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Trial Examination 2006

VCE Further Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
A – Core	1	1	15
Section	**Number of modules**	**Number of modules to be answered**	**Number of marks**
B – Modules	6	3	45

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or CAS and, if desired, one scientific calculator. Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question booklet of 28 pages with a detachable formula sheet in the centrefold.
Working space is provided throughout this booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Please ensure that you write your **name** and your **teacher's name** in the space provided on this page. Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 VCE Further Mathematics Units 3 & 4 Written Examination 2.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions.

SECTION A – DATA ANALYSIS – CORE MATERIAL

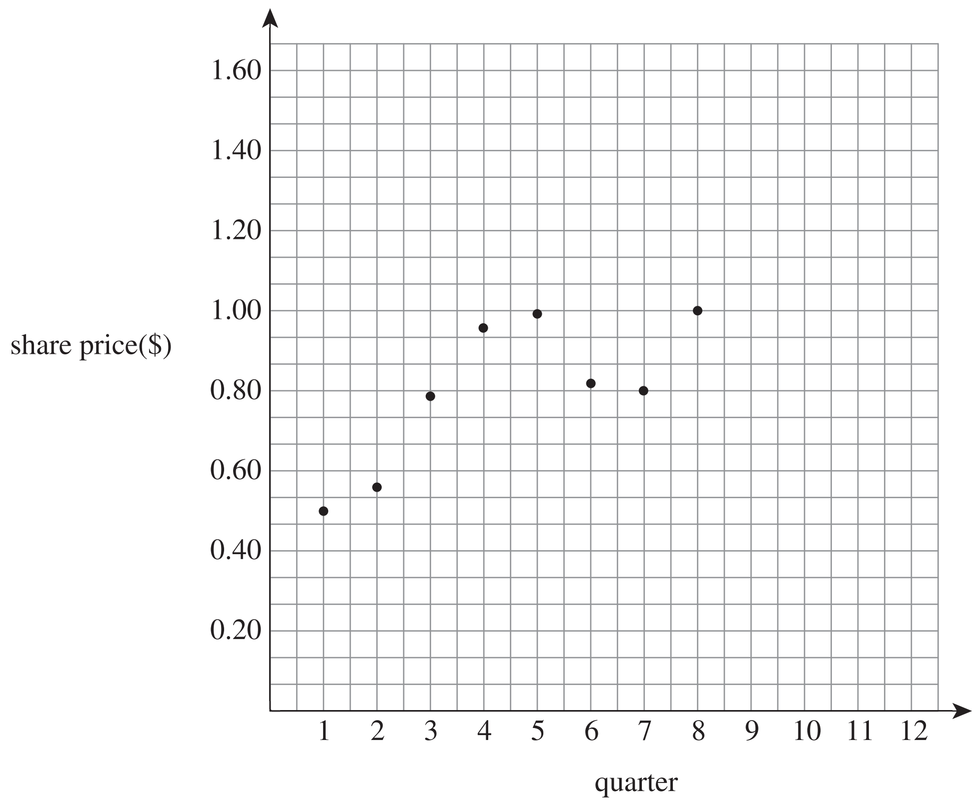
Question 1

The quarterly share price of a company listed on the Australian Stock Exchange (ASX) over a three year period is shown in the table below.

Quarter (1 – 12)		Share price
2003	Q1 (1)	$0.49
	Q2 (2)	$0.55
	Q3 (3)	$0.78
	Q4 (4)	$0.95
2004	Q1 (5)	$0.99
	Q2 (6)	$0.82
	Q3 (7)	$0.80
	Q4 (8)	$1.00
2005	Q1 (**9**)	**$1.05**
	Q2 (**10**)	**$0.87**
	Q3 (**11**)	**$1.35**
	Q4 (**12**)	**$1.58**

a. On the scatterplot below, the points representing the quarterly share price for 2003 and 2004 has been plotted, with **the quarter** on the horizontal axis and **the share price** on the vertical axis.

Plot the points representing the data for the remaining four quarters (as shown in bold in the table above).



2 marks

b. Use the data in the table provided on the previous page to determine

i. the equation of the least squares regression line that will enable the share price to be predicted from the relevant quarter. Write the missing values, correct to two decimal places, in the spaces provided.

share price = ______ + ______ $\times$ the relevant quarter

ii. the value of Pearson's product moment correlation coefficient. Write your answer correct to four decimal places.

$r =$

2 + 1 = 3 marks

c. **i.** Use the least squares regression line to predict the share price of the company at the end of the second quarter in 2006.

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____

ii. Is this extrapolation or interpolation?

____

1 + 1 = 2 marks

d. **i.** For each additional quarter, what is the average increase in the share price according to the equation of the least squares regression line? Give your answer to the nearest whole cent.

____

____

ii. What percentage of the variation in the share price can be explained by the variation in time (or the numerical value of the quarter)? Give your answer to the nearest whole percentage.

____

____

1 + 1 = 2 marks

e. **i.** Construct a neat and clear boxplot of the 12 quarterly share prices provided.

ii. Does the boxplot reveal the existence of any outliers? Explain your answer.

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____

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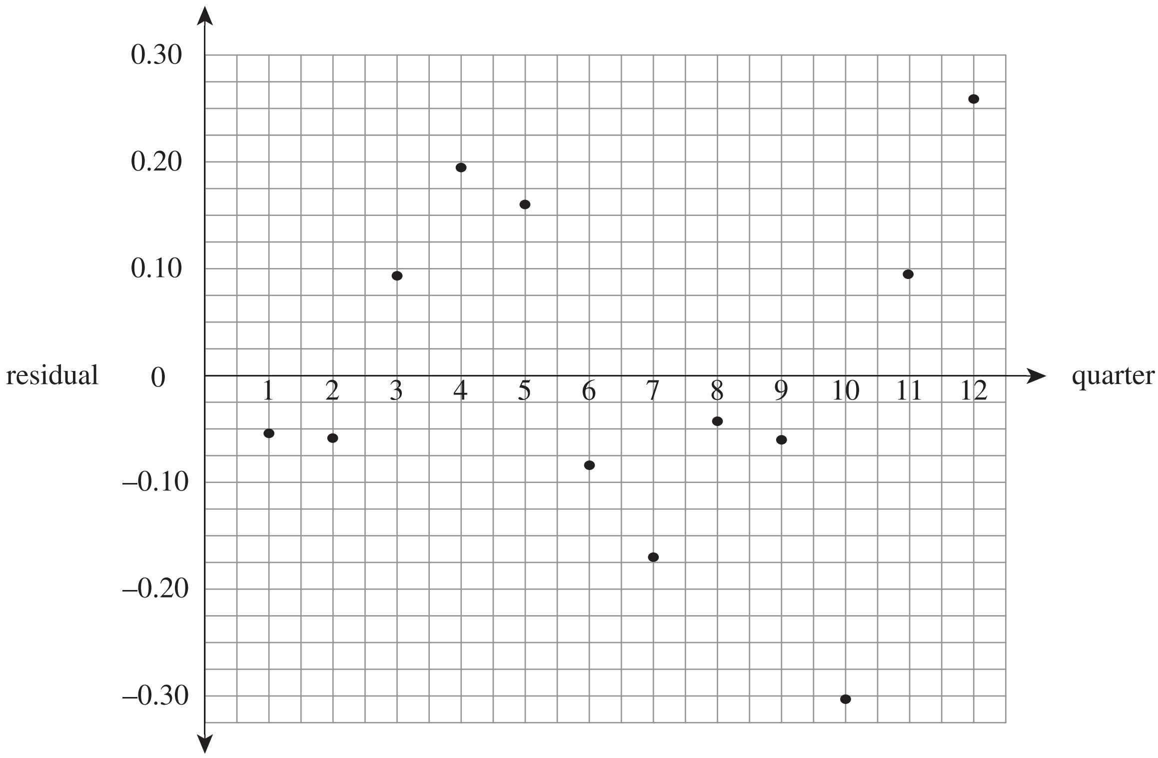
2 + 2 = 4 marks

f. A residual plot has been constructed for the share price of the company over the three year period.



From the residual plot is it reasonable to assume that the original data probably has a linear relationship? Explain your answer.

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____

____

2 marks
Total 15 marks

END OF SECTION A – CORE

SECTION B – MODULES

Module 1: Number patterns

Question 1

Brian and Daphne purchase an old hotel that they wish to improve and operate. Since it requires considerable maintenance and has not been in operation for some time, they know that it will take some time before many of the rooms are booked. Brian believes that they can book out 20% of the rooms in the first month and then increase this by a further 8% of the original number of rooms every month.

a. If this is occurs, what type of sequence will be formed by the monthly booking percentages?

1 mark

b. Under this scenario, in which month would 68% of the rooms be booked?

1 mark

Each room brings in revenue of $5000 per month when fully booked.

c. If Brian is correct, what would the mean revenue per room be in the first six months of operation?

2 marks

Daphne believes that a different type of sequence might be necessary to describe the booking percentages. She decides to focus on the percentage of rooms that are unbooked. She notices that this number reduces by the same proportion during the first three months.

month	1	2	3
rooms unbooked (%)	80	72	64.8

d. During which month would 68% or more of rooms be booked under this scenario? Write your answer correct to the nearest month.

2 marks

e. If this pattern continues indefinitely, what would be the total amount of money that each room would be expected to lose due to not being booked?

2 marks

Question 2

In order to carry out the repairs, Daphne and Brian require a loan. They decide to borrow \$100 000 and are offered a loan where they would pay 1.0% in interest every month. Following this interest being charged, a repayment of \$2000 would be made into the loan, reducing the balance.

a. Find a difference equation for t_{n+1}, the balance at the end of the $(n + 1)^{\text{th}}$ month of the loan, in terms of t_n.

2 marks

b. They decide, instead, to take out a loan for which the following difference equation applies:

$$t_{n+1} = 1.008(t_n - 1000);\ \ t_0 = 100\ 000$$

How much is owing after two months?

2 marks

c. How much is being repaid each month under this proposal?

1 mark

d. Daphne and Brian realise that this loan will take a long time to repay so they decide that they should try to pay it off faster by re-investing their tax deductions into the loan as extra repayments.
The process will now be (for month $n+1$):

- Repay the \$950 minimum repayment.
- Pay back 3% of this loan balance from month $n - 1$ in addition to this.
- Calculate interest at the same rate as indicated by the difference equation in part **b**.

Repayments and interest calculations are to occur in the same order as they have been done in parts **b** and **c**. Determine a difference equation to do this.

2 marks

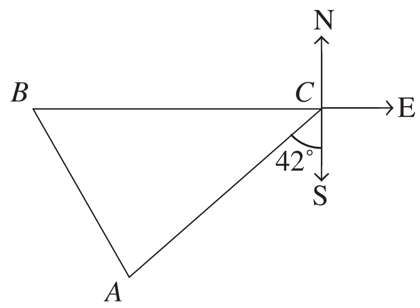
Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

A golf course is situated near a new housing estate. A large triangular block of land encloses the golf course.



The details of two of the boundaries are:

boundary	bearing	distance
B to *C*	East	1.24 km
C to *A*	S 42° W	1.40 km

a. Determine the angle *ACB*.

1 mark

b. Find the length *AB*. Write your answer in kilometres, rounded to two decimal places.

1 mark

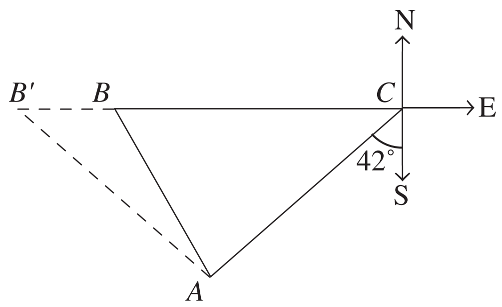
c. Calculate the area of the land. Write your answer rounded to the nearest 100 m^2.

1 mark

d. How far south of point *B* is point *A*? Write your answer in metres, rounded to the nearest 10 metres.

1 mark

An extension to the golf course is proposed. Under the proposal, boundary *BC* is to be extended by 310 m.



e. Find the length AB′ in kilometres. Write your answer in kilometres, rounded to two decimal places.

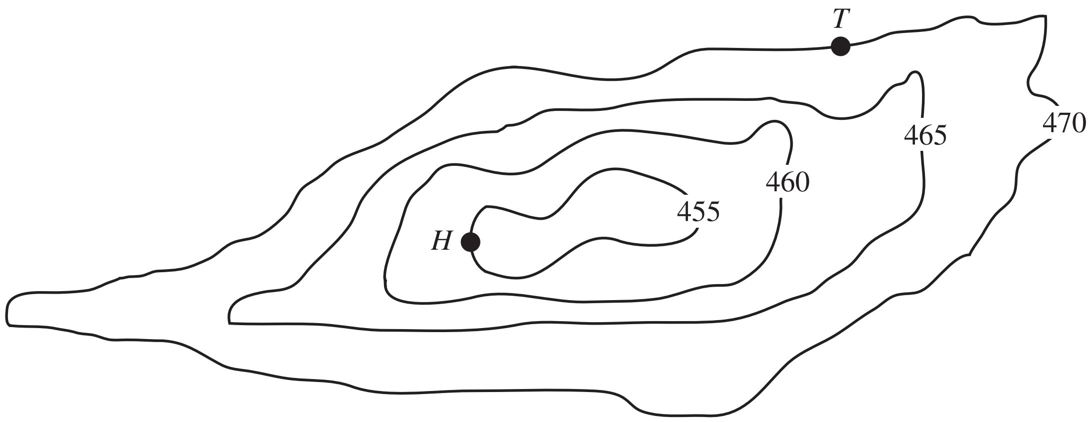
1 mark

f. Find the increase in the area of the proposed golf course with the extension. Write your answer rounded to the nearest 100 m^2.

3 marks

Question 2

The contour map below shows the layout of one hole of the golf course. It has contours drawn at intervals of five metres. There is a golf tee at *T* and a golf hole at *H*.



a. What is the difference in height (in metres) between the golf tee and the golf hole?

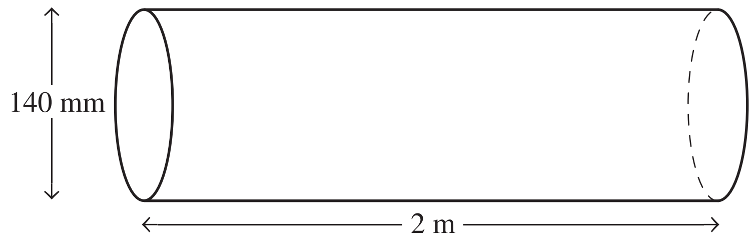
1 mark

b. The average slope between the golf tee and the golf hole is 3°11'. Find the horizontal distance between the golf hole and the golf tee. Write your answer in metres, rounded to the nearest metre.

1 mark

Question 3

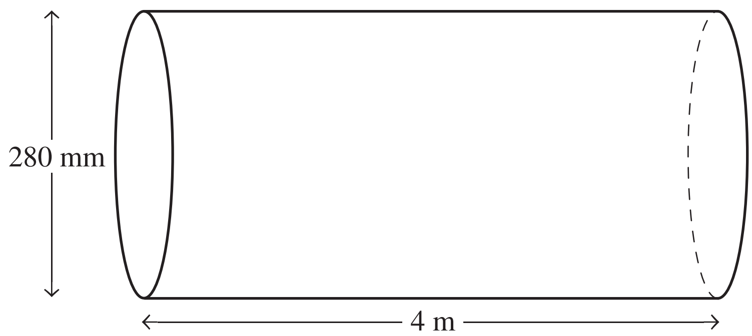
A standard pipe used to water the golf course has a diameter of 140 mm. To replace a leak, a length of pipe two metres long is ordered.



a. Find the volume of the new section of pipe. Write your answer in cm^3, rounded to the nearest cm^3.

2 marks

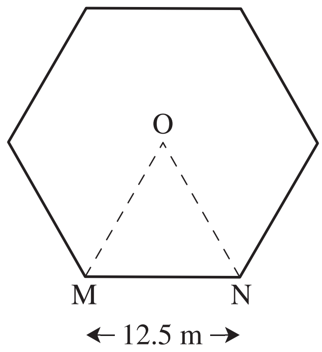
An incorrect piece of pipe was delivered to the golf course. Its length was 4 m and its diameter was 280 mm.



b. What is the ratio of the volume of the incorrect pipe to the volume of the correct pipe?

1 mark

The new pipe is to be used to water a section of lawn constructed in the shape of a regular hexagon. The length of *MN* is 12.5 m.



c. Calculate the area of the lawn. Write your answer correct to the nearest m^2.

2 marks
Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

To operate a certain shoe factory it is necessary to have three basic raw materials: leather uppers, rubber soles and the binding to hold it together. The quantities of each required will depend on the shoes being made. The details for quantities per pair, for each of the standard and deluxe shoes, are shown in the table below.

	leather (cm^2)	rubber (g)	binding (m)
standard	6	45	2
deluxe	8	50	5

The following quantities of materials are available each week: 7200 cm^2 of leather, 50 000 g of rubber and 4000 m of binding. Let x be the number of pairs of standard shoes and y be the number of pairs of deluxe shoes.

a. Write down the constraints involving x and y.

2 marks

b. If each standard pair makes a profit of \$12 and each deluxe pair makes a profit of \$16, write down an equation for the profit in terms of x and y.

1 mark

Trial Examination 2006

VCE Further Mathematics Units 3 & 4

Written Examination 2

Formula Sheet

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

FURTHER MATHEMATICS FORMULAS

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series:	$a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}[2a + (n - 1)d] = \dfrac{n}{2}(a + l)$
geometric series:	$a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}$, $r \neq 1$
infinite geometric series:	$a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}$, $\lvert r \rvert < 1$

Module 2: Geometry and trigonometry

area of a triangle:	$\dfrac{1}{2}bc\sin A$
Heron's formula:	$A = \sqrt{s(s - a)(s - b)(s - c)}$ where $s = \dfrac{1}{2}(a + b + c)$
circumference of a circle:	$2\pi r$
area of a circle:	πr^2
volume of a sphere:	$\dfrac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\dfrac{1}{3}\pi r^2 h$
volume of a cylinder:	$\pi r^2 h$
volume of a prism:	area of base × height
volume of a pyramid:	$\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

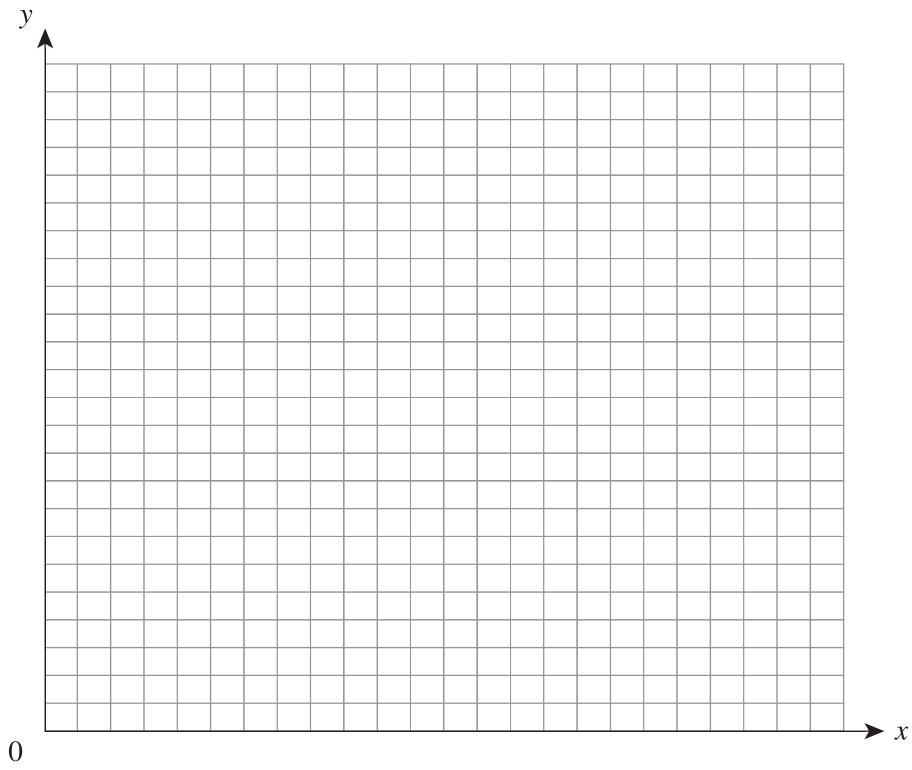
Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

c. On the set of axes below sketch a graph of these constraints and indicate the feasible region.



3 marks

d. Determine the coordinates of the corners of the required region. It may be assumed that quantities may be averaged out from week to week and that the coordinates may be given to one decimal place.

3 marks

e. Hence determine the greatest possible profit that may be obtained and the corresponding quantities of each of the supplements.

2 marks

Question 2

The managers of the shoe factory are expanding their business so the number of shoes that they can make per week will be progressively increased until a capacity of 50 000 pairs per week is reached. The table below gives the number of shoes made in each of the first four months during this time.

week (t)	1	2	3	4
production (p)	10 000	30 000	36 667	40 000
deficit (d)				

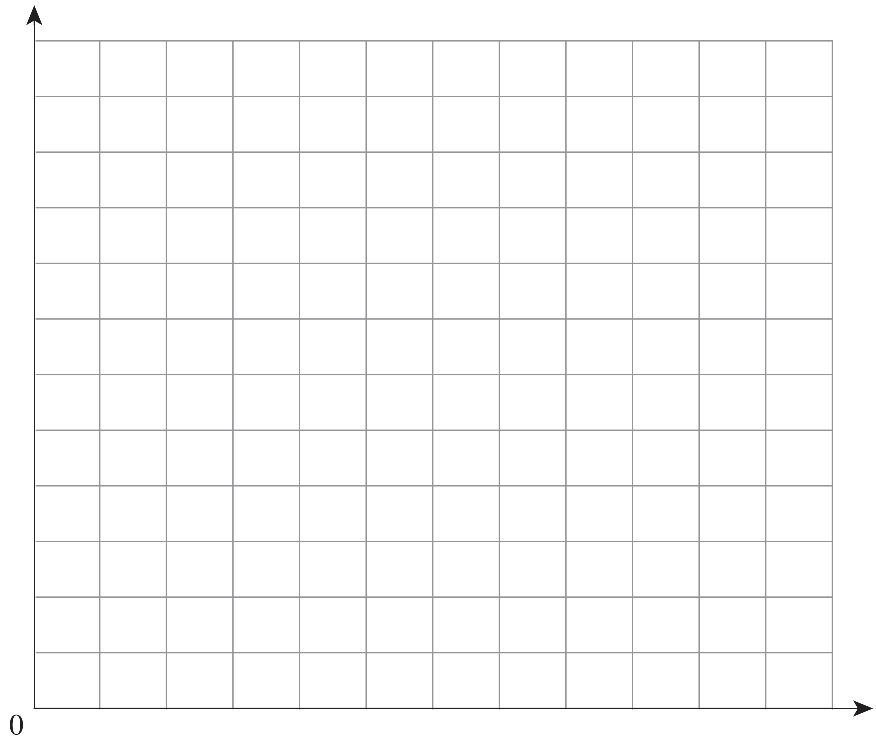
a. The deficit is the number of pairs that the factory is making below capacity. Fill in the last row of the table.

1 mark

b. It has been determined that the relationship between deficit and week number is non-linear. One of the managers has also stated that a relationship of the form $d = kt^2$ will not be successful either. Do you agree with this? Give reasons.

1 mark

c. The same manager believes that the relationship is of the form $d = \frac{k}{t}$. Plot a straight line to show that this is true and thus determine the value of k. Label the axes.



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2 marks
Total 15 marks

END OF MODULE 3

Module 4: Business–related mathematics

Question 1

Jason wants to buy a computer system. Its retail price is $3495. One option for Jason to finance the computer is to use a hire-purchase plan. The terms of the plan offered by a finance company are:

- 10% deposit with fortnightly instalments over three years at an interest rate of 8% per annum.

Give all answers correct to the nearest dollar.

a. How much will Jason need to withdraw from his savings account to pay the deposit?

1 mark

b. Calculate the total interest charge and the fortnightly repayments.

2 marks

c. What is the total cost of the computer?

1 mark

d. A personal loan is advertised at 13.5% per annum. For Jason to compare the interest rate he needs to convert the hire-purchase flat rate of interest to the effective interest rate. Calculate the effective interest rate correct to one decimal place.

1 mark

Question 2

Mary is 28 years old and is about to start a new job on a salary of $52 000 per year. She intends to invest 5% of her salary into a superannuation fund. The employer has to pay an additional 9% of her salary as part of the federal governments' Compulsory Superannuation Scheme. The government taxes the compulsory employer contribution at 15%, and the remainder is deposited into her fund.

a. Show that the amount (to the nearest dollar) that is contributed to the superannuation fund at the end of each month is $548.

3 marks

b. If this amount is contributed to the superannuation fund for 32 years, until Mary reaches 60, calculate the amount (to the nearest \$100) in her account if the average interest rate during that time is 6.5% per annum.

2 marks

c. If the average inflation rate during those 32 years was 2% per annum, calculate the salary (to the nearest \$100) that she would need to earn at age 60 to have the same purchasing power as her starting salary of \$52 000.

1 mark

The amount that she contributes will rise due to inflation, and the size of her account in the fund when she retires at age 60 is estimated to be \$881 000. At retirement Mary would cease making contributions and begin drawing a pension from her superannuation fund. She estimates she would require \$70 000 per year.

d. If the fund continued to earn 6.5% per annum, and she drew a monthly pension, calculate

i. how much she would have in her account 20 years after retirement. Write your answer correct to the nearest \$100.

ii. how long her funds would last if her account ended up with a zero balance.

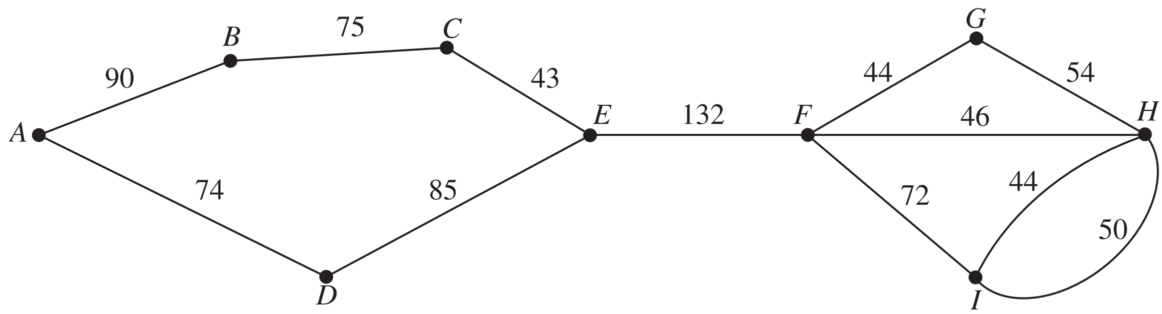
2 + 2 = 4 marks
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

The network diagram below represents the distances, in kilometres, between ports in major cities of the Greek islands. An underwater electricity grid is to be constructed so that each city can access electricity.



a. Find the shortest distance between Port D and Port G.

1 mark

b. On the diagram above, clearly draw the minimum spanning tree. What is the distance of the minimum spanning tree?

2 marks

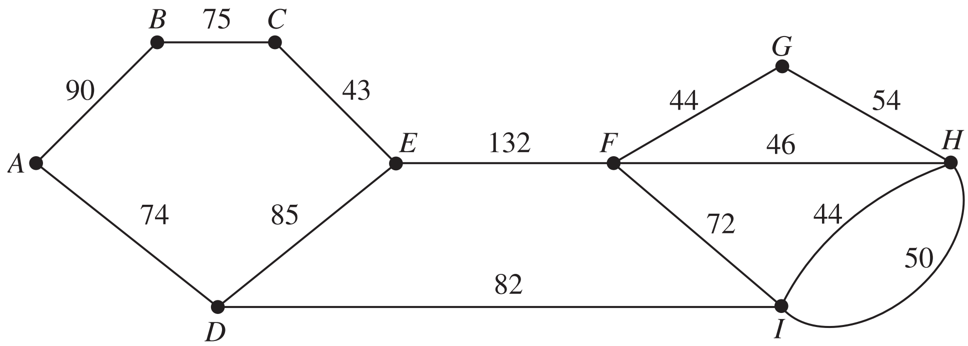
A new route is available for ships to connect Port D directly with Port I. This route is 82 km long.

c. Find the distance saved by travelling from Port D to Port G via the new route.

1 mark

A supply company intends to visit the port in each city. The company does not need to start and finish in the same port.

d. Find the shortest Hamiltonian Path from Port A to Port G. Use the network below to clearly draw your Hamiltonian Path.



1 mark

e. What is the length of the shortest Hamiltonian Path from Port A to Port G?

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1 mark

f. Due to faults with the underwater electricity grid each edge needs to be explored by a maintenance crew. Is it possible to travel along each edge only once and start and finish at the same port? If not, give reasons.

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2 marks

g. Adding one cable (edge) will allow the crew to check all edges, without requiring them to visit the same edge more than once. Between which two vertices must the edge be placed?

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1 mark

h. Write down the mathematical term used to describe the route in part **g**.

---

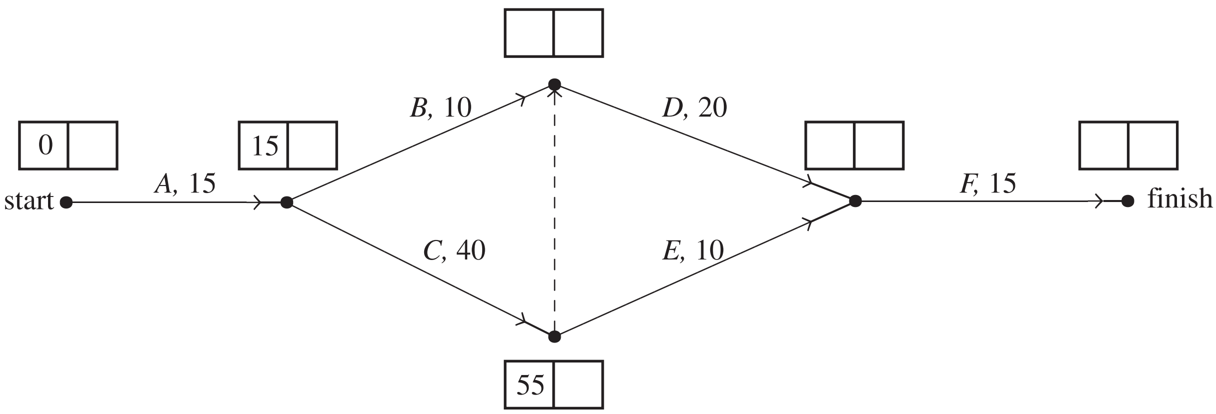
1 mark

Question 2

A company uses an organised approach to check safety aspects prior to each ship leaving a port (e.g. checking life jackets, testing emergency sirens etc.). The activities required for this safety check are shown in the table below.

activity	immediate predecessor	duration (min)
A	–	15
B	*A*	10
C	*A*	40
D	*B*, *C*	20
E	*C*	10
F	*D*, *E*	15

The network diagram below shows the **earliest starting times** for some activities.



a. Find the **earliest start time** for activity *D*.

1 mark

b. Find the overall project completion time.

1 mark

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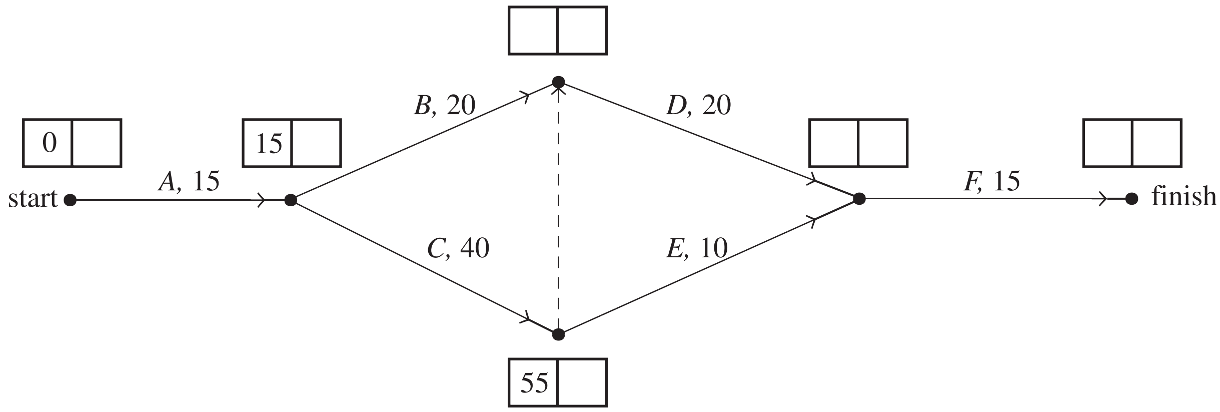
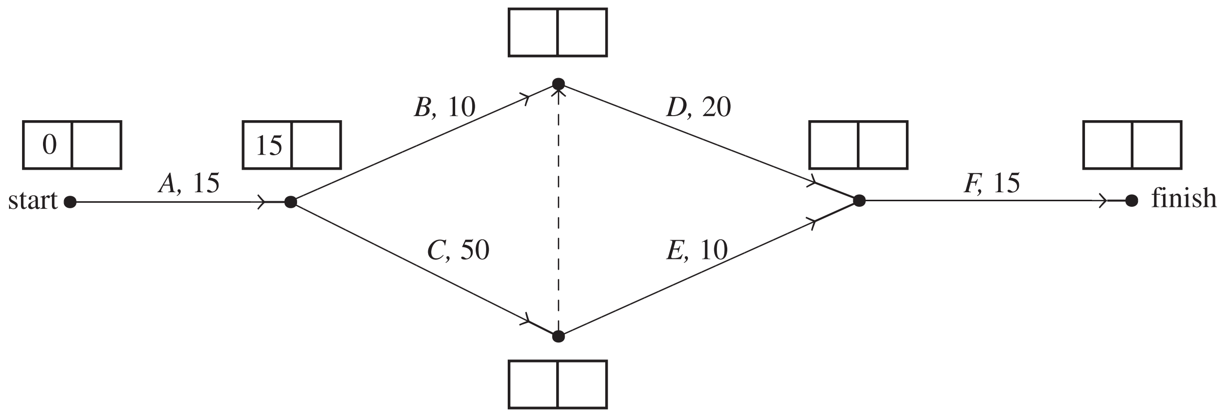
Due to a staff member's illness one activity had to be delayed by 10 minutes. The project manager had to decide between delaying activity *B* by 10 minutes or delaying activity *C* by 10 minutes.

c. Find the overall completion time if activity *B* is delayed by 10 minutes.



1 mark

d. Find the overall completion time if activity *C* is delayed by 10 minutes.



1 mark

e. Which activity (activity *B* or activity *C*) should the project manager delay to guarantee that the overall project is not delayed?

1 mark
Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

Bill operates a swimming pool and leisure goods company. The company manufactures four different types of pool cleaning powder, depending on the strength required.

Their characteristics are as follows (units per kilo).

type	chlorine	sea salt	coagulant	bleach
Envirosafe	0	10	2	1
Industrial	10	1	2	3
Home	3	3	1	2
Slow-release	7	4	3	4

Bill decides to use matrix methods to determine the quantities required of each material. He produces matrix

$M = \begin{bmatrix} C \\ S \\ A \\ B \end{bmatrix}$ to record the amounts required of chlorine, sea salt, coagulant and bleach respectively. The matrix

$T = \begin{bmatrix} E \\ I \\ H \\ R \end{bmatrix}$ records the number of kilos of Envirosafe, Industrial, Home and Slow-release produced.

a. Write a matrix equation relating M and T.

1 mark

b. Bill receives orders for 120 kg of Envirosafe, 40 kg of Industrial, 210 kg of Home and 50 kg of Slow-release. Use matrix methods to determine the quantities of each component required.

2 marks

As summer approaches, demand becomes very high and the company only has the following quantities of materials:

- 45 kg of chlorine
- 64 kg of sea salt
- 21 kg of coagulant
- 26 kg of bleach.

c. Bill wants to utilise all of the materials. How many of each type of pool cleaner can they make?

2 marks

Question 2

The company decides to analyse their sales of pool cleaners. Bill has determined that he has 62% of the local market for pool cleaners. Of this, 21% is for Envirosafe, 20% is for Industrial, 46% is for Home and 13% is for Slow-release.

a. Determine the proportion that each of the four types comprise of the entire market.

1 mark

b. Hence write down a column vector for these proportions in the form:

$$N_0 = \begin{bmatrix} e \\ i \\ h \\ s \\ t \end{bmatrix}$$

where e = the proportion of the total market for **Envirosafe**.

i = the proportion of the total market for **Industrial**.

h = the proportion of the total market for **Home**.

s = the proportion of the total market for **Slow-release**.

t = the proportion of the total market for other brands.

1 mark

An industry body conducts a survey of pool owners. It determines the likely buying habits of customers in 2007 based upon their purchases in 2006. The results are as follows:

	Envirosafe	**Industrial**	**Home**	**Slow-release**	**other brands**
Envirosafe	80%	1%	2%	4%	7%
Industrial	1%	67%	10%	6%	9%
Home	2%	14%	66%	10%	7%
Slow-release	9%	7%	10%	73%	4%
other brands	8%	11%	12%	7%	73%

Thus, for example, 14% of those buying Industrial in 2006 will buy Home in 2007.

c. Produce a transition matrix based on this data.

1 mark

TEVFMU34EX2_QA_06.FM

For the following questions it should be assumed that intentions for the following year will be realised and that these same transitions will occur in following years.

d. Using matrix methods, determine the likely proportion of sales for each of the four types of pool cleaner and for other brands for 2007.

1 mark

e. Determine the sales for the two years after 2007 for each type of cleaner using matrix methods.

2 marks

f. If it is assumed that the transition matrix is applicable for the years preceding 2006, determine the number of each type of cleaner sold for 2005. Comment on the whether the results are sensible and realistic for this year and preceding years.

2 marks

g. Is it likely that the proportions of sales for each type of cleaner will become stable over time or will they continue to fluctuate? If stability is achieved, give the proportions of each type to the nearest 1%. Otherwise, provide evidence of continued fluctuation.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOKLET