end{matrix}$

(1 mark)

$$S_{2010} = T^2 S_{2008}$$
$$= \begin{bmatrix} 52 \cdot 5562 \\ 3 \cdot 3282 \\ 8 \cdot 1156 \end{bmatrix}$$

In 2010, 3 rep. players won't coach.

(1 mark)

c. To find a steady state choose any two consecutive numbers for the power of *T*.

eg. $T^{20} S_{2008} = \begin{bmatrix} 52 \cdot 5425 \\ 3 \cdot 3338 \\ 8 \cdot 1237 \end{bmatrix}$

$T^{21} S_{2008} = \begin{bmatrix} 52 \cdot 5425 \\ 3 \cdot 3338 \\ 8 \cdot 1237 \end{bmatrix}$

(1 mark)

The steady state matrix has been found. Note that it may have been found earlier (i.e. for values less than 20). The steady state matrix tells us that in the long term 53 rep. players (out of the 64) will still be coaching. This represents $\left(\frac{53}{64} \times 100\right)\% = 82.8125\%$ of rep. players coaching. So the club believes that the payment scheme has been successful.

(1 mark)
Total 15 marks