THE HEFFERNAN GROUP

P.O. Box 1180
Surrey Hills North VIC 3127
ABN 20 607 374 020
Phone 9836 5021
Fax 9836 5025

FURTHER MATHEMATICS TRIAL EXAMINATION 2 2001 SOLUTIONS

Section A – Core - solutions

Question 1

a. The dependent variable is cordial sales. **(1 mark)**

b. i. For the winter season,

we have, $\dfrac{11+11+13+17+19+24+26+26+32+37+45+63}{12}$

$=\dfrac{324}{12}$

$=27$

So, mean cordial sales for the winter season were $27 000. **(1 mark)**

ii. The median of the cordial sales for the winter is $25 000, that is halfway between $24 000 and $26 000. **(1 mark)**

c. Using the 1_Var Stats list on the calculator, we see that $Q_1 = 15$ and $Q_3 = 34.5$ So the interquartile range is $34.5 - 15 = 19.5$ **(1 mark)**

Question 2

a. i. Before any regression analysis can be conducted on this data, it must be established that the relationship between the fruit juice content and the price is linear. **(1 mark)**

ii. This is best achieved by drawing a scatterplot for the data. **(1 mark)**

b. The association between fruit juice content and price may be described as having linear form, that is, the points shown on the scatterplot tend to form a straight line. **(1 mark)**

c. Use a calculator to find *r*. Use Table 1 to enter the data. Doing this we find that *r* = 0.96 (correct to 2 decimal places). **(2marks)**

d. Again using your calculator, we find that:
price = 1.56 + 0.02 × fruit juice content **(2 marks)**

e. When we have 35% fruit juice content, we have price = 1.56 + 0.02 × 35 = $2.26 **(1 mark)**

f. The price increases by $0.02 or 2 cents for every 1% increase in fruit juice content. **(1 mark)**

g. This is given by the coefficient of determination, r^2, where $r^2 = 0.9279$ (to 4 places) **(1 mark)**

So, 92.79% of the variation in the price of one litre bottles of cordial can be explained by the variation in the fruit juice content of the cordial. **(1 mark)**

Total 15 marks

Section B : Modules – solutions

Module 1 – Number patterns and applications

Question 1

a. i. In 1987 there were 7600 + 1500 + 1500 + 1500 + 1500 + 1500 + 1500 = 16 600 tickets sold. **(1 mark)**

ii. Now, we have an arithmetic sequence and the general formula for an arithmetic sequence is given by $t_n = a + (n-1)d$ In this case, $a = 7600$ and $d = 1500$

So, we have $t_n = 7600 + (n-1) \times 1500$

$t_n = 7600 + 1500n - 1500$

So, $t_n = 6100 + 1500n$ **(1 mark)**

iii. We want to find the smallest whole value of n for which

$$6100 + 1500n > 30\,000$$
$$1500n > 23\,900$$
$$n > \frac{23\,900}{1500}$$
$$n > 15.9\dot{3}$$

So, in the 16th year, that is, in 1996, the number of tickets first exceeded 30 000 **(1 mark)**

b. In 2000, the number of tickets sold was given by

$$t_{20} = 6100 + 1500 \times 20 = 36100$$ **(1 mark)**

The ratio of the number of artists performing compared to the number of tickets sold was

1 : 95

OR x : 36 100 where x is the number of artists performing at the festival in 2000.

So, $x = \frac{36100}{95}$

therefore, $x = 380$

So, there were 380 artists performing at the festival in 2000. **(1 mark)**

Question 2

a. i. The amount of rubbish generated at the 1982 festival according to the festival organizers is given by $1.05 \times 120 = 126$ cubic metres. **(1 mark)**

ii. The amount of rubbish generated at the festival according to the festival organizers forms a geometric sequence with $a = 120$ and $r = 1.05$.

So, we have, $R_n = 120 \times (1.05)^{n-1}$ **(1 mark)**

iii. We need to find the smallest whole value of n for which

$$120 \times (1.05)^{n-1} > 200$$

Consider $120 \times (1.05)^{n-1} = 200$

$$(1.05)^{n-1} = 1.\dot{6}$$
$$10^{0.0212(n-1)} = 10^{0.2218}$$

So, $0.0212(n-1) = 0.2218$

$$n - 1 = 10.4623$$
$$n = 11.4623$$

So, the smallest whole value of n for which $120 \times (1.05)^{n-1} > 200$ is 12.
So in the 12th year of the festival, that is, in 1992, the amount of garbage first exceeded 200 cubic metres. **(1 mark)**

iv. The total quantity of rubbish generated at the first 20 festivals is given by

$$S_{20} = \frac{120(1.05^{20} - 1)}{0.05} \quad \textbf{(1 mark)}$$
$$= 3967.91$$

So, to the nearest cubic metre, 3968 cubic metres of rubbish was generated at the first 20 festivals. **(1 mark)**

Question 3

a. $R_{n+1} = 1.03R_n + 6$ where $R_1 = 120$

So,
$$R_1 = 120$$
$$R_2 = 1.03 \times 120 + 6$$
$$= 129.6$$
$$R_3 = 1.03 \times 129.6 + 6$$
$$= 139.488$$
$$R_4 = 1.03 \times 139.488 + 6$$
$$= 149.67264$$

So, according to the council, in 1984, to the nearest metre, there was 150 cubic metres of rubbish generated. **(1 mark)**

b. For the general first order difference equation given by $t_n = at_{n-1} + b$ a solution is given by $t_n = a^{n-1}t_1 + b\frac{(a^{n-1} - 1)}{a - 1}$

So, we have $a = 1.03$ and $b = 6$

And so, $$R_n = 1.03^{n-1} \times 120 + 6 \times \frac{(1.03^{n-1} - 1)}{0.03} \quad \textbf{(1 mark)}$$
$$= 120 \times 1.03^{n-1} + 200(1.03^{n-1} - 1)$$
$$= 120 \times 1.03^{n-1} + 200 \times 1.03^{n-1} - 200$$
$$= 320 \times (1.03)^{n-1} - 200 \quad \text{as required} \quad \textbf{(1 mark)}$$

c. Enter the equation generated in part **b.** together with the equation for the geometric sequence in **Question 2** as functions Y_1 and Y_2 in your graphics calculator. Enter a third function $Y_3 = Y_1 - Y_2$ Using the TABLE function scroll down until Y_3 exceeds 50.
This occurs when $n = 17$ So, in 1997 the difference between the estimates of council and of organizers first exceeds 50 cubic metres.

(2 marks)

Total 15 marks

Module 2 – Geometry and trigonometry

Question 1

a. i. $\angle DEF = 45°$ since in ΔDEF , we have $90° + 45° + \angle DEF = 180°$ **(1 mark)**

ii. Since $\angle DEF = 45°$, ΔDEF is an isosceles triangle so, $DF = 4$ metres. **(1 mark)**

iii. Now $DG = DF - FG = 4 - 3 = 1$ metre as required **(1 mark)**

b. ΔEFG is a right-angled-triangle. So, $(EG)^2 = 3^2 + 4^2$

so, $EG = \sqrt{25} = 5$ metres **(1 mark)**

c. Using the sin rule, we have, $\dfrac{\sin(\angle DEG)}{1} = \dfrac{\sin 45°}{5}$

$\sin(\angle DEG) = 0.1414$

$\angle DEG = 8°$ to the nearest degree **(1 mark)**

d. The area of overlap is a triangle so we have,

Now, $\angle DGE = 180° - 45° - 8°$ (from part **c.**)

$= 127°$ **(1 mark)**

area required $= \dfrac{1}{2} bc \sin A$

$= \dfrac{1}{2} \times 1 \times 5 \times \sin 127°$

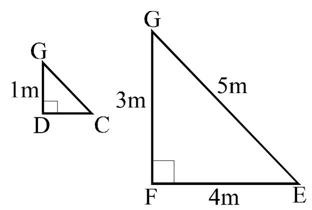
$= 2.0$ square metres (to 1 decimal place) **(1 mark)**

Question 2

a. Now, $\Delta' s\, CDG$ and EFG are similar triangles (angle, angle, angle).

Since $\angle FEG = \angle DCG$ and since $\angle EFG = \angle CDG$, we have,

$\dfrac{CG}{EG} = \dfrac{DG}{FG}$ **(1 mark)**



so, $\dfrac{CG}{5} = \dfrac{1}{3}$

$CG = \dfrac{5}{3} = 1\dfrac{2}{3}$ **(1 mark)**

b. In ΔEFG, $\cos(\angle FEG) = \dfrac{4}{5}$

So, $\angle FEG = 37°$ (to the nearest degree) **(1 mark)**

Since $\Delta' s\, CDG$ and EFG are similar triangles (angle, angle, angle) $\angle FEG = \angle DCG$

So, $\angle DCG = 37°$ (to the nearest degree) **(1 mark)**

c. Now in ΔABC, $AC = 9$ metres and $BC = 6 + 5 + 1\dfrac{2}{3} = 12\dfrac{2}{3}$ metres. (from part **a.**)

(1 mark)

Also, $\angle ACB = \angle DCG = 37°$ (from part **b.**)

So, using the cosine rule, we have, $(AB)^2 = (12\dfrac{2}{3})^2 + 9^2 - 2 \times (12\dfrac{2}{3}) \times 9 \times \cos 37°$

$AB = 7.7$ metres(correct to 1 decimal place) **(1 mark)**

Question 3

The ratio of the area of the drawing of the sail to the actual sail is 1.2 : 1920

which when simplified, becomes 1 : 1600 **(1 mark)**

So if the ratio of the areas is 1 :1600, then the ratio of the distances 1 : $\sqrt{1600}$

which is 1 : 40 So the distance scale used is 1 : 40 **(1 mark)**

Total 15 marks

Module 3 – Graphs and relations

Question 1

a. i. The dance school allocates $45 \times 60 = 2700$ minutes of teaching per week for private lessons in tap dance.

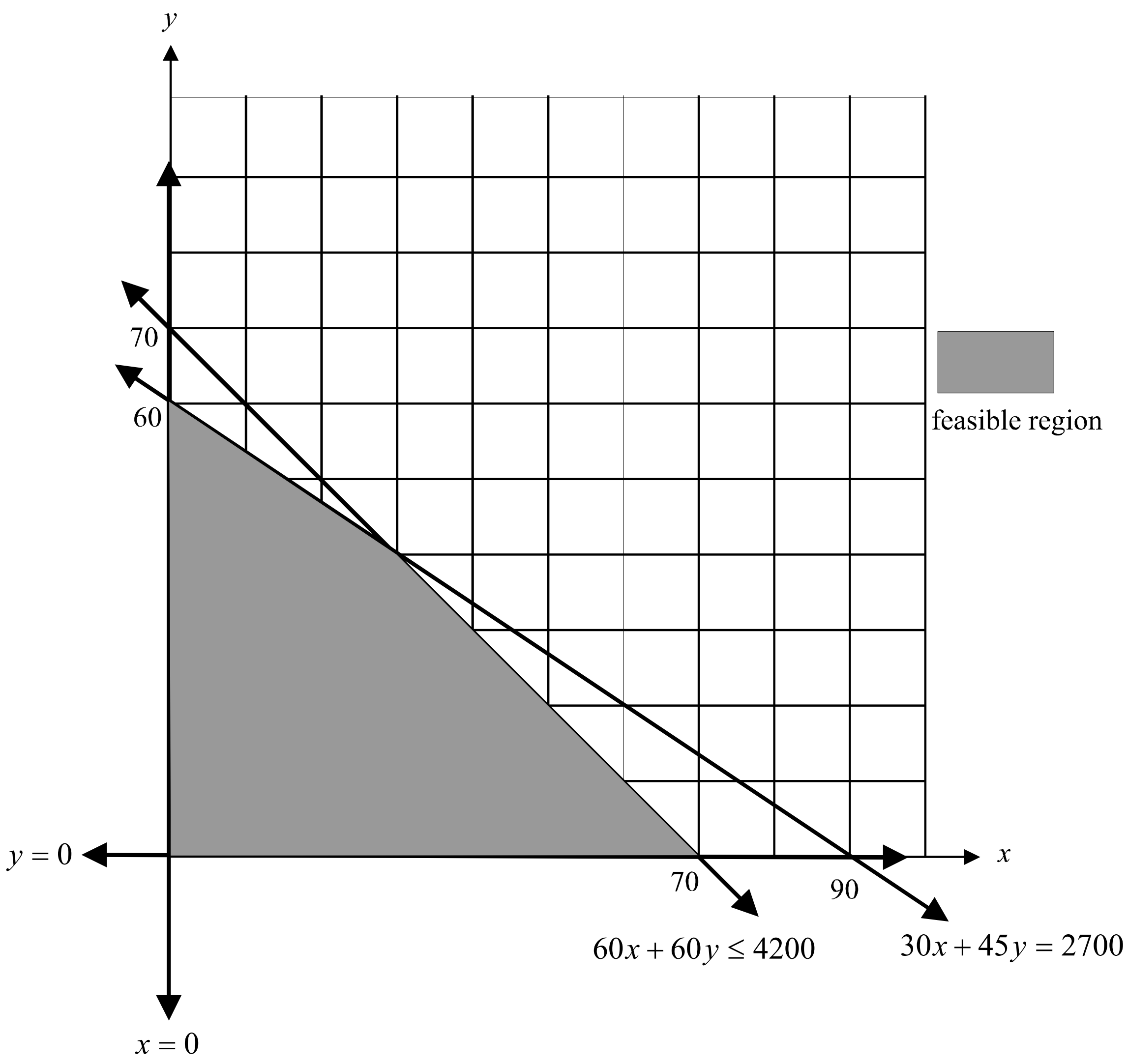
ii. The dance school allocates $70 \times 60 = 4200$ minutes of teaching per week for private lessons in jazz ballet. **(1 mark)** for both answers correct

b. The dance school allocates 2700 minutes of teaching per week to tap dance lessons. If there are only junior tap dance lessons, and they run for 30 minutes, then the maximum number of those lessons which could be offered is $2700 \div 30 = 90$ **(1 mark)**

c. i. $y \geq 0$ **(1 mark)**

ii. $60x + 60y \leq 4200$ **(1 mark)**

d. shown below **(1 mark)** for each line



e. The feasible region with boundaries included is shown above. **(1 mark)**

Question 2

a. $P = 15x + 12y$ **(2 marks)**

b. The maximum profit will occur at a corner point of the feasible region of the graph on page 5.

The corner points are located at (0, 0), (0, 60), (70, 0) and at the intersection of the lines with equations $60x + 60y = 4200$ and $30x + 45y = 2700$

Solving these 2 equations simultaneously, we have

$$60x + 60y = 4200 \quad \text{\_\_\_\_\_\_\_(A)}$$

$$30x + 45y = 2700 \quad \text{\_\_\_\_\_\_\_(B)}$$

(B)$\times$ 2 gives $60x + 90y = 5400$ _______(C)

(C)-(A) gives $30y = 1200$

$$y = 40$$

In (A) gives $x = 30$

So the 4th corner point is (30, 40). **(1 mark)**

At (0,0), $P = 0$

At (0, 60), $P = 15 \times 0 + 12 \times 60 = 720$

At (30, 40), $P = 15 \times 30 + 12 \times 40 = 930$

At (70, 0), $P = 15 \times 70 + 12 \times 0 = 1050$ **(1 mark)**

So, the maximum profit occurs when there are 70 junior lessons and no senior lessons conducted. **(1 mark)**

c. From part **b.** we know that for maximum profit there should be 70 junior lessons conducted per week. So, the profit would be $70 \times \$15 = \$1050$ **(1 mark)**

Total 15 marks

Module 4 : Business related mathematics

Question 1

a. **i.** $\text{simple interest} = \dfrac{P \times r \times T}{100}$

$= \dfrac{1500 \times 4 \times 10}{100}$

$= 600$

The simple interest earned is \$600. **(1 mark)**

ii. For the money to double, the simple interest received would have to amount to \$1500.

So, $\text{simple interest} = \dfrac{P \times r \times T}{100} = 1500$

$$\frac{1500 \times 4 \times T}{100} = 1500$$

$$T = \frac{1500 \times 100}{1500 \times 4}$$

$$T = 25$$

It would take 25 years. **(1 mark)**

b. **i.** $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

$A = 1500 \times (1.04)^{10}$

$A = 2220.37$ **(1 mark)**

Mick would have received \$2220.37 and so the interest he received would be

$\$2220.37 - \$1500 = \$720.37$ **(1 mark)**

ii. $3000 = 1500 \times (1.04)^n$ **(1 mark)**

$2 = (1.04)^n$

$10^{0.3010} = 10^{0.0170n}$

$n = \dfrac{\log_{10} 2}{\log_{10} 1.04}$

$n = 17.673$ (to 3 decimal places)

So, it would take 18 years (to the nearest whole year). **(1 mark)**

Question 2

a. When the digger was new, the value of the digger was \$20 000. **(1 mark)**

b. The graph which forms the straight line is the graph which shows flat rate dpreciation. **(1 mark)**

c. From the graph, we note that in 1 year the digger drops in value by \$2000.
So, the annual flat rate of depreciation is \$2000. **(1 mark)**

d. We have $17\,000 = 20\,000 \times R^1$ **(1 mark)**

So, $R = \dfrac{17\,000}{20000} = 0.85$ **(1 mark)**

Question 3

a. Since the interest is compounding quarterly, after 4 years there will have been 16 quarters, so $n = 16$ **(1 mark)**

b. The annuities formula can be used to calculate this.

We have $A = 40\,000 \times 1.02^{16} - \dfrac{2000(1.02^{16} - 1)}{0.02}$

$= 17\,632.86$ **(1 mark)**

c. The loan will be repaid when $A = 0$

So, we have, $0 = 40\,000 \times 1.02^{n} - \dfrac{2000(1.02^{n} - 1)}{0.02}$ **(1 mark)**

$$40\,000 \times 1.02^{n} = 100\,000(1.02^{n} - 1)$$
$$40\,000 \times 1.02^{n} = 100\,000 \times 1.02^{n} - 100\,000$$
$$60\,000 \times 1.02^{n} = 100\,000$$
$$1.02^{n} = \frac{5}{3}$$

So, $10^{0.0086n} = 10^{0.2218}$

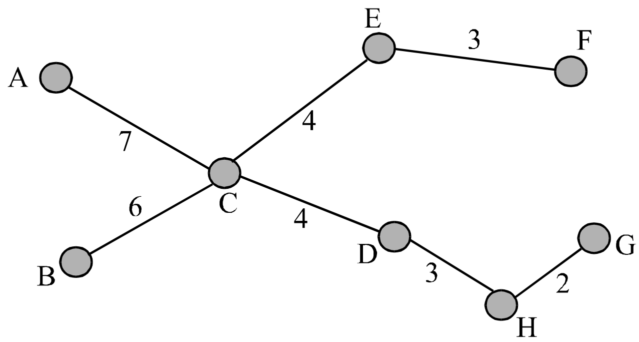
$$n = 25.7963$$

So, it will take 26 quarters (to the nearest quarter). **(1 mark)**

Total 15 marks

Module 5 : Networks and business mathematics

Question 1
a. Two possible circuits are AEFGDHBCA and ACBHDGFEA. **(1mark)**
b. The shortest route from winery A to winery H passes through wineries A, C, D and H.
The length of the shortest route is therefore 7 + 4 + 3 = 14 kilometres **(1 mark)**
c. The minimal-length spanning tree is shown below.



(2 marks)
d. The minimum length of pipes required to connect each of the wineries to at least one other winery is given by the total of the distances shown on the minimal-length spanning-tree.
The required answer is 7 + 6 + 4 + 4 + 3 +3 + 2 = 29 kilometres. **(1 mark)**
e. What the tour leader is describing is an Euler circuit. An Euler circuit exists if the degree of each of the vertices is even. From Figure 1, we see that vertex E and H have odd degrees; that is 5 and 3 respectively. By constructing a road between winery E and winery H, each would then have an even degree; that is 6 and 4 respectively and hence an Euler circuit would exist for the network. **(1 mark)**

Question 2
a. The earliest start time for activity I is 10 months and for activity J is 15 months. The latest start time for activity H is 12 months. **(3 marks)**
b. The critical path for the network is A, C, G, I, J. **(1 mark)**
c. The completion time for the project is 2 + 2 + 6 + 5 + 2 = 17 months **(1 mark)**
d. The slack time for activity E is 7 – 3 = 4months. **(1 mark)**

Question 3
Step 1
Create a matrix from the table given.

$$\begin{bmatrix} 6 & 3 & 2 & 5 \\ 4 & 2 & 3 & 7 \\ 6 & 3 & 3 & 3 \\ 8 & 2 & 9 & 7 \end{bmatrix}$$

Step 2

Use row reduction to obtain the next matrix.

$$\begin{bmatrix} 4 & 1 & 0 & 3 \\ 2 & 0 & 1 & 5 \\ 3 & 0 & 0 & 0 \\ 6 & 0 & 7 & 5 \end{bmatrix}$$

(1 mark)

Step 3

The 1st row contains only 1 zero. Box this and cross out the other zeros in that column,

$$\begin{bmatrix} 4 & 1 & \boxed{0} & 3 \\ 2 & 0 & 1 & 5 \\ 3 & 0 & \cancel{0} & 0 \\ 6 & 0 & 7 & 5 \end{bmatrix}$$

Step 4

The 2nd row contains only 1 zero. Box this and cross out the other zeros in that column.

$$\begin{bmatrix} 4 & 1 & \boxed{0} & 3 \\ 2 & \boxed{0} & 1 & 5 \\ 3 & \cancel{0} & \cancel{0} & 0 \\ 6 & \cancel{0} & 7 & 5 \end{bmatrix}$$

Step 5

The 3rd row contains only 1 zero. Box this.

$$\begin{bmatrix} 4 & 1 & \boxed{0} & 3 \\ 2 & \boxed{0} & 1 & 5 \\ 3 & \cancel{0} & \cancel{0} & \boxed{0} \\ 6 & \cancel{0} & 7 & 5 \end{bmatrix}$$

Complete allocation is not possible since the first column contains no zeros.

Step 6

Since there are no zeros in column 1, we need to perform a column reduction.

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 1 & 0 & 0 & 0 \\ 4 & 0 & 7 & 5 \end{bmatrix}$$

Step 7

Now we start the process again.
Row contains only 1 zero. Box it and cross out any other zeros in the column.

$$\begin{bmatrix} 2 & 1 & \boxed{0} & 3 \\ 0 & 0 & 1 & 5 \\ 1 & 0 & \cancel{0} & 0 \\ 4 & 0 & 7 & 5 \end{bmatrix}$$

Step 8

Row 4 contains only 1 zero. Box it and cross out any other zeros in the column.

$$\begin{bmatrix} 2 & 1 & \boxed{0} & 3 \\ 0 & \not{0} & 1 & 5 \\ 1 & \not{0} & \not{0} & 0 \\ 4 & \boxed{0} & 7 & 5 \end{bmatrix}$$

Step 9
Rows 2 and 3 each now have only 1 zero. Box them.

$$\begin{bmatrix} 2 & 1 & \boxed{0} & 3 \\ \boxed{0} & \not{0} & 1 & 5 \\ 1 & \not{0} & \not{0} & \boxed{0} \\ 4 & \boxed{0} & 7 & 5 \end{bmatrix}$$

Our optimal allocation is:

Emily should go to winery C
Trent should go to winery A
Jo should go to winery D
Sally should go to winery B

(2 marks)
(1 mark for each 2 correct allocations)
Total 15 marks