The Mathematical Association of Victoria

Trial Examination 2023

GENERAL MATHEMATICS

Written Examination 2

STUDENT NAME: _____

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Content Area	*Number of questions*	*Number of questions to be answered*	*Number of marks*
Data Analysis	8	8	24
Recursion and Financial Modelling	4	4	12
Matrices	4	4	12
Networks and Decision Maths	4	4	12
			Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference (which may be annotated), one approved technology(calculator or software) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 28 pages
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

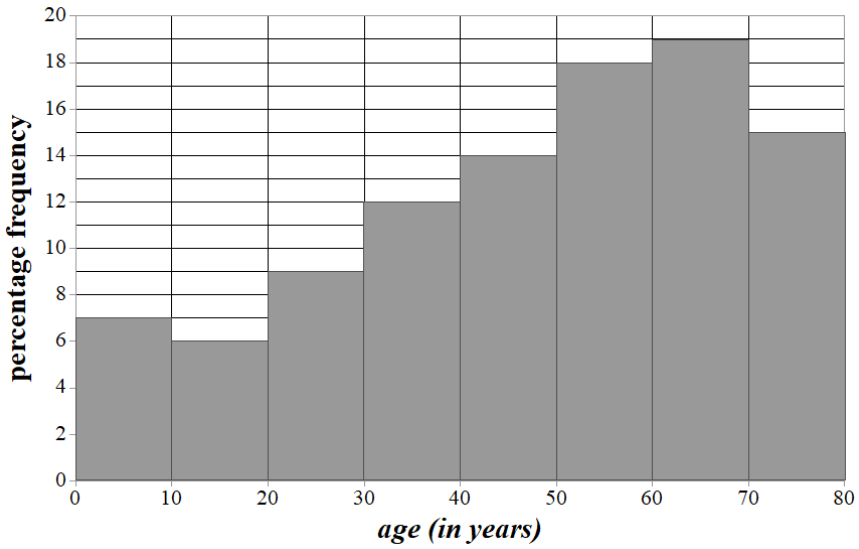
Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (3 marks)

The *age (in years)* of passengers travelling to any destination and on any length trip on cruise ships worldwide during the past decade is shown in the histogram below:



a. Describe the shape of the distribution of *age (in years)* shown in the histogram. 1 mark

b. In what class interval of *age (in years)* does the Q_1 value for this data fall? 1 mark

c. In 2019 there were 27 508 900 passengers travelling on cruise ships worldwide.

Using the values shown in the histogram above determine the number of these passengers that would have been expected to be less than 40 years.

Round your answer to two significant figures. 1 mark

Question 2 (3 marks)

A survey of cruise passengers found that the mean *money spent* by cruise passengers in the city of their departure port was $385, with a standard deviation of *money spent* of $115. The *money spent* data was normally distributed.

a. What percentage of cruise passengers are expected to spend more than $500 in the city of their departure port? 1 mark

Recently a cruise ship that had a capacity of 1998 passengers departed from Station Pier, Port Melbourne.

b. How much money would be expected to be spent, in total, in Melbourne by these passengers?

1 mark

c. One passenger travelling on the cruise ship spent $98 in Melbourne prior to departure.

Calculate the standardized score for *money spent* by this passenger.

Round your answer to one decimal place. 1 mark

TURN OVER

Question 3 (7 marks)

Twenty Victorian passengers on a cruise ship were randomly selected and asked six questions.

The variables listed are:

Passenger: the passenger's unique identifying number in the survey.

Postcode: the postcode where the passenger normally lives.

Age: the passenger's age, in years, at the start of the cruise.

Cabin type: type of cabin they are travelling in (interior, ocean-view, balcony or suite).

Previous passenger: has the passenger previously been on a cruise (yes or no).

Money spent: how much money the passenger spent in the departure port before the cruise, in dollars.

Annual income: the passenger's estimated annual income, in thousands of dollars.

The results of the survey are shown below in Table 1:

Table 1

Passenger	***Postcode***	***Age***	***Cabin type***	***Previous passenger***	***Money spent ($)***	***Annual Income (in $000's)***
1	3228	60	suite	yes	150	110
2	3175	55	balcony	no	200	78
3	3084	39	interior	no	220	180
4	3551	33	interior	no	250	92
5	3141	29	ocean view	yes	260	244
6	3168	19	interior	no	280	30
7	3199	47	interior	no	280	140
8	3137	55	interior	no	290	128
9	3810	52	balcony	no	290	176
10	3371	51	balcony	no	300	44
11	3065	34	balcony	yes	340	370
12	3149	65	balcony	no	350	220
13	3350	52	balcony	yes	380	300
14	3977	35	ocean view	no	390	100
15	3124	45	interior	yes	400	334
16	3025	54	interior	no	480	80
17	3142	29	balcony	yes	550	250
18	3691	72	suite	yes	600	210
19	3498	45	ocean view	no	640	135
20	3134	66	interior	yes	900	50

a. How many of the variables in Table 1 are numerical? 1 mark

On a cruise ship, interior and ocean-view cabins do not have opening windows, whereas balcony and suite cabins do have opening windows.

b. Complete the following **percentaged** two-way table for the variables *age group* (50 years or less, more than 50 years) and *opening windows* (yes or no) from the table.

1 mark

		age group	
		50 years or less	more than 50 years
opening windows	yes		
	no		
	Total	100%	100%

c. Explain why the data in the two-way table supports the contention that there is an association between whether a passenger's cabin has *opening windows* and their *age group*. 1 mark

d. Passenger 20 spent $900 in the departure port before their cruise.

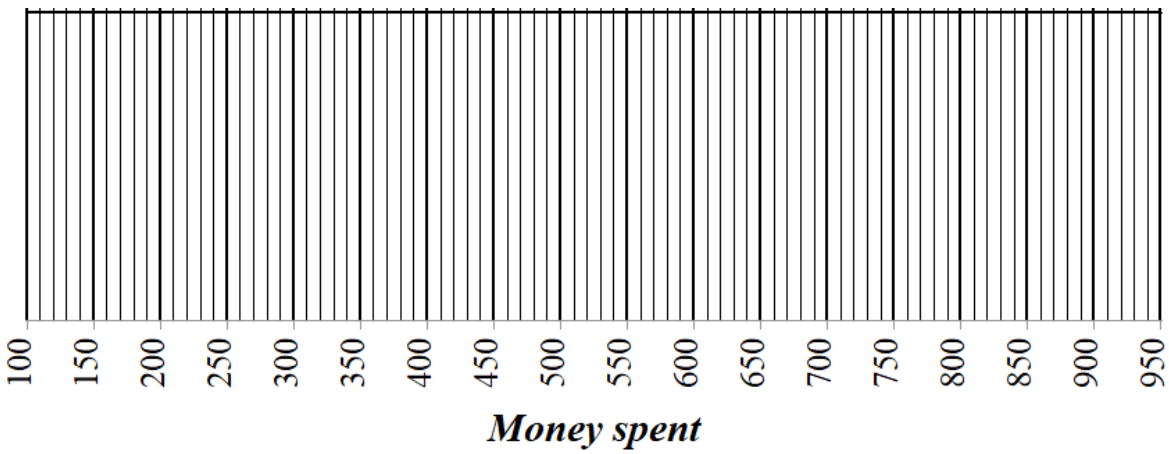
Show why the amount of *Money spent* by Passenger 20 would be considered an outlier for the *Money spent* data from the twenty surveyed passengers. 2 marks

Question 3- continued

TURN OVER

e. Construct a boxplot of the *Money spent* data for these twenty passengers on the graph below.

2 marks



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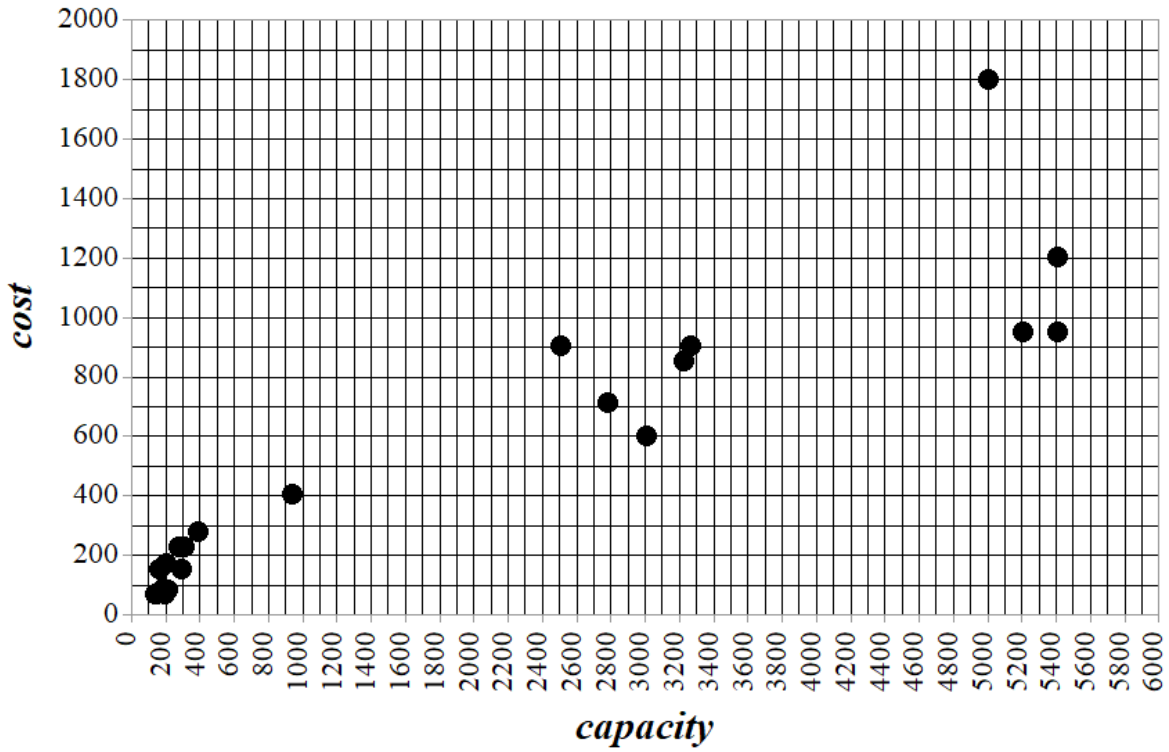
Question 4 (5 marks)

There were 23 new cruise ships delivered worldwide to cruise companies during 2022. A scatterplot of the 23 brand new cruise ships comparing their *cost* (in $millions) and *capacity* (number of passengers that can be carried) is shown below:



The equation of the least squares line for the association between *cost* and *capacity* is

$$cost\ (\text{in \$millions}) = 113.74 + 0.25 \times capacity\ .$$

The correlation coefficient for the association between *cost* and *capacity* is $r = 0.922$, correct to three decimal places.

a. Describe the strength and direction of the association between *cost* and *capacity*. 1 mark

b. Add the least squares line to the scatterplot above. 1 mark

ANSWER ON GRAPH

c. Interpret the slope of the least squares lines in terms of the variables *cost* (in $millions) and *capacity*. 1 mark

d. The Celebrity cruise ship "Beyond" was launched in 2022. The *cost* to build the ship was $900 million and the ship has a *capacity* of 3260 passengers.
Calculate the residual value for the ship the "Beyond".
Round your answer to the nearest whole million dollars. 2 marks

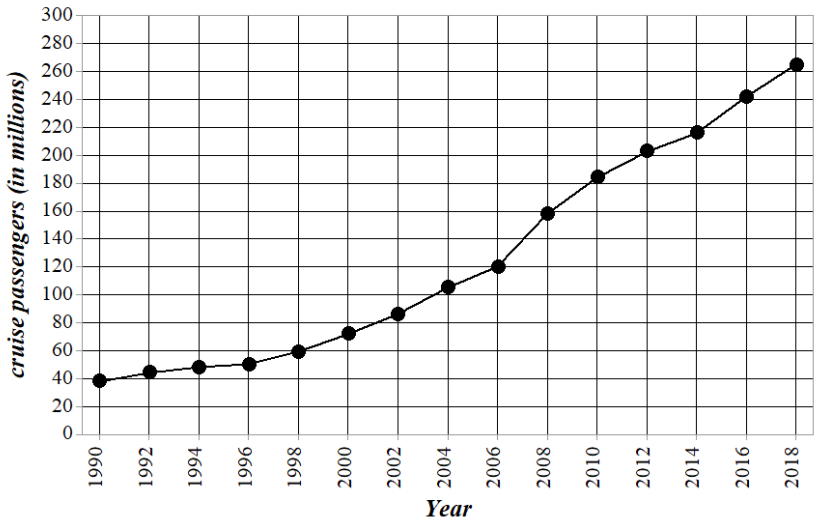
TURN OVER

Question 5 (6 marks)

The annual number of *cruise passengers* worldwide, rounded to the nearest million people, was recorded every second *year* from 1990 through to 2018.

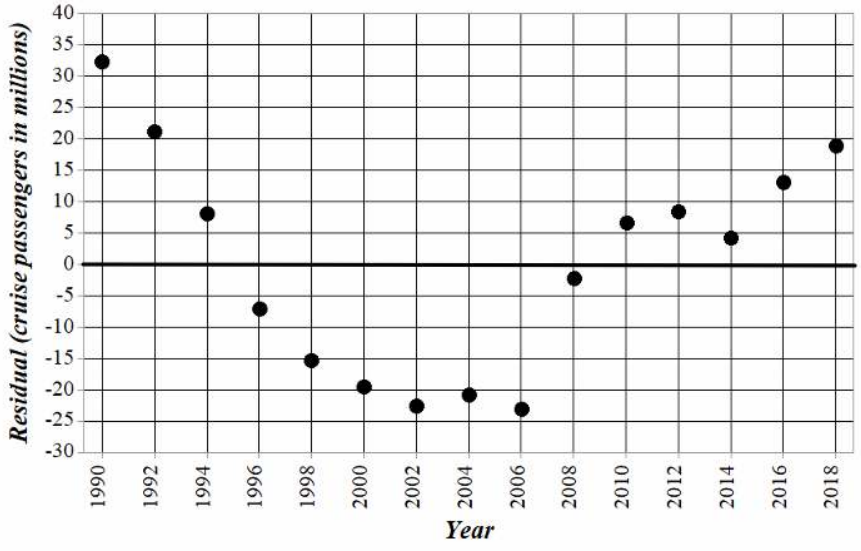
The resulting data and the time series plot are shown below:

year	*cruise passengers* (in millions)
1990	38
1992	44
1994	48
1996	50
1998	59
2000	72
2002	86
2004	105
2006	120
2008	158
2010	184
2012	203
2014	216
2016	242
2018	265



a. Describe the trend in the time series shown. 1 mark

The residual plot associated with a least squares line fitted to this data is shown below:



b. The residual plot shown can be used to test one of the assumptions about the nature of the association between the number of *cruise passengers*, in millions, and the *year*.

i. What is this assumption? 1 mark

ii. The residual plot above does not support this assumption.
Explain why. 1 mark

c. Apply a logarithmic transformation to the variable *cruise passengers* to linearise the data.
Fit a least squares line to the transformed data and write its equation below.
Round the values of the intercept and the slope to three significant figures. 2 marks

d. Use the equation from part **c.**, to predict during which year the number of *cruise passengers*, in millions, is predicted to first exceed 565 million.
Round your answer to the nearest year. 1 mark

TURN OVER

Recursion and Financial Modelling

Question 6 (5 marks)

Mary has a reducing balance loan of $172 500 on her home where the interest compounds monthly. Her monthly repayments are $1700. The first three lines of the amortisation table for Mary's loan are shown below.

Payment Number	Payment ($)	Interest ($)	Principal Reduction ($)	Balance (S)
0	0.00	0.00	0.00	172 500.00
1	1700.00	698.63	1001.37	171 498.63
2	1700.00	694.57	**P**	170 493.20

a. What is the value of **P**, the principal reduction amount for payment number 2? 1 mark

b. Show that the annual interest rate for this loan is 4.86% correct to two decimal places. 1 mark

c. Let A_n be the amount owing, in dollars, of Mary's home loan after n months.
Write a recurrence relation, in terms of A_0, A_{n+1} and A_n, that can model this balance from month to month. . 2 marks

d. To ensure the loan is fully repaid the required final monthly repayment will be lower.
What is the final monthly repayment Mary will make to fully repay the loan?
Give your answer correct to the nearest dollar. 1 mark

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TURN OVER

Question 7 (4 marks)

Mary is planning a renovation to her house, so that she can run a small tailoring business from home. The quote for the renovations is $9500 and she has already saved $5000.

She first considers depositing the $5000 into a saving account, where the interest rate is 1.8% per annum compounding monthly and add a fixed amount each month immediately after the interest is added.

Let B_n be the balance of the savings account, in dollars, n months after it was opened.

The value of B_n can be determined by using the recurrence relation.

$$B_0 = 5000, \quad B_{n+1} = 1.0015B_n + k$$

where k is a fixed amount that is added to the savings account each month immediately after the interest was added.

a. Show, using recursion, that if Mary adds $375 each month the balance of the account would be $5765.57 after two months. 1 mark

b. What is the minimum amount that Mary would need to add to her savings account each month to ensure that she has $9500 after 12 months? 1 mark

Alternatively, she could deposit the $5000 in a term deposit investment that earns 3.35% per annum simple interest with the interest paid at the end of the term deposit.

c. **i.** Show that the value of the term deposit investment after 12 months is $5167.50. 1 mark

In addition to this term deposit of $5000, Mary will open a savings account, with a zero balance, that earns 1.8% per annum interest compounding monthly. She will add a regular monthly payment to this account.

ii. What is the minimum amount that Mary would need to add to her savings account each month to ensure that she has $9500 in total after 12 months with this scenario? 1 mark

TURN OVER

Question 8 (3 marks)

Mary's accountant advises that she takes out a home loan of $182 000 which would allow her to start the renovations immediately. Her loan will have an interest rate of 4.86% per annum, compounding monthly and she will make payments of $2100 every month for the first 12 months.

a. What is the total amount of interest that Mary will pay in the first 12 months of her home loan.
Round your answer correct to the nearest dollar. 2 marks

After the first 12 months she will only pay $1700 per month on the loan with a final smaller payment.

b. How long in months, in total, will it take her to pay off her home loan? 1 mark

Matrices

Question 9 (3 marks)

A nature reserve has been set up for the preservation of the Eastern Barred Bandicoot.

The nature reserve has programs to increase the population and has an education centre where visitors can learn about the programs at the reserve.

The matrix, *P*, below shows the price, in dollars, for adults (*A*), children (*C*) and seniors (*S*) to attend the education centre:

$$P = \begin{bmatrix} 30 \\ 15 \\ 20 \end{bmatrix} \begin{matrix} A \\ C \\ S \end{matrix}$$

a. What is the order of matrix *P*? 1 mark

A family group of two adults, three children and one senior attend the education centre.

The total cost for the family group to attend the centre is given in a 1×1 matrix, *C*.

Matrix *C* can be calculated using a matrix equation, $C = N \times P$.

b. Write down matrix, *N*, below. 1 mark

Large groups attending the education centre are offered a discount.

Adults are offered 10% discount, children receive a 5% discount, but seniors get no further discount.

The new prices, after any discounts are applied, are given in a matrix, *D*, where $D = R \times P$.

c. Write down matrix, *R*, below. 1 mark

TURN OVER

Question 10 (2 marks)

The matrix, W, below shows the food relationship between bandicoots (B), lizards (L), crickets (C) and seeds (S). A "1" represents that the food is consumed by the feeder. For example, the "1" in row 4 and column 1 of the matrix indicates that bandicoots eat seeds.

$$W = \begin{array}{c} \\ B \\ L \\ C \\ S \end{array} \begin{array}{c} \begin{array}{cccc} B & L & C & S \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \\ \\ food \end{array}$$

$$\begin{array}{c} \\ feeder \end{array}$$

a. From the matrix above, what feeders consume crickets? 1 mark

The matrix W^2 is shown below:

$$W^2 = \begin{array}{c} \\ B \\ L \\ C \\ S \end{array} \begin{array}{c} \begin{array}{cccc} B & L & C & S \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ \mathbf{2} & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \\ \\ food \end{array}$$

$$\begin{array}{c} \\ feeder \end{array}$$

The element "2" in row 4 column 1 is bolded.

b. Explain the meaning of the element bolded in W^2 in the context given. 1 mark

Question 11 (4 marks)

The nature reserve has a breeding program with the intention of increasing the bandicoot population.

The matrix, B_{2018}, below shows the number of bandicoot joeys (J), young adults (Y) and mature adults (M) placed at the nature reserve at the start of 2018. A fourth row is in the matrix to represent the bandicoots that die (D).

$$B_{2018} = \begin{bmatrix} 3000 \\ 1000 \\ 1000 \\ 0 \end{bmatrix} \begin{matrix} J \\ Y \\ M \\ D \end{matrix}$$

Bandicoots are an endangered species and breeding can be difficult.

The move to the nature reserve caused the bandicoots to stop breeding for two years after they were moved.

The bandicoots mature from joeys to young adults to mature adults or die from year to year according to the transition matrix, T, shown below:

$$T = \begin{array}{c} \\ J \\ Y \\ M \\ D \end{array} \begin{array}{c} \textit{this year} \\ \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.6 & 0.5 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 \\ 0.2 & 0.1 & 0.4 & 1 \end{bmatrix} \end{array} \begin{array}{l} J \\ Y \\ M \\ D \end{array} \quad \textit{next year}$$

a. Complete the matrix, B_{2020}, below showing the number of bandicoots at each stage of life at the start of 2020. 1 mark

$$B_{2020} = \begin{bmatrix} \underline{\quad\quad} \\ \underline{\quad\quad} \\ 1520 \\ \underline{\quad\quad} \end{bmatrix} \begin{matrix} J \\ Y \\ M \\ D \end{matrix}$$

Question 11- continued

TURN OVER

b. What percentage of the mature adult bandicoots at the start of 2020, were young adults at the start of 2019?

Round your answer to the nearest whole percent. 1 mark

The staff at the nature reserve were very concerned about the depletion of bandicoots and so, during 2020, they restocked the reserve with bandicoots.

The number of bandicoots used in the restocking ensured that the numbers of live bandicoots at each stage did not change from 2020 to 2021.

c. How many of each of bandicoot joeys, young adults and mature adults did the staff need to add to the reserve to preserve numbers at each life stage? 2 marks

Question 12 (3 marks)

During 2021 the bandicoots resumed breeding, but individual bandicoots only live approximately three years. The female bandicoots have three life breeding phases (pre-breeding, independent adult and senior adult), each phase being approximately one year long.

The number of female pre-breeding bandicoots (*P*), independent adult (*I*) and senior adult (*S*) bandicoots at the start of 2021 is shown below in F_{2021}:

$$F_{2021} = \begin{bmatrix} 10 \\ 500 \\ 850 \end{bmatrix} \begin{matrix} P \\ I \\ S \end{matrix}$$

The number of female bandicoots at the start of each subsequent year can be predicted using the matrix equation $F_{n+1} = L \times F_n$ where *L* is the Leslie matrix shown below:

$$L = \begin{array}{c} \textit{this year} \\ \begin{matrix} P & I & S \end{matrix} \\ \begin{bmatrix} 0 & 1.3 & 2.5 \\ 0.6 & 0 & 0 \\ 0 & 0.4 & 0 \end{bmatrix} \end{array} \begin{matrix} P \\ I \\ S \end{matrix} \quad \textit{next year}$$

a. Explain the meaning of the element of value 2.5 in the Leslie matrix. 1 mark

b. What is the predicted number of female independent adults at the start of 2023? 1 mark

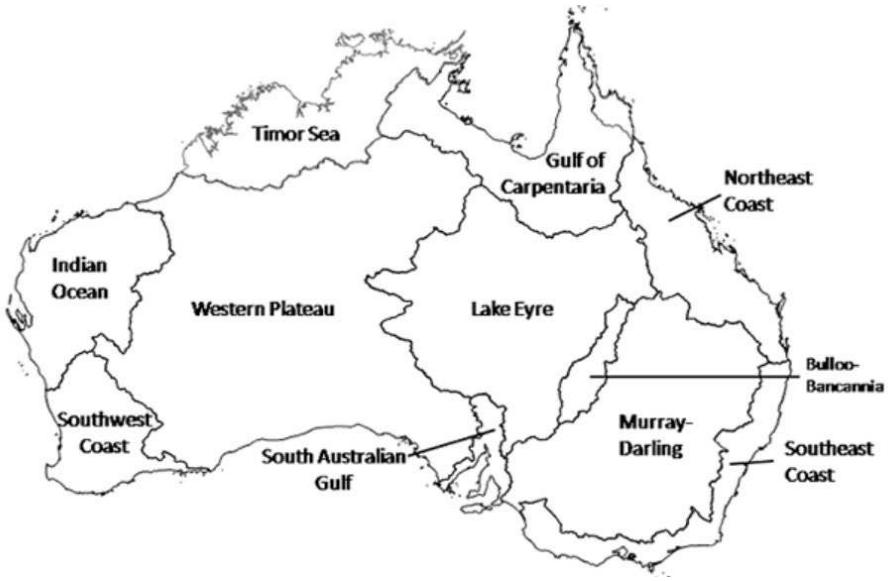
c. How many years, after 2021, will it be before female pre-breeding, independent adult and senior adult bandicoot group's numbers **all** increase from year to year? 1 mark

TURN OVER

Networks and decision mathematics

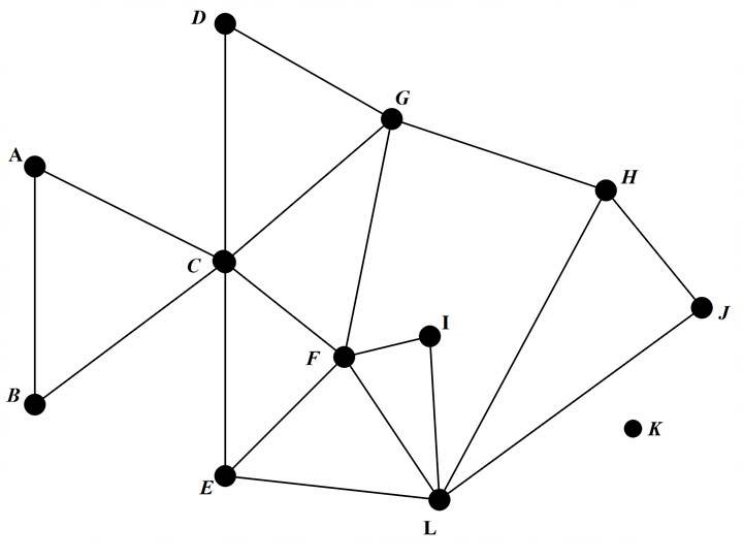
Question 13 (3 marks)

Mainland Australia is divided into eleven water drainage divisions with a twelfth division in Tasmania. The eleven mainland divisions are displayed below:



An incomplete network diagram displays the borders between each division.

The regions are represented by the vertices and the edges represent borders between each division.



Key to network:

- **A.** Indian Ocean
- **B.** Southwest Coast
- **C.** Western Plateau
- **D.** Timor Sea
- **E.** South Australian Gulf
- **F.** Lake Eyre
- **G.** Gulf of Carpentaria
- **H.** Northeast Coast
- **I.** Bulloo-Bancanni
- **J.** Southeast Coast
- **K.** Tasmania
- **L.** Murray Darling

a. One of the edges from Lake Eyre is missing. Add this edge to the network. 1 mark

ANSWER ON GRAPH

Question 13 - continued

b. What is the degree of the Murray Darling vertex? 1 mark

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The adjacency matrix for this network is

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
A	**0**	1	1	0	0	0	0	0	0	0	0	0
B	1	**0**	1	0	0	0	0	0	0	0	0	0
C	1	1	**0**	1	1	1	1	0	0	0	0	0
D	0	0	1	**0**	0	0	1	***X***	0	0	0	0
E	0	0	1	0	**0**	0	1	0	0	0	0	1
F	0	0	1	0	0	**0**	1	1	***Y***	1	0	1
G	0	0	1	1	1	1	**0**	1	0	0	0	0
H	0	0	0	0	0	1	1	**0**	0	0	0	1
I	0	0	0	0	0	1	0	0	**0**	0	0	1
J	0	0	0	0	0	1	0	1	0	**0**	0	1
K	0	0	0	0	0	0	0	0	0	0	**0**	0
L	0	0	0	0	1	1	0	1	1	1	0	**0**

c. Write down the values of X and Y. 1 mark

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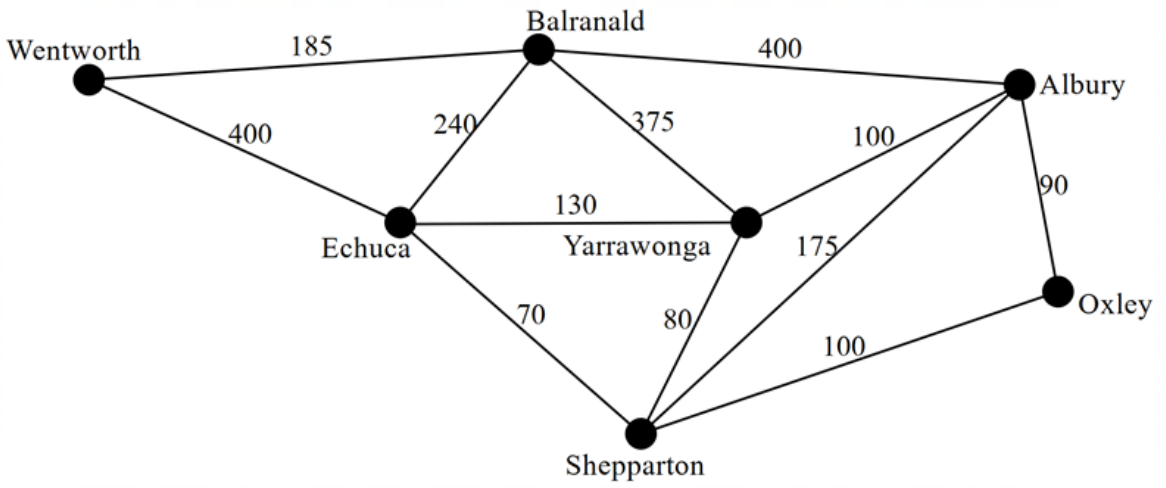
TURN OVER

Question 14 (4 marks)

The Murray-Darling drainage division is the area where water from rainfall, creeks and smaller rivers drain into the Murray and Darling Rivers.

Dennis works for a water company and regularly has to check the rivers at Wentworth (*W*), Balranald (*B*), Albury (*A*), Echuca (*E*), Yarrawonga (*Y*), Oxley (*O*) and Shepparton (*S*).

He may only travel along approved safe roads as shown in the network below. The weight on the edges represents the distance, in kilometres, between the towns.



a. What is the shortest distance between Wentworth and Oxley? 1 mark

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_____

On one trip, starting from and returning to his home in Yarrawonga, Dennis would like to check the rivers at all seven towns without going to any town more than once.

b. **i.** State a route that Dennis could take to check all rivers, starting and finishing at Yarrawonga. 1 mark

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_____

_____

Question 14 - continued

ii. What is the mathematical name given to the route Dennis should take? 1 mark

_____

Dennis' manager is based in Albury.

After the floods, she decides that she needs to check all of the roads for safety, starting and finishing in Albury.

c. Explain why she is able to do this without travelling along any of the roads more than once. 1 mark

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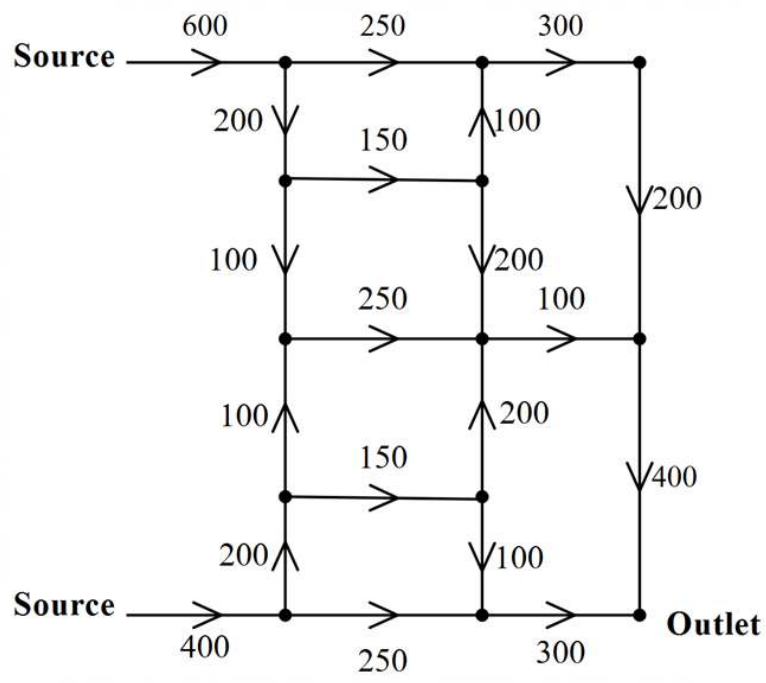
TURN OVER

Question 15 (2 marks)

Stormwater from one town enters a system of channels at one of two source points and flows through the channels into the river from an outlet.

On the directed graph below the numbers represent the maximum rate, in kilolitres per minute, at which the stormwater can flow through each pipe.



a. What is the maximum rate, in kilolitres per minute, at which the stormwater can flow through the drainage system and out of Outlet? 1 mark

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b. Add the minimum cut to the diagram above. 1 mark

ANSWER ON GRAPH.

Question 15 - continued

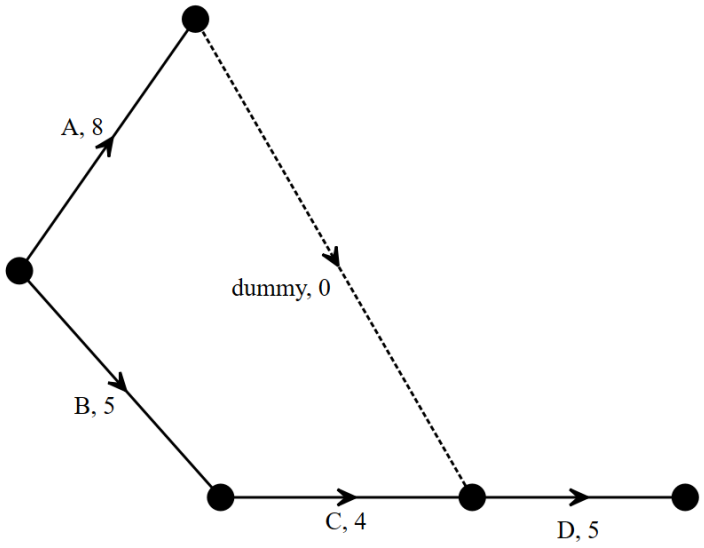
Question 16 (3 marks)

The water company needs to build some new drainage channels. This project involves eight activities A to H with a minimum completion time of 18 weeks.

The table below shows the immediate predecessor(s) and the duration, in weeks, for each activity.

Activity	Immediate predecessor(s)	Duration
A	-	8
B	-	5
C	B	4
D	A, C	5
E	A	7
F	D, E	3
G	A	6
H	D, E, G	2

An **incomplete** diagram of the activity network is provided below:



a. What is the earliest starting time for activity H? 1 mark

_____

b. What is the latest starting time for activity C? 1 mark

_____

_____

TURN OVER

A contractor can be employed to reduce the time in weeks on any activity or activities that currently have a duration of more than four weeks. The contractor can be employed for a maximum of three weeks, so the maximum total reduction of duration of any activities is three weeks.

c. Which activity or activities should be reduced in order to minimise the overall time for the project?
Include the amount of time each activity should be reduced by to achieve maximum total overall time reduction. 1 mark

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_____

END OF QUESTION AND ANSWER BOOK