Neap:

Trial Examination 2007

# VCE Further Mathematics Units 3 & 4

Written Examination 1

## Suggested Solutions

## SECTION A – DATA ANALYSIS – CORE MATERIAL

**Question 1**

$$\frac{250 + 300 + 225 + 400 + 195 + 260 + 420}{7} = 292.86 \approx 293 \text{ (Closest to \$293).}$$

**Answer D**

**Question 2**

Range = \$420 – \$195 = \$225 (not \$215)

Therefore Option **D** is false. All other statements are true and correct. Therefore the correct answer is option **D**.

**Answer D**

**Question 3**

There are 25 results, therefore 25 students sat the test.

**Answer C**

**Question 4**

$Q_1 = 25$

$Q_3 = 43.5$

Interquartile range = $43.5 - 25$

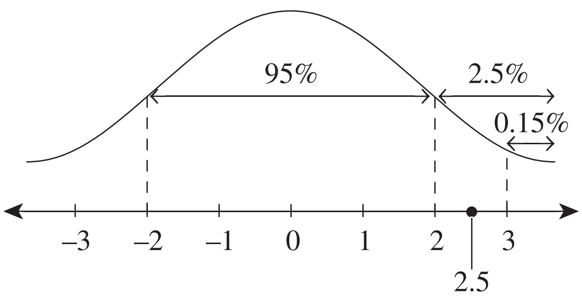
$= 18.5$

This is closest to 19, therefore option **A** is correct.

**Answer A**

**Question 5**

$z = 2.4$




The student's test mark was in the top 2.5% but not in the top 0.15%. Therefore option **E** is correct.

**Answer E**

**Question 6**

$$z = \frac{x - \bar{x}}{\sigma}$$

$$-1.3 = \frac{x - 62}{3.95}$$

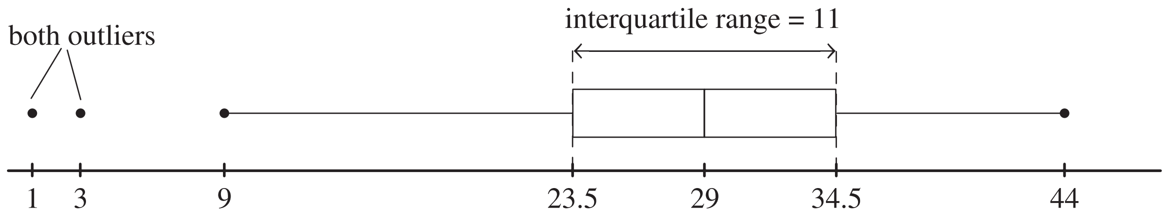
$$-5.135 = x - 62$$

$$\Rightarrow x = 56.87$$

$$\approx 57$$

**Answer B**

**Question 7**




**Answer B**

**Question 8**

$$b = r \times \frac{s_y}{s_x}$$

$$= 0.865 \times \frac{17.42}{4.76}$$

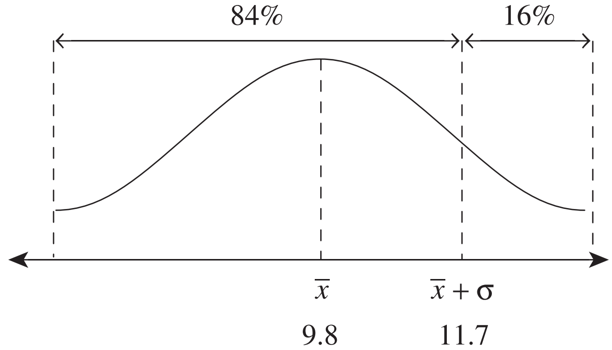
$$\cong 3.17$$

**Answer C**

**Question 9**

$\bar{x} = 9.8$

$\sigma = 1.9$




$< 11.7 \Rightarrow 84\%$ of results

**Answer E**

**Question 10**

Using a graphics calculator, enter the temperature ranges into the L1 column and the average rainfall data into the L2 column. Once this has been done you can then use the graphics calculator to come up with Pearson's product–moment correlation coefficient $r = -0.9552$ .

**Answer B**

**Question 11**

From the equation it can be seen that for every 1°C in the temperature range there will be a corresponding decrease of 10.69 cm in the average rainfall. This information coincides with option C.

**Answer C**

**Question 12**

| | | | | |
|---|---|---|---|---|
| 1997 | 5 | | | |
| 1998 | 8 | 7.5 | | |
| 1999 | 6 | | 8.5 | 8 |
| 2000 | 11 | | | |
| 2001 | 9 | | | |
| 2002 | 10 | | | |
| 2003 | 5 | | | |
| 2004 | 12 | | | |
| 2005 | 9 | | | |
| 2006 | 13 | | | |

**Answer B**

**Question 13**

Spring 2007 is quarter number 8. Therefore deseasonalised sales for spring 2007 can be calculated the following way:

$$\begin{aligned} \text{sales} &= 185 + 38.9 \times 8 \\ &= 496.2 \text{ or } 496.200 \\ \therefore\ \text{deseasonalised sales} &= 1.02 \times \$496\,200 \\ &= \$506\,124 \end{aligned}$$

**Answer E**

## SECTION B – MODULES

### Module 1: Number patterns

**Question 1**

The numbers are equally spaced. They have a common difference of –4. Thus the sequence is arithmetic with a negative common difference.

**Answer D**

**Question 2**

$$a = 32$$

$$r = \frac{24}{32} = \frac{3}{4}$$

$$S_\infty = \frac{32}{1 - \frac{3}{4}} = 128$$

**Answer E**

**Question 3**

If 80% are killed, 20% remain. Thus $r = 0.2$

$$t_n = ar^{n-1}$$

$$= 2(0.2)^{n-1}$$

**Answer E**

**Question 4**

$$ar = 36 \text{ ...(1)}$$

$$ar^3 = 16 \text{ ...(2)}$$

$$\frac{(2)}{(1)} = r^2 = \frac{16}{36}$$

$$r = \pm\frac{4}{6} = \pm\frac{2}{3}$$

$$t^3 = t_2 \times r$$

$$= 36 \times \frac{2}{3} \quad \text{or} \quad 36 \times \frac{-2}{3}$$

$$= 24 \quad \text{or} \quad -24$$

**Answer A**

**Question 5**

We already know that $f_1 = 2$

$$f_2 = 2f_1 - 3$$
$$= 1$$
$$f_3 = 2(1) - 3 = -1$$

**Answer B**

**Question 6**

It is immediately clear that this sequence is alternating. Each term is multiplied by a negative value. It is also clear that a positive value is added. Option **A** is thus not possible. Likewise, option **C** is not an option as the values would be too large. All of the remaining options correctly calculate $t_2$. However, only option **D** correctly determines $t_3$ from $t_2$.

**Answer D**

**Question 7**

$$t_3 = t_2 - t_1 + 1$$
$$= 2 - 4 + 1 = -1$$
$$t_4 = t_3 - t_2 + 1$$
$$= -1 - 2 + 1 = -2$$
$$t_5 = t_4 - t_3 + 1$$
$$= -2 - (-1) + 1$$
$$= 0$$

**Answer C**

**Question 8**

$$S_5 = t_3$$
$$\frac{5}{2}[2a + 4d] = a + 2d$$
$$5(a + 2d) = a + 2d$$
$$4(a + 2d) = 0$$
$$a = -2d$$

$$t_3 = a + 2d$$
$$= -2d + 2d$$
$$= 0$$

**Answer A**

**Question 9**

The amount grown, $g_n$, is given by the geometric sequence:

$$g_n = 40\left(\frac{3}{4}\right)^{n-1}$$

Thus $t_n = t_{n-1} + 40\left(\frac{3}{4}\right)^{n-1}$ is the total height of the tree.

**Answer C**

## Module 2: Geometry and trigonometry

**Question 1**

$a^2 = h^2 - b^2$

$a^2 = 130^2 - 40^2$

$a^2 = 15\ 300$

$a \approx 123.69$ m

**Answer D**

**Question 2**

$$\frac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$$

$$\frac{40}{130} = \cos\theta$$

$$72.08° \approx \theta$$

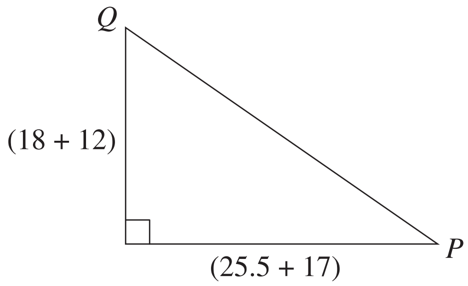
**Answer C**

**Question 3**

The vertical distance between $Q$ and $P$ is $18 + 12 = 30$ m

The horizontal distance between $Q$ and $P$ is $25.5 + 17$ m

Therefore, a simpler version of the problem is:




Thus option **A** is correct.

**Answer A**

**Question 4**

The original plan and the new plan demonstrate similarity. When two shapes demonstrate similarity, doubling a distance results in the quadrupling of the area.

**Answer D**

**Question 5**

$$\text{Area} = \frac{\text{top} + \text{bottom}}{2} \times \text{height}$$

$$\text{Area} = \frac{11 + 6}{2} \times 12$$

$$\text{Area} = 102 \text{ m}^2$$

$$\text{Volume} = \text{end area} \times \text{height}$$

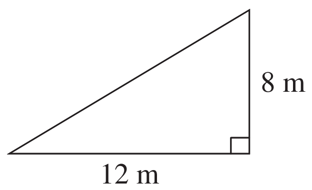
$$\text{Volume} = 102 \times 8$$

$$\text{Volume} = 816 \text{ m}^3$$

**Answer D**

**Question 6**

Base

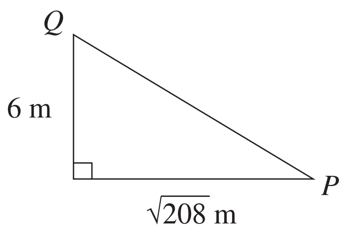



$$h^2 = a^2 + b^2$$

$$h^2 = 8^2 + 12^2$$

$$h^2 = 208$$

$$h = \sqrt{208}$$




$$h^2 = a^2 + b^2$$

$$h^2 = 6^2 + (\sqrt{208})^2$$

$$h^2 = 244$$

$$h \approx 15.62 \text{ m}$$

**Answer A**

**Question 7**

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{29.4}{\sin QPR} = \frac{37.4}{\sin 41°}$$

$$\frac{\sin QPR}{29.4} = \frac{\sin 41°}{37.4}$$

$$\sin QPR = \frac{29.4 \times \sin 41°}{37.4}$$

$$QPR \approx 31.04°$$

**Answer E**

**Question 8**

$$\angle QRP = 180° - (\angle PQR + \angle QPR)$$

$$\angle QRP = 180° - (41° + 31°)$$

$$\angle QRP = 108°$$

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$\text{Area} = \frac{1}{2} \times 29.4 \times 37.4 \times \sin 108°$$

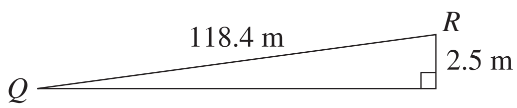
$$\text{Area} \approx 522.87 \text{ m}^2$$

$$\frac{522.87}{175} = 2.9878...$$

$$\approx 3$$

**Answer C**

**Question 9**




$$\frac{\text{opposite}}{\text{hypotenuse}} = \sin\theta$$

$$\frac{2.5}{118.4} = \sin\theta$$

$$1.2099° \approx \theta$$

**Answer A**

## Module 3: Graphs and relations

**Question 1**

There is an element of trial and error with this question. The point $(0, 2)$ does not work. Thus option **A** is false. The point $(1, 2)$ fails also, ruling out option **B**. Both $(6, 2)$ and $(-3, -4)$ are successful, however, and thus option **C** is correct.

**Answer C**

**Question 2**

The differences presented are simply variations in the $x$ intervals and the end-points. There is one open end point on the graph. It is the top end of the $y = 2x + 2$ line. All others are closed. Option **D** is thus correct.

**Answer D**

**Question 3**

It is necessary only to count the number of times that the graph crosses a horizontal line at $T = 30$.
It is four times.

**Answer E**

**Question 4**

If the entire first equation is multiplied by –2, then the resulting equation is the second equation. Thus they represent identical lines.

**Answer D**

**Question 5**

It is required that:

cost = revenue

$3n + 2000 = 5n$

Thus option **C** is correct.

**Answer C**

**Question 6**

Options **A** and **C** both have the correct gradient to be the negative-slope line. However, it is only the region below this line that is included, not even the line itself. Thus neither option is correct as they either select above the line or include the line. The other line is $y = \frac{x}{2}$. The shaded region is, again, below the line although this time the line itself is included.

**Answer D**

**Question 7**

The slope of the line can be obtained by re-arranging

$6x - 8y = 12$

$8y = 6x - 12$

$y = \frac{3}{4}x - \frac{3}{2}$

The lines quoted in options **A** and **B** are indentically sloped. They are both correct statements, as in option **D**. It can be seen from the re-arrangement that the $y$ intercept is $-\frac{3}{2}$. This proves option **C** to be true and option **E** to be false.

**Answer E**

**Question 8**

Some of the options that are listed lack credibility. Option **A** is untrue since the $y$-values are not equally spaced. For options **D** or **E** to be true, it would be necessary that the $y$-values become more closely spaced as $x$-values increase. This is not what the table shows. Thus compare options **B** and **C**.

| $x$ | 1 | 3 | 5 | 7 |
|---|---|---|---|---|
| $x^2$ | 1 | 9 | 25 | 49 |
| $x^3$ | 1 | 27 | 125 | 343 |
| $y$ | 40 | 56 | 88 | 136 |

From this table it is evident that $y$ increases at a rate proportional to that of $x^2$. Failing this, it would be possible to just plot $y$ against $x^2$.

**Answer B**

**Question 9**

Energy: $40x + 30y \geq 500$

Emissions: $20x + 10y \leq 250$

If both of these inequations are divided by 10, the result is that shown in option **C**.

**Answer C**

## Module 4: Business-related mathematics

**Question 1**

*Compound interest*

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$A = 40\ 000\left(1 + \frac{9.5}{100}\right)^3$$

$A = 52\ 517.30$

$I = 52\ 517 - 40\ 000$

$\quad = 12\ 500$

Therefore the total is closest to \$12 500.

**Answer C**

**Question 2**

*Simple interest*

$$R = 100 \times \frac{I}{(P \times t)}$$

$$R = 100 \times \frac{8700}{(21\ 500 \times 10)}$$

$R = 4.05$

Therefore the interest rate is closest to 4%.

**Answer C**

**Question 3**

*Compound interest*

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$3P = P\left(1 + \frac{7}{100}\right)^n$$

$$3 = (1.07)^n$$

*Method 1:*

Graph $y_1 = 3$

and $y_2 = (1.07)^x$

find the point of intersection $n = 16.24$

*Method 2:*

Using a graphics calculator, use the TVM Solver:

```
N=
I%=7
PV=-1
PMT=0
FV=3
P/Y=1
C/Y=1
PMT:END BEGIN
```

Answer: 16 years

**Answer D**

**Question 4**

*Loan repayment*

Using a graphics calculator, use the TVM Solver:

```
N=18
I%=8.82
PV=12000
PMT=-248
FV=
P/Y=12
C/Y=12
PMT:END BEGIN
```

Answer: $FV = -8936.65$

**Answer C**

**Question 5**

*Depreciation*

$$\begin{aligned} \text{book value} &= \text{cost price} - \text{unit cost} \times \text{number of units} \\ &= 35\,000 - 0.22 \times 65\,000 \\ &= 20\,700 \end{aligned}$$

**Answer A**

**Question 6**

*Depreciation*

book value = cost price – annual depreciation × number of years

$$1000 = 5350 - 0.175 \times 5350 \times n$$

$$n = \frac{4350}{0.175 \times 5350}$$

$$= 4.65$$

$$\approx 5 \text{ years}$$

**Answer B**

**Question 7**

*Mortgage loan*

*Part 1*

Using a graphics calculator, use the TVM Solver:

```
N=12
I%=7.52
PV=142000
PMT=-1500
FV=
P/Y=12
C/Y=12
PMT:END BEGIN
```

$FV = -134\,420.70$

The amount owing after 12 months is $134 420.70.

*Part 2*

Using a graphics calculator, use the TVM Solver:

```
N=
I%=8.02
PV=134420.7
PMT=-1500
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

$N = 137.15$

137 + 12 – 144 = 5 months

**Answer A**

**Question 8**

*Bank balance*

$$\begin{aligned}\text{July minimum} &= 1970\\ \text{August minimum} &= 4217\\ \text{September minimum} &= 4217\end{aligned}$$

$$\begin{aligned}\text{Interest} &= 1970 \times 0.0275 \times \frac{1}{12} + 4217 \times 0.0275 \times \frac{1}{12} + 4217 \times 0.0275 \times \frac{1}{12}\\ &= \$23.84\end{aligned}$$

**Answer A**

**Question 9**

*Percentage change*

Let the cost price be $100.

$$\begin{aligned}\text{marked price} &= 100 \times 2\\ &= 200\\ \text{discounted price} &= \text{marked price} \times \left(1 - \frac{\text{percentage discount}}{100}\right)\\ &= 200 \times \left(1 - \frac{25}{100}\right)\\ &= 150\\ \text{average profit} &= 100 \times 0.5 + 50 \times 0.5\\ &= 75\\ \text{percentage profit} &= \frac{75}{100} \times 100\\ &= 75\%\end{aligned}$$

**Answer D**

## Module 5: Networks and decision mathematics

### Question 1

Vertices $B$, $C$, $D$ and $F$ all have an odd degree.

| Vertex | Degree |
|---|---|
| $B$ | 3 |
| $C$ | 3 |
| $D$ | 1 |
| $F$ | 1 |
| **Total** | **8** |

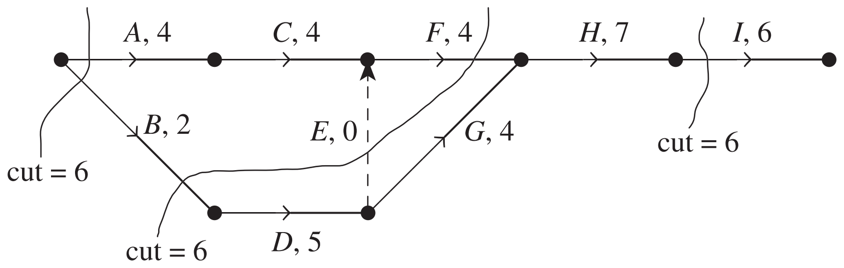
**Answer C**

### Question 2

Euler circuits must contain vertices with an even degree. Options **A**, **B**, **C** and **D** do not generate a network where every vertex has an even degree. However, option **E** does generate a network where all vertices have an even degree.

**Answer E**

### Question 3




The 'maximum flow' is always equal to the 'minimum cut'. The minimum cut is shown above. It happens to occur three times and is equal to 6 litres.

**Answer B**

### Question 4

Cut 1 = 15

Cut 2 = 13

Cut 3 = 15

Based on the capacity of each of the three cuts, only option **A** holds true.

**Answer A**

**Question 5**

Option **A** is true because Doris and Richard joined four activities in total. This is more than Ian's three activities.

Option **B** is false because Ian joined three activities and Ritsa only joined two.

Option **C** is false because Annmarie participated in the same number of activities as Ritsa, Richard and Doris.

Option **D** is false because Ritsa and Annmarie did not share an activity.

Option **E** is false because swimming and reading were more popular than participating in the Computer Club.

**Answer A**

**Question 6**

Option **A** is false because Mary did not defeat any player who defeated Mark.

Option **B** is true because Mary defeated Peter and Sarah, who both defeated Mark.

Option **C** is false because Sarah did not defeat Mary.

Option **D** is false because there are eight edges, hence eight tennis matches were played.

Option **E** is false because Mary did not defeat Newton.

**Answer B**

**Question 7**

The path $C$, $F$, $I$ is the critical path (in other words all other paths from the starting vertex to the finishing vertex take less time to complete). The path $C$, $F$, $I$ takes 15 hours to complete because $4 + 6 + 5 = 15$ hours.

**Answer D**

**Question 8**

There are four ways that three hours can be deducted from two activities. The new completion times of activities $H$ and $I$ after these four possible reductions is summarised in the table below.

| **New *H*** | **New *I*** |
|---|---|
| 0 | 5 |
| 1 | 4 |
| 2 | 3 |
| 3 | 2 |

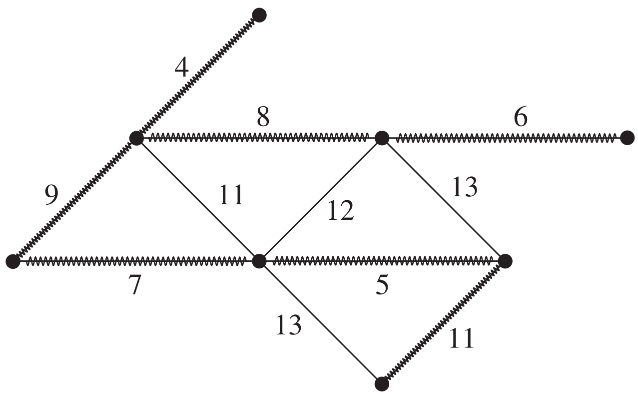
The table above is then used to calculate the critical path for each of the four new possible values of $H$ and $I$. This has been done in the table below.

| **New *H*** | **New *I*** | **Length of path *A, D, H*** | **Length of path *C, F, I*** | **Length of critical path** |
|---|---|---|---|---|
| 0 | 5 | 11 | 15 | 15 |
| 1 | 4 | 12 | 14 | 14 |
| 2 | 3 | 13 | 13 | *13* |
| 3 | 2 | 14 | 12 | 14 |

The length of the shortest critical path is 13. Therefore the minimum project completion time is 13 hours.

**Answer B**

**Question 9**




The edges selected in the above diagram are chosen by use of Prim's algorithm.

$4 + 9 + 7 + 8 + 6 + 5 + 11 = 50$ km

**Answer B**

## Module 6: Matrices

### Question 1

$$BA = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & -3 \\ -2 & -1 & 2 \\ -1 & 10 & 1 \end{bmatrix}$$

**Answer B**

### Question 2

$$\begin{array}{cccc} A & X & = & B \\ (2 \times 3) & (m \times n) & & (3 \times 2) \end{array}$$

$$\neq$$

There is clearly a problem with this equation. For it to be possible, matrix *B* must have the same number of rows as matrix *A*, but matrix *B* has more rows than matrix *A*.

**Answer A**

### Question 3

$$R = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$-2R = \begin{bmatrix} -6 & -2 \\ -4 & 2 \end{bmatrix}$$

**Answer D**

### Question 4

It is easy to find the determinants of the matrices in options **A** and **B**. They are –2 and 1, respectively. Use of the calculator is required for the other three matrices. The result of this is that the matrix in option **D** has a determinant of zero, so it is the singular matrix.

**Answer D**

### Question 5

It is required that the price of standard T-shirts becomes 112% of the previous price, and that the price of monogrammed T-shirts becomes 108% of the previous price. The price of neither should be influenced by the previous price of the other. Thus only diagonal elements should be non-zero. Option **D** achieves all of these objectives.

**Answer D**

**Question 6**

The subtraction of these two matrices would not result in any form of totalling. Neither would the results be in the form of a percentage. This leaves only options **A** and **B**. Since matrix $W$ represents selling prices and matrix $V$ represents cost prices, $W - V$ gives the profit, not the loss.

**Answer A**

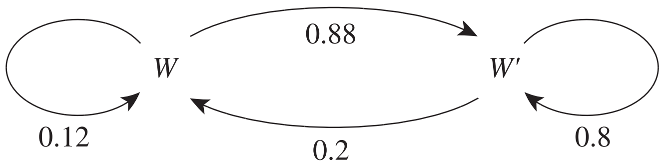
**Question 7**

The best process would be to form $2 \times 2$ matrices from the coefficients for each of these four sets of equations. The determinants of these coefficient matrices would be 1, 0, 0 and 1 for these sets of equations (left to right as shown in the question). Given that no singular coefficient matrix can give a unique solution, this means that two of the sets of equations have unique solutions.

**Answer C**

**Question 8**

$W$ and $W'$ represent "require work" and "not require work".




Thus the transition matrix is

$$\begin{bmatrix} 0.12 & 0.20 \\ 0.88 & 0.80 \end{bmatrix}$$

Note that $\begin{bmatrix} 0.80 & 0.88 \\ 0.20 & 0.12 \end{bmatrix}$ is also possible but this matrix is not provided as one of the options to this question.

**Answer D**

**Question 9**

It is worth noting that the sum of each of the three columns of the matrix is 1.0. Thus it is clear that no other company influences the market share of $P$, $Q$ or $R$. This eliminates option **A**.

All other options require the calculation of a steady-state matrix:

$$T^{50}S_0 \approx \begin{bmatrix} 0.143 \\ 0.536 \\ 0.321 \end{bmatrix}$$

Thus $Q$ has increased its market share at the expense of $P$.

**Answer D**