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**FURTHER MATHEMATICS
TRIAL EXAMINATION 1
SOLUTIONS
2009**

Section A – answers

Core

**Module 1
Number
patterns**

**Module 2
Geometry
&
trig**

Section B – answers

**Module 3
Graphs
&
relations**

**Module 4
Business
related
maths**

**Module 5
Networks
&
decision
maths**

**Module 6
Matrices**

1. E	1. D	1. B	1. D	1. A	1. E	1. B
2. A	2. D	2. D	2. B	2. C	2. D	2. C
3. B	3. B	3. E	3. D	3. E	3. D	3. D
4. B	4. B	4. D	4. B	4. D	4. C	4. E
5. A	5. E	5. C	5. C	5. C	5. C	5. D
6. D	6. C	6. D	6. C	6. B	6. B	6. D
7. C	7. A	7. C	7. D	7. B	7. A	7. A
8. D	8. E	8. D	8. E	8. E	8. B	8. C
9. B	9. C	9. A	9. C	9. B	9. E	9. C
10. C						
11. E						
12. C						
13. E						

Core - solutions

Question 1

The minimum value is 1 (the outlier).

The maximum value is 16.

The range is $16 - 1 = 15$.

The answer is E.

Question 2

For this distribution, a score is an outlier if it is less than

$$\begin{aligned}Q_1 - 1.5 \times IQR \\&= 8 - 1.5 \times (12 - 8) \\&= 8 - 1.5 \times 4 \\&= 8 - 6 \\&= 2\end{aligned}$$

The answer is A.

Question 3Method 1

$$1 \quad 1 \quad 1 \quad | \quad 2 \quad 2 \quad 3 \quad \textcircled{3} \quad 3 \quad 3 \quad 4 \quad | \quad 4 \quad 4 \quad 5$$

Q_1 median Q_3

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 4 - 1.5 \\ &= 2.5 \end{aligned}$$

The answer is B.

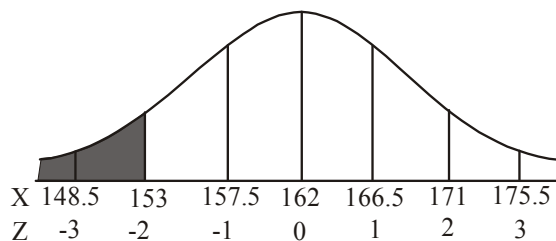
Method 2 – use a calculator

Enter the 13 numbers.

Calculate 1 – variable stats.

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 4 - 1.5 \\ &= 2.5 \end{aligned}$$

The answer is B.

Question 4

$$\Pr(X < 153) = 0.025$$

Note that 95% of heights lie between 2 standard deviations either side of the mean; that is between 153cm and 171cm. So 5% lie outside this; half below 153cm and the other half above 171cm.

$$\text{Now } 0.025 \times 200 = 5.$$

The answer is B.

Question 5

The variable “number of passengers” is a discrete variable; that is you count the number of passengers you don’t measure them. It is also the independent variable; that is, it does not depend on the total weight.

The variable “total weight” is a continuous variable; that is you measure it; you don’t count it. It is the dependent variable; that is, it is dependent on how many passengers there are.

The answer is A.

Question 6

Enter the data into a calculator.

$r = 0.9787$ correct to 4 decimal places.

The answer is D.

Question 7

For every extra passenger on board there will be an increase of 81.1429kg on board the plane.

The answer is C.

Question 8

The correlation coefficient r , is 0.7.

This suggests that an increase in the average number of kilometers run in a week is associated with an increase in the number of injuries.

Options A, B, C are incorrect.

Option E is incorrect because whilst an association between the two variables has been established, it does not prove that running causes injuries.

Since $r = 0.7$, $r^2 = 0.49$.

This is the coefficient of determination. It is true to say that 49% of the variation in the number of injuries can be accounted for by the variation in the average number of kilometers run.

The answer is D.

Question 9

Residual value = actual value – predicted value

The largest residual value occurs for the piece of data for which $x = 7$. That is, it is the piece of data that is furthest from the trend line.

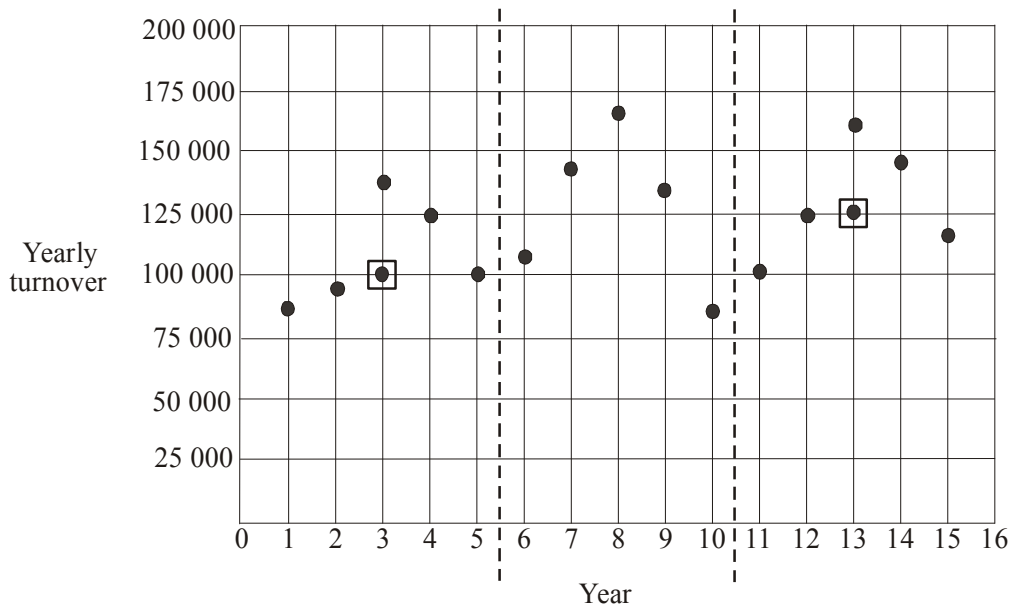
The residual value is $3 - 1.5 = 1.5$.

The answer is B.

Question 10

There is a clear pattern in the turnover of the small business. It is a cyclic trend because each cycle occurs over the course of 5 years (unlike a seasonal trend which occurs over the course of 1 year).

The answer is C.

Question 11

The median in the bottom one third of data points is (3,100 000). The median in the top one third of data points is (13,125,000).

The slope is given by

$$\begin{aligned} & \frac{125\,000 - 100\,000}{13 - 3} \\ &= \frac{25\,000}{10} \\ &= 2500 \end{aligned}$$

The answer is E.

Question 12

The three mean moving average for June is given by $\frac{30+25+23}{3} = \frac{78}{3} = 26$.

The answer is C.

Question 13

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{\text{seasonal index}}$$

$$32\,500 = \frac{\text{actual sales}}{1.23}$$

$$\text{actual sales} = 1.23 \times 32\,500$$

$$= 399\,750$$

The answer is E.

SECTION B**Module 1: Number patterns****Question 1**

Method 1 – using a calculator

Generate the sequence on your calculator. The twentieth term is 137.
The answer is D.

Method 2

$$t_n = a + (n-1)d$$

$$t_{20} = 4 + (20-1) \times 7 \\ = 137$$

The answer is D.

Question 2

Method 1 – trial and error

$$\text{If } r = 2, 3 \times 2 = 6, 6 \times 2 = 12$$

$$\text{If } r = 3, 3 \times 3 = 9, 9 \times 3 = 27$$

$$\text{If } r = 4, 3 \times 4 = 12, 12 \times 4 = 48$$

So $m = 12$

The answer is D.

Method 2

Because the sequence is geometric,

$$\frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\text{So, } \frac{m}{3} = \frac{48}{m} \quad (\text{cross multiply})$$

$$m^2 = 144$$

$$m = \pm 12$$

Only the option 12 is offered.

The answer is D.

Question 3

$$t_n = t_{n-2} + t_{n-1}, \quad t_1 = -3, t_2 = 1$$

$$t_1 = -3$$

$$t_2 = 1$$

$$t_3 = -3 + 1 = -2$$

$$t_4 = 1 - 2 = -1$$

$$t_5 = -2 - 1 = -3$$

$$t_6 = -1 - 3 = -4$$

The answer is B.

Question 4

We have a geometric sequence with $a = 18$ and $r = 1.02$.

We want the sum of the first nine terms.

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{since } r > 1$$

$$S_9 = \frac{18(1.02^9 - 1)}{0.02}$$

$$= 175.5833\dots$$

The closest answer is 175.58.

The answer is B.

Question 5

We need to find the sum of the first twenty terms of the arithmetic sequence 1,2,3,4,...

Method 1

$$S_n = \frac{n}{2}(a + l)$$

$$S_{20} = \frac{20}{2}(1 + 20)$$

$$= 210$$

The answer is E.

Method 2

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{20} = \frac{20}{2}(2 \times 1 + (20-1) \times 1)$$

$$= 10(2 + 19)$$

$$= 210$$

The answer is E.

Question 6

For the budget for the $(n+1)$ th year, the previous year's budget is B_n .

3.5% of the previous year's budget is $\frac{3.5}{100}B_n = 0.035B_n$.

The previous year's budget plus 3.5% of the previous year's budget is given by

$$B_n + 0.035B_n = 1.035B_n$$

$$\text{So } B_{n+1} = 1.035B_n + 6000 \quad B_1 = 42\,000$$

The answer is C.

Question 7

The sequence is not arithmetic because there is no common difference between successive terms.

This eliminates options C and D.

Since the first term is negative, the second positive, the third negative, option B is not correct.

Similarly with option E since,

$$\begin{aligned}t_2 &= 1.3 \times -10 - 23 \\ &= -36\end{aligned}$$

From the graph, $t_2 > 0$.

The answer is A.

Question 8

We have a geometric sequence with $a = 10$ and $r = 0.96$.

$$\begin{aligned}S_{\infty} &= \frac{a}{1-r} \\ &= \frac{10}{1-0.96} \\ &= \frac{10}{0.04} \\ &= 250 \text{ mg}\end{aligned}$$

The answer is E.

Question 9

$$t_{n+1} = 0.5t_n - 1 \quad t_1 = 1000$$

Generate the sequence on your calculator.

1000, 499, 248.5, 123.25, 60.625, ...

The sequence is neither arithmetic nor geometric so options B and D are true.

The sequence is decreasing so option A is true.

$t_3 = 248.5$ so option E is true.

The sequence has positive and negative numbers. The tenth term, the 11th term and the rest of the terms are negative.

So option C is incorrect.

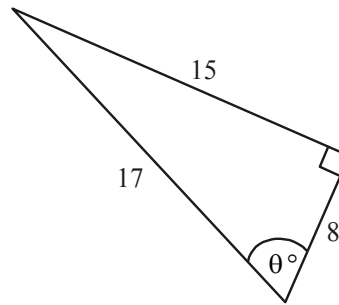
The answer is C.

Module 2: Geometry and trigonometry

Question 1

$$\begin{aligned}\cos \theta^{\circ} &= \frac{\text{adj}}{\text{hyp}} \\ &= \frac{8}{17}\end{aligned}$$

The answer is B.



Question 2

The sum of the interior angles of a rectangular polygon with n sides is given by

$$S = \{180(n-2)\}^{\circ}$$

For a regular pentagon, $n = 5$

$$\text{so } S = \{180(5-2)\}^{\circ}$$

$$= 540^{\circ}$$

The answer is D.

Question 3

Because the cross-section is an equilateral triangle with sidelength 4cm, we have

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc \sin A \\ &= \frac{1}{2} \times 4 \times 4 \times \sin 60^{\circ} \\ &= 6.9282\dots\end{aligned}$$

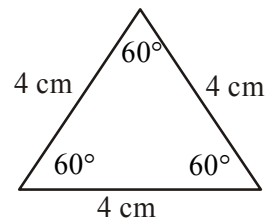
$$\text{Area of rectangular face} = 10 \times 4 = 40\text{cm}^2$$

$$\text{Total surface area} = 2 \times 6.9282\dots + 3 \times 40$$

$$= 133.8564\dots$$

The closest answer is 133.86.

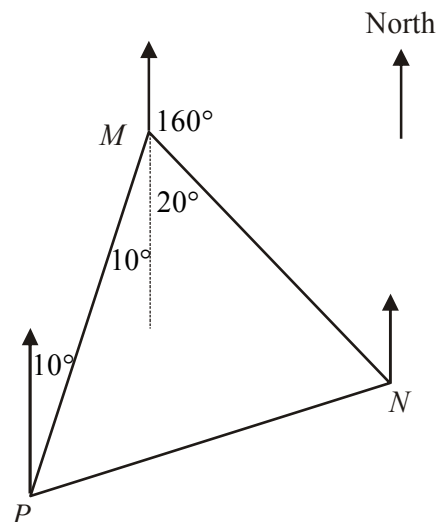
The answer is E.



Question 4

The bearing of P from M is 190° .

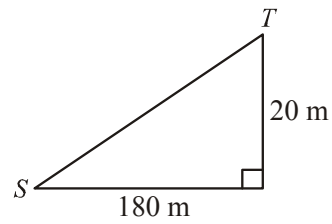
The answer is D.



Question 5

$$\begin{aligned} \text{average slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{20}{180} \\ &= \frac{1}{9} \end{aligned}$$

The answer is C.

**Question 6**

The triangles are similar.

Scale factor = k

$$\begin{aligned} &= \frac{12}{8} \\ &= 1.5 \end{aligned}$$

So, area of $\triangle PQR = 1.5^2 \times \text{area of } \triangle ABC$

$$= 2.25 \times 30$$

$$= 67.5 \text{ cm}^2$$

The answer is D.

Question 7

$$\begin{aligned} \text{In } \triangle ABC, \\ (AC)^2 &= 2^2 + 2^2 \\ &= 8 \end{aligned}$$

$$AC = 2\sqrt{2}$$

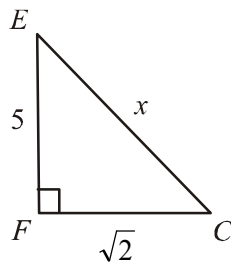
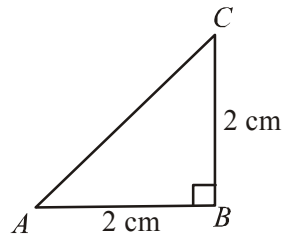
$$\text{So } CF = \sqrt{2}$$

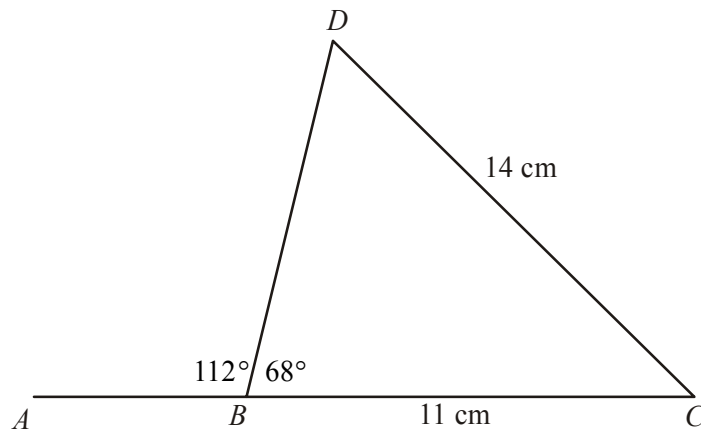
In $\triangle CEF$

$$\begin{aligned} x^2 &= 5^2 + (\sqrt{2})^2 \\ &= 27 \end{aligned}$$

$$x = \sqrt{27}$$

The answer is C.



Question 8

$$\begin{aligned}\angle CBD &= 180^\circ - 112^\circ \\ &= 68^\circ\end{aligned}$$

$$\frac{\sin(\angle BDC)}{11} = \frac{\sin(68^\circ)}{14}$$

$$\sin(\angle BDC) = 0.7285\dots$$

$$\begin{aligned}\angle BDC &= \sin^{-1}(0.7285\dots) \\ &= 46.7609\dots\end{aligned}$$

$$\begin{aligned}\text{So } \angle BCD &= 180^\circ - 68^\circ - 46.7609\dots^\circ \\ &= 65.2390\dots^\circ\end{aligned}$$

The closest answer is 65.2°.

The answer is D.

Question 9

Draw a diagram.

Use the bearing to find $\angle CML$

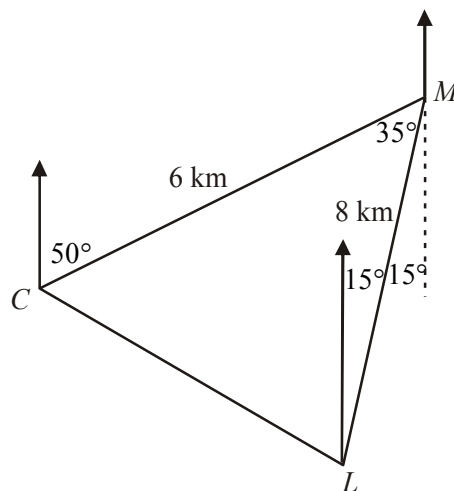
(since $35^\circ + 15^\circ = 50^\circ$ alternate angles).

$$\begin{aligned}(CL)^2 &= 6^2 + 8^2 - 2 \times 6 \times 8 \cos(35^\circ) \\ &= 21.3614\dots\end{aligned}$$

$$CL = 4.6218\dots$$

The closest answer is A.

The answer is A.



Module 3: Graphs and relations**Question 1**Method 1

$$\begin{aligned} \text{From the graph, gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

The answer is D.

Method 2

The line passes through $(-2,0)$ and $(0,2)$.

Let $(-2,0) = (x_1, y_1)$ and $(0,2) = (x_2, y_2)$.

$$\begin{aligned} \text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 0}{0 - (-2)} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

The answer is D.

Question 2

$$C = 30 + 4n$$

For each call made, the monthly cost is increased by \$4.

The answer is B.

Question 3

From the graph, the line drops below 10km/hr between 1.5 and 3 days and again between 5 and 7.5 days. In total it is below 10km/hr for approximately $1.5 + 2.5 = 4$ days.

The answer is D.

Question 4

We are looking for not just an increase in wind speed but the steepness of the increase. The steepest increase occurs between 3 – 4 days.

The answer is B.

Question 5

Let x = cost of hiring an overnight DVD.

Let y = cost of hiring a weekly DVD .

$$2x + 3y = 24 \cdot 40 \quad -(1)$$

$$3x + 5y = 38 \cdot 50 \quad -(2)$$

$$(1) \times 3 \quad 6x + 9y = 73 \cdot 20 \quad -(3)$$

$$(2) \times 2 \quad 6x + 10y = 77 \cdot 00 \quad -(4)$$

$$(4) - (3) \quad y = 3 \cdot 80$$

$$\text{In (1)} \quad 2x + 3 \times 3 \cdot 80 = 24 \cdot 40$$

$$2x + 11 \cdot 40 = 24 \cdot 40$$

$$2x = 13 \cdot 00$$

$$x = 6 \cdot 50$$

So one overnight and one weekly DVD would cost $\$3 \cdot 80 + \$6 \cdot 50 = \$10 \cdot 30$.

The answer is C.

Question 6

Option A is correct.

Option B is correct.

Option C is incorrect because in the feasible region, the maximum value of y is 10 (occurs at the corner point $(5,10)$).

Options D and E are both correct.

The answer is C.

Question 7

The gradient of the straight line is $\frac{2}{1} = 2$.

The relationship between y and x is therefore given by $y = 2x^3$.

The answer is D.

Question 8

Let C = total costs for the cabinet maker in dollars

$$C = 2\,800 + 70 \times 30$$

$$= 4\,900$$

The cabinetmaker sells the 30 stands to the retailer for a total of \$6 000 (\$4 900 in costs and \$1 100 in profit).

For each stand, the retailer pays $\$6\,000 \div 30 = \200 .

The answer is E.

Question 9

The constraint due to labour is $20x + 15y \leq 2\,400$.

The constraint due to fabric is $80x + 120y \leq 14\,400$.

The answer is C.

Module 4: Business-related mathematics**Question 1**

$$\begin{aligned} \text{simple interest} &= \frac{PrT}{100} \\ 40 &= \frac{P \times 2 \cdot 5 \times 2}{100} \\ 40 \times 100 &= 5P \\ \frac{4000}{5} &= P \\ P &= 800 \end{aligned}$$

The answer is A.

Question 2

Compound interest.

$$\begin{aligned} A &= PR^n & R &= 1 + \frac{r}{100} = 1 + \frac{1.6}{100} = 1.016 \quad \text{Note that the rate per annum is 3.2\%} \\ & & & & & \text{so the rate per 6 months is 1.6\%.} \\ &= 13000 \times 1 \cdot 016^8 \\ &= 14760 \cdot 23 \end{aligned}$$

The answer is C.

Question 3

For a perpetuity

$$P = \frac{100Q}{R}$$

where Q is the amount paid per annum.

Gwen receives \$350 per week which is $\$350 \times 52 = \$18\,200$ per year. The interest per annum is 5%.

$$\begin{aligned} \text{So } P &= \frac{100 \times 18\,200}{5} \\ &= \$364\,000 \end{aligned}$$

The answer is E.

Question 4

Using the reducing balance method,

$$V = P \times \left(1 - \frac{r}{100}\right)^t$$

$$800 = 1500 \times \left(1 - \frac{r}{100}\right)^3$$

Method 1 – trial and error

$$\text{If } r = 7\%, \quad 1500 \times \left(1 - \frac{7}{100}\right)^3 = 1206.5355$$

$$\text{If } r = 15\%, \quad 1500 \times \left(1 - \frac{15}{100}\right)^3 = 921.1875$$

$$\text{If } r = 17\%, \quad 1500 \times \left(1 - \frac{17}{100}\right)^3 = 857.6805$$

$$\text{If } r = 19\%, \quad 1500 \times \left(1 - \frac{19}{100}\right)^3 = 797.1615$$

$$\text{If } r = 23\%, \quad 1500 \times \left(1 - \frac{23}{100}\right)^3 = 684.7995$$

The closest answer is 19.

The answer is D.

Method 2 – using indices

$$800 = 1500 \times \left(1 - \frac{r}{100}\right)^3$$

$$\frac{800}{1500} = \left(1 - \frac{r}{100}\right)^3$$

$$\left(\frac{8}{15}\right)^{\frac{1}{3}} = \left(\left(1 - \frac{r}{100}\right)^3\right)^{\frac{1}{3}}$$

$$0.8109... = 1 - \frac{r}{100}$$

$$0.8109... - 1 = -\frac{r}{100}$$

$$-0.1890... = -\frac{r}{100}$$

$$0.1890... = \frac{r}{100}$$

$$r = 100 \times 0.1890...$$

$$= 18.90...$$

The closest answer is 19.

The answer is D.

Question 5

Between quarter 1 and 2 the price of the bicycle increased by \$10.

This represents a percentage increase of $\left(\frac{10}{500} \times \frac{100}{1}\right)\% = 2\%$.

The answer is C.

Question 6Method 1

For flat interest we have

$$r = \frac{100I}{Pt}$$

Now, $r = 5$

$$\begin{aligned} I &= \text{total interest paid} \\ &= 1000 + 26 \times x - 3500 \\ &= 26x - 2500 \end{aligned}$$

where x is the fortnightly payment.

$$\begin{aligned} P &= 3500 - 1000 \\ &= 2500 \end{aligned}$$

$$\text{So, } 5 = \frac{100(26x - 2500)}{2500 \times 1}$$

$$\frac{5 \times 2500}{100} = 26x - 2500$$

$$125 = 26x - 2500$$

$$2625 = 26x$$

$$x = 100.96$$

The answer is B.

Method 2

Amount owing = \$3500 - \$1000 = \$2500.

Fortnightly payment is given by

$$\frac{\$2500 \times 1.05}{26} = \$100.96$$

The answer is B.

Question 7

$$\begin{aligned} \text{the effective rate of interest} &= \frac{2n}{n+1} \times \text{flat rate} \\ &= \frac{2 \times 26}{26+1} \times 5\% \\ &= 9.63\% \end{aligned}$$

The answer is B.

Question 8

Use *TVM* solver

$$N = ?$$

$$I\% = 5.5$$

$$PV = -120000$$

$$PMT = 750$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

$$N = 289.0434\dots$$

This is the number of months that the annuity will last for.

$$\text{So } 289.0434\dots \div 12 = 24.08\dots$$

The annuity will last for 24.08... years.

The closest answer is 24.

The answer is E.

(Negative because Nigel gave this money to the bank.)

(Positive because the bank gives this money each month to Nigel.)

Question 9

Use *TVM* solver to find the quarterly repayments Paul has to make.

$$N = 16$$

$$I\% = 6$$

$$PV = 60000$$

$$PMT = ?$$

$$FV = 0$$

$$P/Y = 4$$

$$C/Y = 4$$

$$PMT = -4\,245.90 \text{ (Negative because Paul has to pay it to the bank).}$$

Use *TVM* solver to find the future value (i.e. the principal remaining to be paid off on the loan) of the loan after 8 repayments.

$$N = 8$$

$$I\% = 6$$

$$PV = 60\,000$$

$$PMT = -4\,245.90$$

$$FV = ?$$

$$P/Y = 4$$

$$C/Y = 4$$

$$FV = -31\,784.52$$

So after 8 repayments Paul still owes \$31 784.52.

This means he has paid \$60 000 - \$31 784.52 = \$28 215.48 off the principal.

The answer is B.

Module 5: Network and decision mathematics**Question 1**

All 5 vertices have an even degree.
The answer is E.

Question 2

A Hamiltonian path passes through each vertex once starting and finishing on a different vertex.
Only option D offers this.
The answer is D.

Question 3

A tree contains no circuits. The graph is not a tree. Option A is incorrect.
A complete graph has each pair of vertices connected by an edge.
Not all edges are connected by an edge eg. M and S or P and R . Option B is incorrect.
The graph has plenty of circuits but no loops. Option C is incorrect.
The graph does contain an Euler path since exactly two vertices N and Q have odd degrees and the rest have even degrees. Option D is correct.
The answer is D.

Question 4

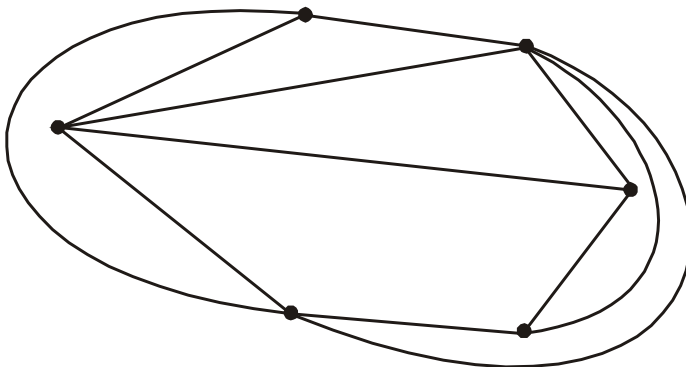
There are no loops so the leading diagonal of the matrix contains only zeroes.
Only vertices A and D have two vertices connecting them.
All other vertices are connected to each other vertex just once.
Matrix C represents this.
The answer is C.

Question 5

The directed edges running across the cut in the direction from start to finish are
 $7 + 5 + 2 + k + 7 = 28$. So $k = 7$.
The answer is C.

Question 6

The graph can be redrawn



to reveal a planar graph.

Since the graph is a connected, planar graph, Euler's formula can be applied.

$$v - e + f = 2$$

where $v = 6$

$$e = 11$$

So $6 - 11 + f = 2$

$$f = 7$$

The answer is B.

Question 7

	<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>
<i>F1</i>	3	4	1	1
<i>F2</i>	4	7	3	6
<i>F3</i>	6	10	6	8
<i>F4</i>	8	5	9	4

Use the Hungarian algorithm.

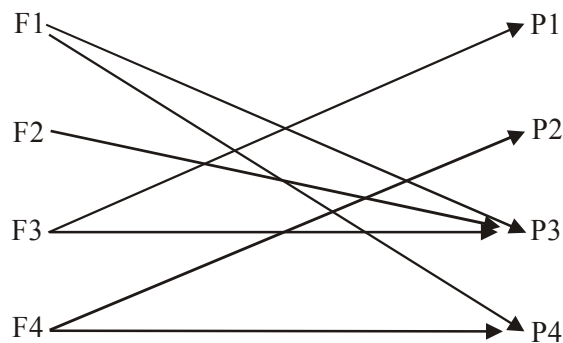
Subtract the minimum element in each row from each of the elements in that row.

	<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>
<i>F1</i>	2	3	0	0
<i>F2</i>	1	4	0	3
<i>F3</i>	0	4	0	2
<i>F4</i>	4	1	5	0

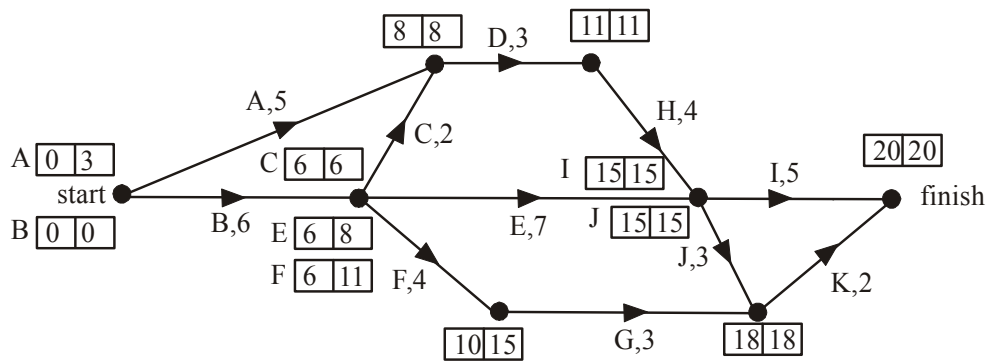
Do the same for column 2 because it has no zeroes.

	<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>
<i>F1</i>	2	2	0	0
<i>F2</i>	1	3	0	3
<i>F3</i>	0	3	0	2
<i>F4</i>	4	0	5	0

Use a bipartite graph.



The allocation is *F3* produces *P1*, *F4* produces *P2*, *F1* produces *P4* and *F2* produces *P3*.
The answer is A.

Question 8

There are two critical paths B, C, D, H, I and B, C, D, H, J, K .
The answer is B.

Question 9

From Question 8 we saw that there were 7 activities that were critical. For the other 4, A has 3 days of slack time, E has 2, F has 5 and G has 5.
In total there are 15 days of slack time in the project.
The answer is E.

Module 6: Matrices**Question 1**

$$\begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix} \\ = \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$$

The answer is B.

Question 2

A is a (2×4) matrix.

B is a (4×1) matrix.

The matrix product AB is of order (2×1) .

The answer is C.

Question 3

The matrix product in Option A is not defined $(1 \times 3) \times (1 \times 3)$.

Similarly the matrix product in option B is not defined $(3 \times 1) \times (3 \times 1)$.

The matrix in option C has 3 elements not one and is not really a matrix product (or product of matrices).

Option D is correct; $(1 \times 3) \times (3 \times 1)$. This product gives a (1×1) matrix which gives the total that Pete spent.

Option E; $(3 \times 1) \times (1 \times 3)$ whilst defined, gives a (3×3) matrix as an answer.

The answer is D.

Question 4

The system of equations

$$2x + y - z = 3$$

$$3y - z = 4$$

$$5x + 3z = 8$$

can be written as

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -1 \\ 5 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$$

The answer is E.

Question 5

$$\det \begin{bmatrix} a & a \\ 2 & 3 \end{bmatrix} = 3a - 2a$$

$$= a$$

So $a = 5$.

The answer is D.

Question 6

Express each one as a matrix equation in the form

$$AX = B$$

The first set

$$x + y = 5$$

$$x + y = 3$$

can be expressed as the matrix equation

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$\det A = 0$ so this set has no unique solution.

For the second set, $A = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$

$$\det A = 1 \times -1 - 3 \times 0 = -1$$

A unique solution exists.

For the third set, $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$\det A = 1 \times 1 - 0 \times -1 = 1$$

A unique solution exists.

For the fourth set, $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$

$$\det A = 2 \times 1 - 3 \times -1 = 5$$

A unique solution exists.

There are 3 unique solutions.

The answer is D.

Question 7

First, a transition matrix has column totals of 1. We can eliminate options D and E immediately.

Options B and C can be eliminated because the leading diagonal of both matrices doesn't contain zeroes meaning that they can return to the same place as last year.

The answer is A.

Question 8

Let the number of mining workers in town A in 2008 be n so the number of mining workers in town B in 2008 is also n . Let the number of mining workers in town B in 2009 be x .

So,

$$\begin{bmatrix} 0.6 & 0.8 \\ 0.4 & 0.2 \end{bmatrix} \begin{bmatrix} n \\ n \end{bmatrix} = \begin{bmatrix} 560 \\ x \end{bmatrix}$$

Multiplying, we get

$$0.6 \times n + 0.8 \times n = 560$$

$$1.4n = 560$$

$$n = \frac{560}{1.4}$$

$$= 400$$

The number of mining workers in town B (and town A) in 2008 was 400.

Note that we're not interested in finding the value of x , it was just introduced to fill a hole in the matrix.

The answer is C.

Question 9

The price matrix $\begin{bmatrix} 980 & 920 \\ 860 & 810 \\ 750 & 720 \end{bmatrix}$ is a (3×2) matrix.

We want to produce another (3×2) matrix so we need to multiply $(3 \times 2) \times (2 \times 2)$ to get a (3×2) matrix.

We can therefore eliminate options D and E.

We want the first column of our price matrix to remain the same and the second to be multiplied by 0.8 (20% reduction in price). Only option C offers this.

For example $980 \times 1 + 920 \times 0 = 980$ is element_{1,1}. Also $980 \times 0 + 920 \times 0.8 = 736$ is element_{1,2}.

The answer is C.