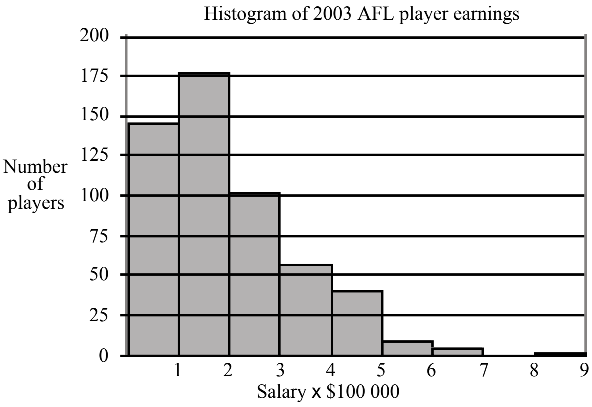
FURTHER MATHEMATICS EXAM 2: SOLUTIONS

Core: Data Analysis

Question 1

a.



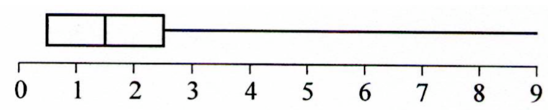
For labelling **[A1]**, for x-axis scale [A1], for correct columns **[A1]**

b.

$$IQR = Q_3 - Q_1$$
$$= 250000 - 50000 \qquad \textbf{[A1]}$$
$$= \$200000$$

Interquartile range for AFL player earnings.

c. Boxplot of AFL player Earnings for either 2003 & 2004



Annual Salary (× $100 000) [A1]

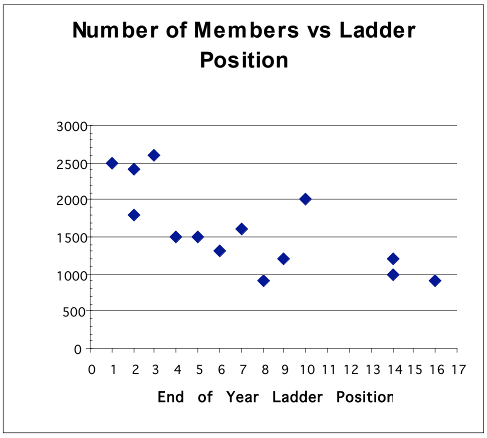
There are outliers. **[A1]**
This is indicated by the whisker being more than 1.5 times the length of the InterQuartile Range (IQR). [A1]

$$\text{Upper limit} = Q_3 + 1.5 \times IQR$$
$$= 250000 + 1.5 \times 200000$$
$$= 250000 + 300000$$
$$= 550000$$

There are at least 5 player that are outliers with salaries in excess of $600 000.

d. The two distributions are skewed and outliers affect the mean and standard deviation. **[A1]**
Also in 2004 there were a few more outliers that would affect the mean and standard deviation. **[A1]**

Question 2



a.

```
LinReg
 y=ax+b
 a=-88.20940468
 b=2236.367848
 r²=.5428271057
 r=-.7367680135
```

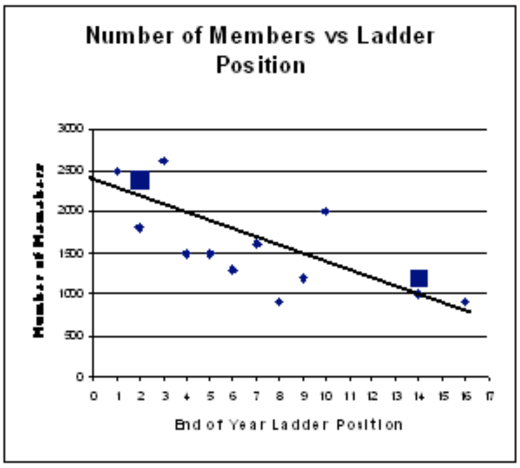
strength of the relationship, $r = -0.74$
direction of relationship – negative **[A1]**

b. "We can conclude from this that | 54 | % of the variation in the number of members can be explained by the variation in:

| End of year ladder position |

The other 46% variation in number of members is due to other factors." **[A2]**

c.



[A2]

d. Number of Members = – 100 x position on the ladder + 2417
Number of Members = – 100 x 12 + 2417
= – 1200 + 2417
= 1217 members

[M1][A1]

Module 1: Number patterns

Question 1

a. a = 192 and t_3 = a + 2d = 432;

192 + 2d = 432

2d = 432 – 192

2d = 240

d = 120

t_2 = 192 + 120 = **312** **[A1]**

b. Substituting a = 192 and d = 120 in

$t_n = a + (n-1)d$ **[M1]**

Then $A_n = 192 + (n-1)\times 120$ can be simplified to $A_n = 72 + 120n$ **[A1]**

c. $A_5 = 192 + (5-1)\times 120 = 672$ **[A1]**

d. Solve $192 + (n-1)\times 120 > 800$

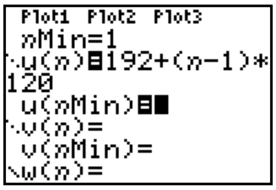
$72 + 120n > 800$

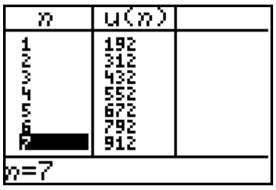
$120n > 728$

$n > 6.066$

In the 7th week she will first exceed 800 sandwiches. **[A1]**

Or using the calculator:





Question 2

a. a = 192 and $t_3 = ar^2 = 432$

$192\times r^2 = 432$

$r^2 = 2.25$

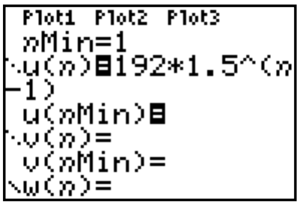
r = 1.5

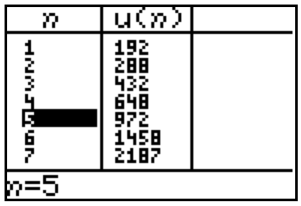
$t_2 = ar = 192 \times 1.5 = 288$ **[M1]**

b. $r = \dfrac{t_1}{t_2} = \dfrac{288}{192} = 1.5$ **[A1]**

c. Solve $192\times 1.5^n > 800$

Using trial and error or the calculator:-





In the 5th week the number of sandwiches sold will first exceed 800. **[H1]**

Question 3

a. Using the difference equation

$C_6 = 1.2\times C_5 - k$

$716 = 1.2 \times 680 - k$ **[M1]**

$716 - 816 = -k$

$-100 = -k$

$k = 100$ **[A1]**

b. $C_7 = 1.2 \times C_6 - 100$

$= 1.2 \times 716 - 100$

$= 759.2$ **[M1]**

$C_8 = 1.2 \times C_7 - 100$

$= 1.2 \times 759.2 - 100$

$= 811.04$

In the 8th week the predicted sales will be more than 800. **[A1]**

Question 4

a. 716 – 680 = 36

740 – 716 = 24

756 – 740 = 16

The sequence is 36, 24, 16, … **[A1]**

b. Maximum number = 680 + the sum to infinity of the sequence 36, 24, 16,…

36, 24, 16, … is a geometric sequence with $a = 36$ and $r = \frac{24}{36} = \frac{2}{3}$

$$S_\infty = \frac{a}{1-r} = \frac{36}{1-\frac{2}{3}} = \frac{36}{\frac{1}{3}} = 108$$ **[M1]**

Maximum number = 680 + 108 = 788 **[A1]**

Module 2: Geometry & Trigonometry

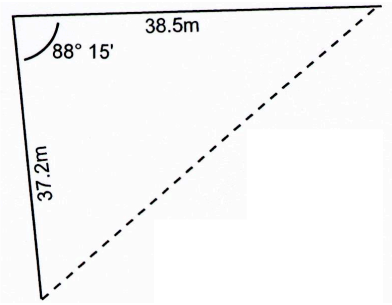
Question 1

a. The angle is the angle between the two given bearings.

356°30'T and 268°15' T where

$$\begin{array}{r} 356°30'T \\ -268°15'T \\ \hline 88°15' \end{array}$$ **[A1]**

b.



For length of BD use

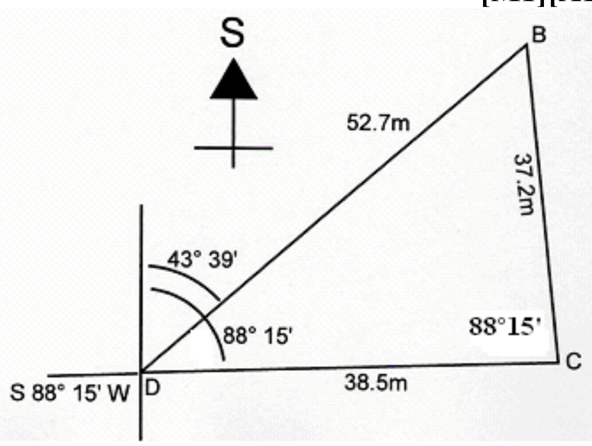
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 37.2^2 + 38.5^2 - 2 \times 37.2 \times 38.5 \times \cos 88.25°$

$\sqrt{c^2} = \sqrt{2778.6155}$

$c = 52.7126 = 52.7m$

[M1][A1]



[A1]

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{52.7}{\sin 88.25°} = \frac{37.2}{\sin B}$

$B = \sin^{-1}\left(\frac{37.2 \times \sin 88.25°}{52.7}\right)$

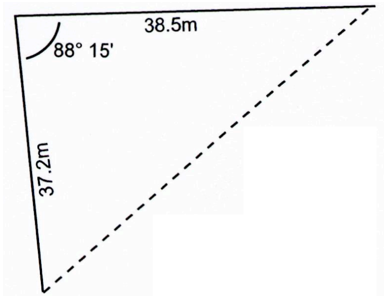
$B = 44.86°$

[M1]

Direction of D from B = 88.25°– 44.86°= 43.39°

Bearing = 180° + 43° = 223° T. **[A1]**

c.

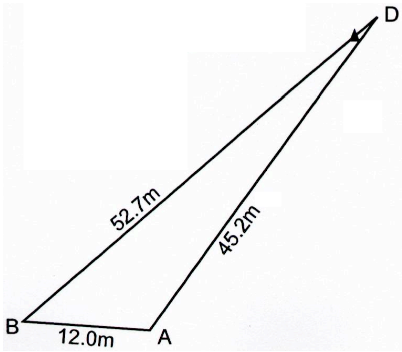


$A_{triangle} = \frac{1}{2}ab\sin C°$

$= \frac{1}{2} \times 37.2 \times 38.5 \times \sin 88.25°$ **[M1][A1]**

$= 715.766 = 716m^2$

d.



$s = \frac{a+b+c}{2}$

$= \frac{12.0 + 52.7 + 45.2}{3} = 54.95$

$A_{triangle} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54.95(54.95-12)(54.95-52.7)(54.95-45.2)}$

$= \sqrt{51774.749}$

$= 227.5407 = 228m^2$

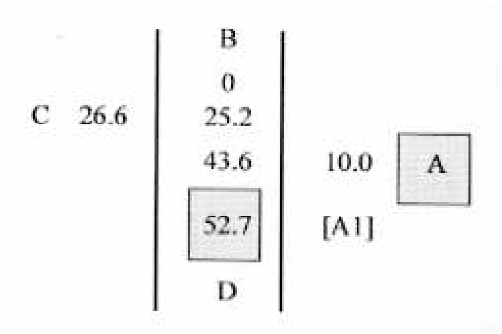
[M1][A1]

e. Area of Block = 715.766+227.5407

= 943.3067=943 m^2 **[A1]**

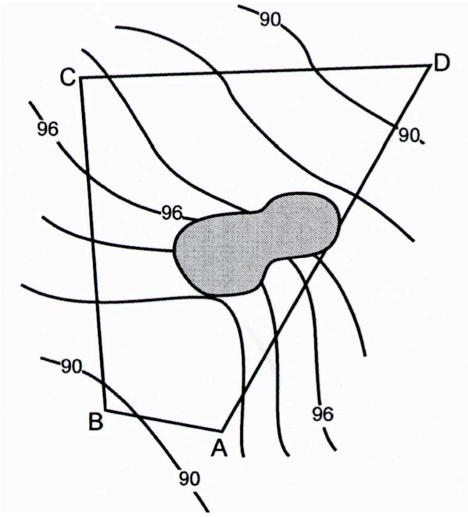
Question 2.

a.



[A1]

b.



[A1]

c. Any line that connects two ALTERNATE contour lines. **[A1]**

Module 3: Graphs and relations

Question 1

a. $\frac{x}{6}+\frac{y}{7}\le 50$

Lowest common denominator is $6 \times 7 = 42$.
Multiply both sides by 42:-

$7x + 6y \le 50 \times 42$

$7x + 6y \le 2100$

'7' **[A1]**

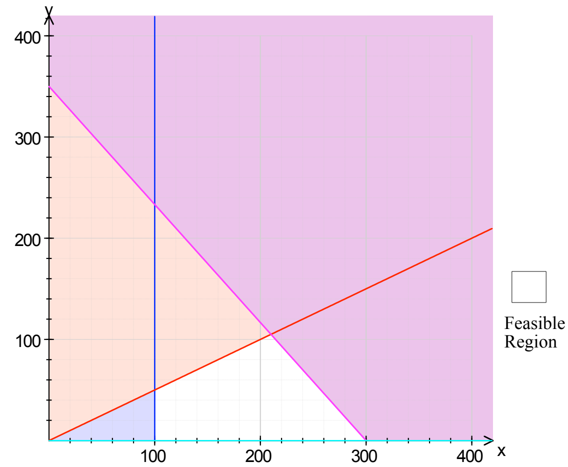
'2100' **[A1]**

b. $x \ge 100$ **[A1]**

c. $x \ge 2y$: divide both sides by 2

$\frac{x}{2} \ge y$ or $y \le \frac{x}{2}$ *or* $y \le \frac{1}{2}x$ **[A1]**

d.



Equation $7x + 6y = 2100$ **[H1]**

Equation $y = 1/2x$ **[H1**

$x = 100$ correct **[A1],** Region correct **[A1]**

e. $P = 24x + 22y$ **[A1]**

f. Four extreme points

Intersection of $y = \frac{1}{2}x$ and $7x + 6y = 2100$

Substitute $y = \frac{1}{2}x$ in $7x + 6y = 2100$:

$7x + 3x = 2100$

$10x = 2100$

$x = 210$

$y = \frac{1}{2}x$ so $y = 105$

Point (210, 105) **[H1]**

The other extreme points can be read from the graph: (100, 50), (100, 0), (300, 0) **[H1]**

(Marks can be awarded if the region is of similar difficulty)

g.

Extreme point	$P = 24x + 22y$
(210, 105)	7350
(100, 50)	3500
(100, 0)	2400
(300, 0)	7200

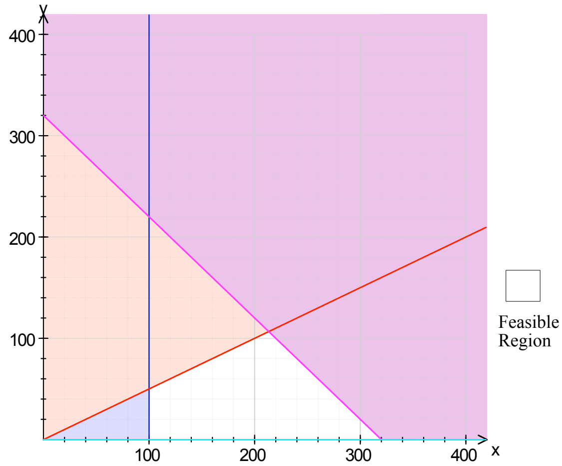
[M1]

Maximum profit when 210 *Standard* and 105 *Junior* footballs are made. [A1]

Question 2

The constraint $7x + 6y \leq 2100$ changes to $x + y \leq 320$

This changes the feasible region:-



Two extreme values change.

New points (320, 0) and the intersection of $y = ½\,x$ and $x + y = 320$

$x + ½\,x = 320$

$1.5\,x = 320$

$x = 213.333$

$y = 106.667$ [M1]

Profit for (213.333, 106.667) is $7466.67

Note: Whole footballs must be made.

Profit for (320, 0) is $7680.

The maximum profit now occurs when 320 Standard footballs are made. [A1]

Module 4 : Business Related Mathematics

Question 1

a. $10\%\ \text{GST} = \dfrac{\textit{GST included price}}{11}$

$= \dfrac{6490}{11}$

$= 590$

Answer = $590 GST [A1]

b. $6490 - \dfrac{8}{100} \times 6490 = 5970.80$ [A1]

He can expect to pay $5971

Question 2

a. Using the simple interest formula:

$I = \dfrac{PrT}{100}$ [M1]

Interest = $\dfrac{4000 \times 5.2 \times 1}{100} = 208$ [A1]

$208 in interest is earned in the account.

b. Monthly interest rate is $\dfrac{4.5}{12} = 0.375$

For the compound interest formula: $A = PR^n$,

$R = 1 + \dfrac{0.375}{100} = 1.00375$

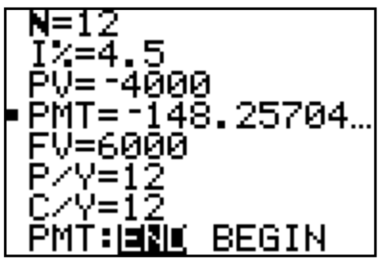
and P = 4000, n = 12 months

Amount accumulated:

$A = 4000 \times 1.00375^{12} = 4183.76$ [M1]

$4183.76 is accumulated. [A1]

c. Using the TVM solver on the calculator:-



[M1]

He will need to add $148.26 each month to his account to have a total of $6000 after a year.

[A1]

Question 3

a. 20% of \$6490 =

$\$6490 \times \frac{20}{100} = \$1298$ **[A1]**

b. \$1298 + 12×\$475 = \$6998 **[A1]**

c. \$6998 - \$6490 = \$508 **[A1]**

d. After paying the deposit of \$1298 John will owe \$6490 – \$1298 = \$5192 **[M1]**

Flat rate of interest =

$\frac{508}{5192} \times \frac{100}{1} = 9.78\%$ **[A1]**

e. Using the formula:

Effective rate of interest =

$\frac{2n}{n+1} \times Flate \quad rate$

where n = 12 payments. **[M1]**

Effective rate of interest

$= \frac{2 \times 12}{12+1} \times 9.784$

$= \frac{24}{13} \times 9.784$ **[A1]**

$= 18.08\%$

Module 5: Networks & decision mathematics

Question 1.

a. The four missing elements of the network.

	A	B	C	D	E	F	G	H	I
A	0	1	1	0	0	0	0	0	0
B	1	0	0	1	0	1	0	0	1
C	1	0	0	1	1	0	**0**	0	0
D	0	1	1	0	1	1	0	0	0
E	0	0	**1**	1	0	1	1	1	0
F	0	1	0	**1**	1	0	1	0	1
G	0	0	0	0	1	**1**	0	1	0
H	0	0	0	0	1	0	1	0	1
I	0	1	0	0	0	1	0	1	0

[**A2** for 4 correct], [**A1** for 3 correct]

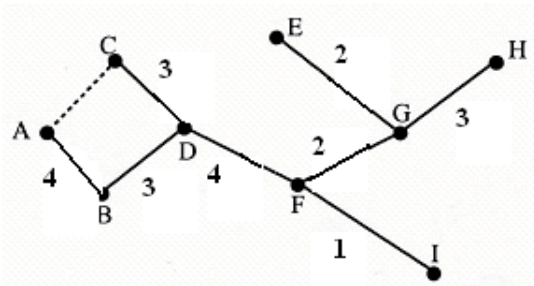
b. Hamiltonian circuit visits each town (or vertex) once only and as a circuit start and finish at the same town.

F – I – B – A – C – D – E – H – G – F

[**M1** for ending at Town F] **[A1]**

c. A – B – D – E – H for 15 kilometres **[A1]**

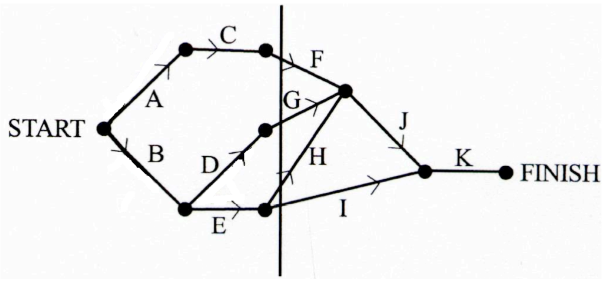
d.



[A2]

Question 2

a.



For A to F **[A1]** For B to G **[A1]**
For E, H and I **[A1]**

b. Critical path of the network is:
B – E – H – J – K **[A1]**

c.

Activity	Earliest start time	Latest start time
A	0	1
B	0	0
C	0.5	1.5
D	1	1.5
E	1	1
F	1	2
G	1.5	2
H	4.5	4.5
I	4.5	5.5
J	5	5
K	6.5	6.5

[A2] for 3 correct answers.
[A1] for 2 correct answers.

d. The earliest completion time for the project is 15.5 hours. **[A1]**

e.
Float time = Latest Finish Time – Earliest Start Time – Activity Time

= 5 – 1 – 3 = 1 hour **[A1]**