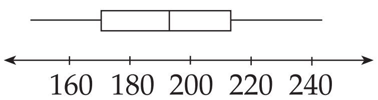
MAV Further Mathematics Examination 2
Answers & Solutions

Solutions

Core : Data analysis

Question 1

a. 5-number summary:
Min = 153
Lower quartile = 174
Median = 196
Upper quartile = 217
Max = 244



Box **[A]**
Whiskers **[A]**

b. i Shape : Symmetrical **[A]**

ii Centre : Attendance figures are centred around the median, 196. **[H]**

iii Spread : Attendance figures are spread over 91 values (range) **[H]**

OR The middle 50% of attendances are spread over 43 values (IQR)

Question 2

a. i Positive correlation. **[A]**

ii As **maximum daily temperature** increases the **attendance** also increases. **[A]**

b. **Attendance** = 79.3 + 4.01 × **max. daily temp.**
Variables correct **[A]**
Figures correct **[A]**

c. "increases by 4 " **[A]**

d. 20°:

Attendance $= 79.3 + 4.01 \times 20$
≈ 160 **[H]**

35°:

Attendance $= 79.3 + 4.01 \times 35$
≈ 220 **[A]**

Correctly plotted line.

(20,160); (35, 220) or sensible points **[H]**

e. **i.** $r^2 = (0.884)^2 \approx 0.781$ **[A]**

ii "78.1% of the variation in **attendance** can be explained by the variation in **maximum daily temperature**."
(sensible) **[H]**

Total 15 marks

Module 1 : Number patterns and applications

Question 1

a. 13,15 or a total of 28 seats **[A]**

The pattern goes up by +2 each time, that is 7, 9, 11, 13, 15

b. Common difference,
$d = t_2 - t_1 = t_3 - t_2 = t_4 - t_3$

Common difference
$= t_2 - t_1 = 9 - 7 = +2$
$= t_3 - t_2 = 11 - 9 = +2$
$= t_4 - t_3 = 13 - 11 = +2$ and so on **[A]**

c. Arithmetic sequence; substitute first term, $a = 7$, common difference, $d = 2$ and $n = 25$ into

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_n = \frac{25}{2}\left(2 \times 7 + (25-1)2\right)$$ **[M]**

$$S_{25} = 12.5(14 + 48)) = 775$$

Total of middle section is 775 seats. Hence total seating capacity for the dress circle is

3 x 775 = 2 325 dress circle seats **[A]**

d. From the ratio 25 shares = 2 325 seats therefore

1 share = 2 325 ÷ 25 = 93 seats **[M]**

total seats = (10 + 25) = 35 shares **[M]**

= 35 × 93 = 3255 seats **[A]**

Or alternatively

Upper circle = 10 shares

= 10 × 93 = 930 seats

Total seats = lower + upper

= 2325 + 930 = 3 255 seats in total

Question 2

a. Common ratio, $r = \frac{r_2}{r_1} = \frac{r_3}{r_2} = \frac{r_4}{r_3}$ if geometric

$$\frac{r_2}{r_1} = \frac{27000}{30000} = 0.9$$

$$\frac{r_3}{r_2} = \frac{24300}{27000} = 0.9$$

$$\frac{r_2}{r_1} = \frac{21870}{24300} = 0.9$$

and so on ⇒ $r = 0.9$ **[A]**

b. Common ratio r = 0.9 is 90% of previous weekly sales or (100% – 90%) a 10% weekly decrease **[A]**

c. Geometric sequence a = 30 000 , $r = 0.9$ and $t_n = 7\,000$

$t_n = ar^{n-1}$

$7000 = 30000 \times 0.9^{n-1}$ **[M]**

$\frac{7000}{30000} = 0.9^{n-1}$

$(n-1)\log 0.9 = \log \frac{7}{30}$

$n - 1 = \frac{\log_e(7 \div 30)}{\log_e 0.9}$

$n - 1 = 13.812..$

$n = 14.812..$

Therefore it will take 15 weeks before the total weekly sales drops below 7000. **[A]**

Or use Trial & Error Technique

$t_n = ar^{n-1}$

$7000 = 30000 \times 0.9^{n-1}$ Or **[M]**

$\frac{7000}{30000} = 0.9^{n-1}$

$0.9^{n-1} = 0.23333....$

try $n = 10$

$0.9^{10-1} = 0.9^9 = 0.387..$

which is greater than 0.23333

$n = 16$

$0.9^{16-1} = 0.9^{15} = 0.205..$ which is less than 0.23333

$n = 14$

$0.9^{14-1} = 0.9^{13} = 0.254..$ which is greater than 0.23333

$n = 15$

$0.9^{15-1} = 0.9^{14} = 0.228.$ which is just less than 0.23333

Question 3

a. $a = 100\% - 5\% = 95\%$ or 0.95

$b = +50$ **[A]**

b. The general form of a difference equation of the form $t_{n+1} = at_n + b$; $t_1 = 30\,000$ is

$t_n = a^{n-1}t_1 + \frac{b\left(1 - a^{n-1}\right)}{1 - a}$ and for

$a = 0.95$

$b = +50$; $t_1 = 30\,000$

$t_n = 0.95^{n-1} \times 30000 + \frac{50\left(1 - 0.95^{n-1}\right)}{1 - 0.95}$ **[M]**

$t_{30} = 0.95^{30-1} \times 30000 + \frac{50\left(1 - 0.95^{30-1}\right)}{1 - 0.95}$

$t_{30} = 7552.13068..$ which is greater than 7000 **[M]** or ALTERNATIVELY **an iteration technique** using the difference equation to generate 30 terms e.g. 30000, 28550, 27172.5, 25863.875, 24620.68125 ……

Therefore the show will still be above the 7000 limit **[A]**

Total 15 marks

Module 2 : Geometry and trigonometry

Question 1

a. $DB^2 = 33^2 + 30^2$ **[M]**

$DB = \sqrt{1989}$

≈ 44.60m **[A]**

b. **i.** $\tan(\angle ABD) = \dfrac{33}{30}$ **[M]**

$\angle ABD = 47.73°$ **[A]**

(Correct to two decimal places)

ii. $\angle DBC = 125° - \angle ABD$ **[A]**

$= 125° - 47.73°$

$= 77.27°$

(Correct to two decimal places)

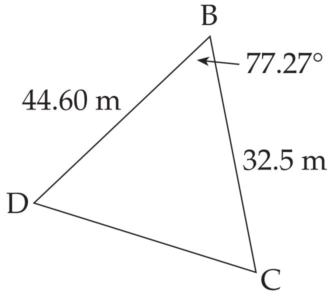
c. Right-angled triangle :

Area $= \dfrac{1}{2} \times \text{base} \times \text{height}$

$= \dfrac{1}{2} \times 30 \times 33$ **[M]**

$= 495 \text{ m}^2$ **[A]**

d.



Area $= \dfrac{1}{2} dc \sin B$

$= \dfrac{1}{2} \times 32.5 \times 44.6 \sin 77.27°$ **[M]**

$= 706.94 \text{ m}^2$ **[A]**

(Correct to two decimal places)

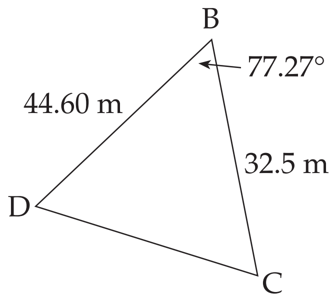
e. $706.94 + 495 = 1201.94\text{m}^2$

The estate agent has underestimated the area of the block by 1.94m^2

(2m^2 acceptable) **[H]**

Question 2

a.



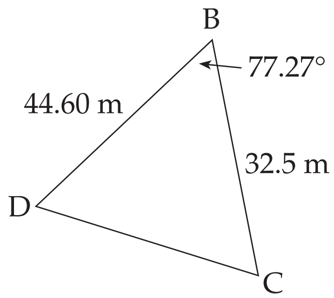
Using cosine rule :

$DC^2 = 44.6^2 + 32.5^2$
$- 2 \times 44.6 \times 32.5 \cos 77.27°$ **[M]**

$DC = 49.06$m **[A]**

(Correct to two decimal places)

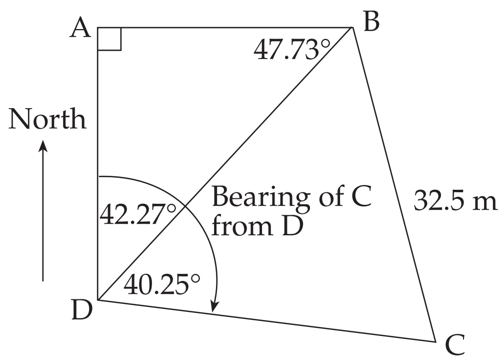
b.



$\dfrac{32.5}{\sin \angle BDC} = \dfrac{49.06}{\sin 77.27°}$

$\sin \angle BDC = \dfrac{32.5 \times \sin 77.27°}{49.06}$ **[M]**

$\angle BDC = 40.25° \approx 40°$ **[A]**



Bearing $= (42.27 + 40.25)°T$

$= 82.52°T$

$\approx 083°T$ **[A]**

(082°T acceptable)

Total 15 marks

Module 3 : Graphs and relations

Question 1

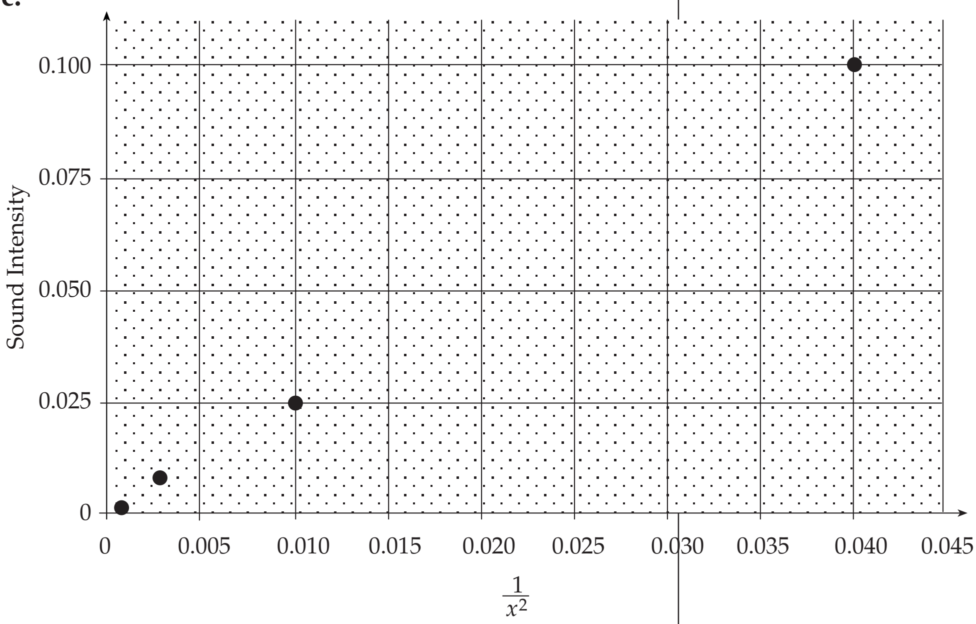
a. X-axis label – Distance from Speakers (metres) **[A]**

b.

Distance from Speakers, x (metres)	5	10	20	40
$\mathbf{\frac{1}{x^2}}$	**0.04**	**0.01**	**0.0025**	**0.000625**
Sound Intensity, I	0.1	0.025	0.00625	0.001563

[A]

c.



[A][A]

d. Yes as it forms a straight line **[A]**

e. Gradient of line using the two points (0.01,0.025) and (0.04,0.1)

$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

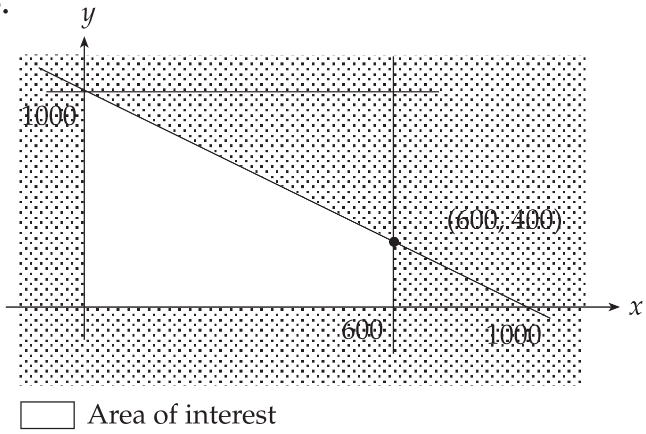
$$k = \frac{0.1 - 0.025}{0.04 - 0.01} = \frac{0.075}{0.03} = 2.5$$ **[M] [A]**

Question 2

a. Constraint 4 $x \le 600$ **[A]**

Constraint 5 $x + y \le 1000$ **[A]**

b.



[M]

Correctly drawing $x = 600$ and $y = 1000$ **[A]**

Correctly drawing $x + y = 1000$ **[A]**

c. $Z = 10x + 5y$ **[A]**

d. Maximum value of objective function occurs at (600,400)

Maximum ticket sales = \$8000 **[A]**

They must sell 600 front section tickets and 400 rear section tickets in order to achieve this maximum ticket sales revenue. **[A]**

Total 15 marks

Module 4 : Business related mathematics

Question 1

a. \$30 000 – \$20 000 = \$10 000 **[A]**

b. Investment A : \$750 (since simple interest) **[A]**

Investment B : \$749 (7% of \$10700) **[A]**

c. Points plotted at (2, 11500) and (2, 11449) **[M]**

d. Investment A drawn as a straight line through the points plotted **[M]**

Investment B drawn as an upward curve through the points plotted **[M]**

e. Year 3 **[A]**

f. Using a trial method or by setting up 2 equations on the calculator **[M]**
e.g. on the TI – 83

$y_1 = 10000 + 750x$

$y_2 = 10000 \times 1.07^x$

After year 7 both investments have reached \$15000 **[A]**

(this can be found from the table of values on the calculator)

Question 2

a. $n = 25 \times 12 = 300$ **[A]**

$R = 1 + \frac{\frac{6}{12}}{100} = 1.005$ **[A]**

b. Substituting all the correct values into the formula

$0 = 180000 \times 1.005^{300} - \frac{Q(1.005^{300} - 1)}{1.005 - 1}$ **[M]**

Solving to give Q = \$1159.74 **[A]**

This could be done using the TVM solver on the TI-83 as follows

N = 300

I% = 6

PV = –180000

PMT (to be solved)

FV = 0

P/Y = 12

C/Y = 12

c. $300 \times 1159.74 - 180000 = \$167922$ **[A]**

d. The extra interest paid because of the higher principal would mean that Bob would pay more to the bank overall. **[A]**

Total 15 marks

Module 5 : Networks and decision mathematics

Question 1

a. It is a connected graph with all vertices having an even degree **[A]**

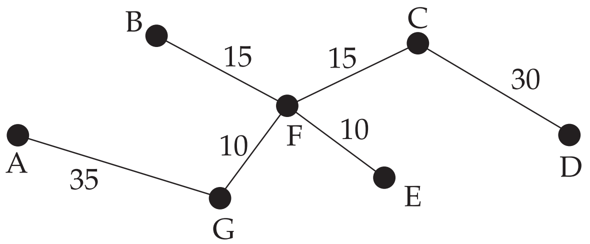
b. **[A]**

$A \to B \to C \to D \to E \to C \to F \to E \to G \to F \to B \to G \to A$

other answers possible

c. $A \to B \to C \to D \to E \to F \to G \to A$ **[A]**

d.



[M]

e. sum of weights =115 **[M]**

Award this mark provided the sum is correct for their graph (must be a tree)

Question 2

a. E drawn from end of activity A.
F drawn from end of activities C and D **[M]**

G drawn from the meeting of the ends of activities E and F **[M]**

Note : arrows must be included

b. Earliest start time for F is 7 **[A]**

Latest start time for D is 3 **[A]**

c. Critical path $A \to C \to F \to G$ **[A]**

d. Project completion time is 20 mins **[A]**

e. Float (slack) time for this activity is 2 minutes therefore they will go 1 minute overtime resulting in a \$10 fine **[A]**

Question 3

Ken to Oval 3 , Mark to Oval 2,
Richard to Oval 1, Tony to Oval 4

Use of Hungarian algorithm or other appropriate method **[M]**

Two coaches assigned to the correct ovals **[A]**

All four coaches correctly assigned **[A]**

Working for Hungarian algorithm (generally accepted method)

1. Subtract the minimum entry in each row from each element in the row

	Oval 1	Oval 2	Oval 3	Oval 4
Ken	5	0	11	3
Mark	5	0	13	3
Richard	0	2	18	0
Tony	3	4	9	0

2. Repeat for any column without a zero entry

	Oval 1	Oval 2	Oval 3	Oval 4
Ken	5	0	2	3
Mark	5	0	4	3
Richard	0	2	9	0
Tony	3	4	0	0

3. Minimum number of lines to cover the zero elements is 3 (through column 2 and rows 3 and 4). Since this is less than the number of rows proceed to 4.

4. The lowest uncovered element is 2. Add this to the covered rows and columns

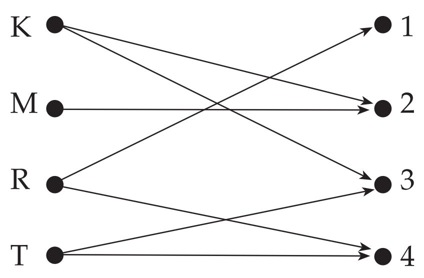
	Oval 1	Oval 2	Oval 3	Oval 4
Ken	5	2	2	3
Mark	5	2	4	3
Richard	2	6	11	2
Tony	5	8	2	2

5. This lowest uncovered element of 2 is now subtracted from all entries

	Oval 1	Oval 2	Oval 3	Oval 4
Ken	3	0	0	1
Mark	3	0	2	1
Richard	0	4	9	0
Tony	3	6	0	0

6. The minimum number of lines needed to cover the zeros is now 4 so a bipartite graph can be drawn with the edges chosen from the zero elements.

From this graph Mark must go to Oval 2 and Richard to Oval 1. This leaves Ken with Oval 3 as his only option and Tony with Oval 4 as his only option.



Total 15 marks