The Mathematical Association of Victoria

Trial Examination 2023

GENERAL MATHEMATICS

Trial Written Examination 2 - SOLUTIONS

Data Analysis

Question 1

a. Negative skew 1M

The data is centred closer to the top of the data than the bottom and there is a tail in the data towards the lower end. This indicates a negative skew.

b. 30 - < 40 years 1M

The data in the histogram shows the percentage frequency. By definition, the value of Q_1 is the 25^{th} percentile. 7% of values are between 1 and 10 years, with another 6% between 10 and 20 years. Another 9% are between 20 and 30 years, with 12% between 30 and 40 years.

This means that a total of 7 + 6 + 9 = 22% are below 30 years and 22 + 12 = 34% are below 40 years. Therefore the 25^{th} percentile is 30 years or above up to 40 years of age.

c. 9 400 000 passengers 1M

As can be seen from the histogram, 7 + 6 + 9 + 12 = 34% of passengers were less than 40 years of age.

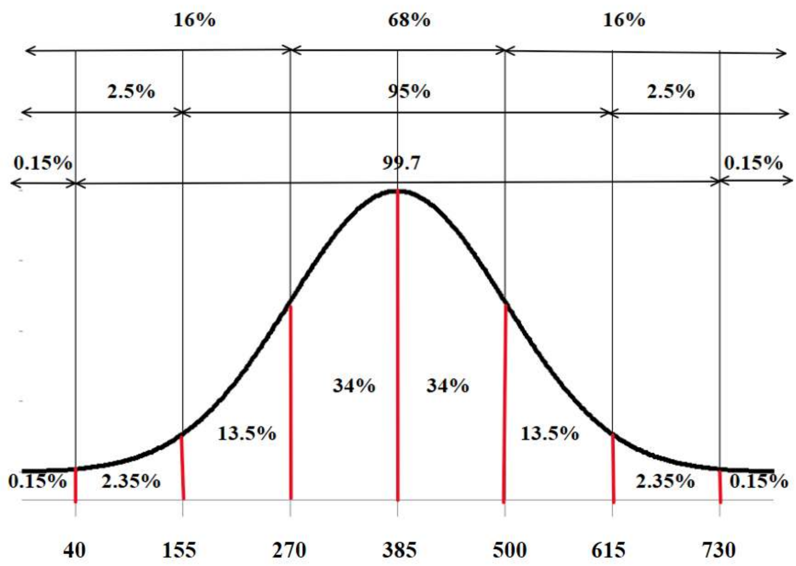
$$\frac{34}{100} \times 27\,508\,900 = 9\,353\,026\,.$$

9 353 026 rounded to two significant figures is 9 400 000.

Question 2

a. 16% ` 1M

The diagram below shows the distribution of *money spent*, based on a mean of \$385 and a standard deviation of \$115:



From this diagram, it can be seen that 16% of cruise passengers are expected to spend more than \$500.

b. \$769 230 1M

The total amount spent would be the mean multiplied by the number of passengers:

$1998 \times 385 = \$769\,230$.

c. −2.5 1M

The z-score is calculated using the mean of \$385, the standard deviation of \$115 and the actual data value of \$98:

$$z - score = \frac{actual\ value - mean}{s_x}$$

$$z - score = \frac{98 - 385}{115} = -2.4956... \approx -2.5.$$

Question 3

a. 3 1M

Of the seven variables there are three (*Age, Money spent* and *Annual Income*) that are numerical.

The *Passenger* number is not numerical as it is just an allocated identifying number and *Postcode* is similar in that it is just an allocated value for a location.

Cabin type and *Previous passenger* are also both nominal as they have words rather than numbers as possible responses.

b. 1M

		age group	
		50 years or less	more than 50 years
opening windows	yes	**20%**	**70%**
	no	**80%**	**30%**
	Total	100%	100%

There were 10 passengers 50 years or less. Of these, two had opening windows (20%) and eight did not (80%). There were also 10 passengers more than 50 years of age. Of these, seven had opening windows (70%) and three did not (30%).

c. There is a difference between the two age groups. 20% of the 50 years or less passengers had opening windows, but a greater percentage of 70% of passengers more than 50 years had opening windows.
OR
There is a difference between the two age groups. 80% of the 50 years or less passengers did not have opening windows, but a smaller percentage of 30% of passengers more than 50 years did not have opening windows. 1M

The answer must refer to a difference in percentages in one row of the table and quote the appropriate percentages in that row.

d. The calculations shown below:

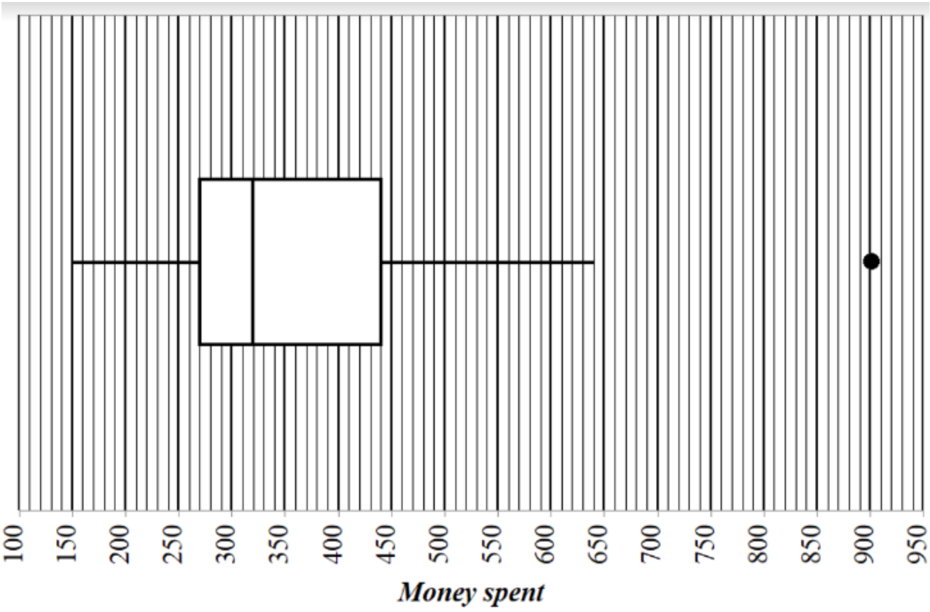
$IQR = 440 - 270 = 170$

Upper fence is $440 + 1.5 \times 170 = 695$

900 > 695, so it is an outlier. 2M

The five-figure summary for the *Money spent* is: Min = 150, $Q_1 = 270$, Med = 320, $Q_3 = 440$, Max = 900. A value is an outlier if it is above the upper fence and as this is a "show that" question, calculations that lead to this conclusion must be shown.

e. Boxplot as shown below: 2M



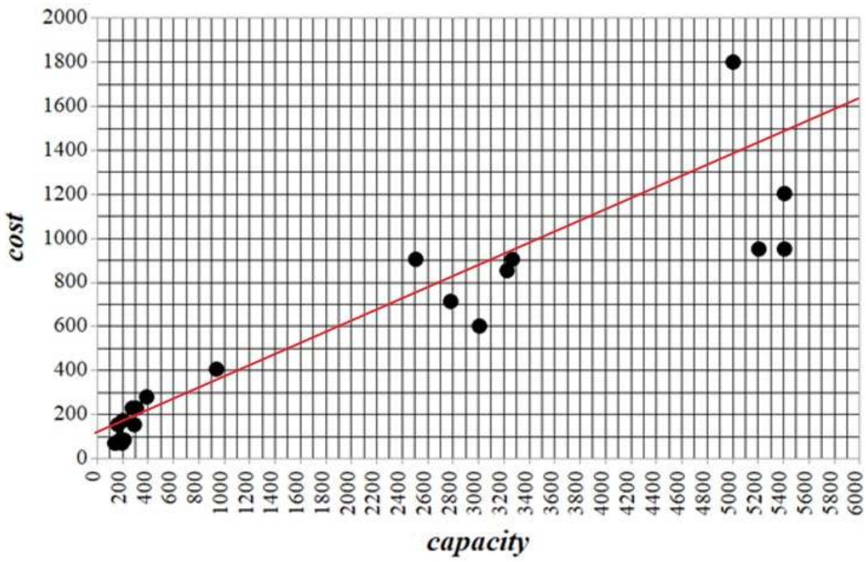
The five-figure summary for the *Money spent* is: Min = 150, Q_1 = 270, Med = 320, Q_3 = 440, Max = 900 as determined in the previous question. The value of $900 was shown to be an outlier and so it needs to be drawn as such, and as the upper whisker extends to the highest value that is not an outlier, that whisker should end at $640.

Question 4

a. Strong, positive association 1M

The *r*-value for the association is given as 0.922, so the association is a strong, positive, linear association. However, this question only asks for strength and direction and so only strong (strength) and positive (direction) are suitable answers.

b. The line shown below: 1M



The line can be drawn using any two points, preferably at either end of the graph:
capacity = 0, $cost = 113.74 + 0.25 \times 0 = 113.74$
capacity = 6000, $cost = 113.74 + 0.25 \times 6000 = 1613.74$
Therefore, this line should go through (0, 113.74) and (6000, 1613.74).
As the scale does not allow this level of accuracy, the line should pass through
(0, 100<cost<150) and (6000, 1600<cost<1650).

c. For every increase of one in passenger capacity, the cost is expected to increase by $250 000 ($0.25 million). 1M

The answer must refer to a unit increase in passenger capacity and refer to an increase in cost. Either $250 000 or $0.25 million would be acceptable.

d. -29 million dollars 2M

The residual is calculated by determining the difference between the actual cost of $900 million and the predicted cost using the least squares line:

$$Predicted\ cost = 113.74 + 0.25 \times 3260 = 928.74$$
$$Residual = 900 - 928.74 = -28.74 \approx -29$$

Question 5

a. Increasing trend 1M

The time series graph increases consistently over time, so there is an increasing trend.

b. **i.** That the association is linear. 1M

A residual plot is used to test the assumption of a linear association between the two variables. If the residual plot is linear, the data would be considered to be best explored using a linear association, but if a clear pattern is seen, the data is more likely to be non-linear.

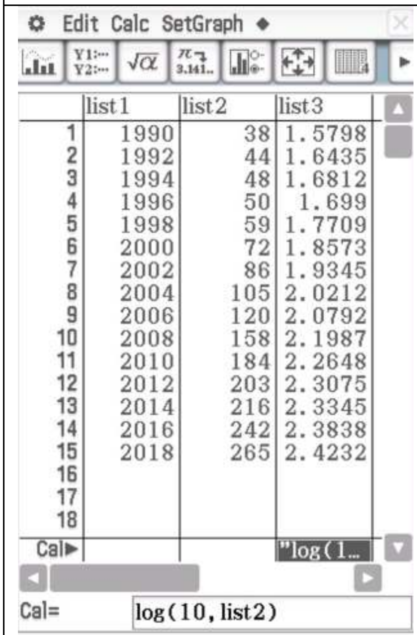
ii. A clear pattern is present. 1M

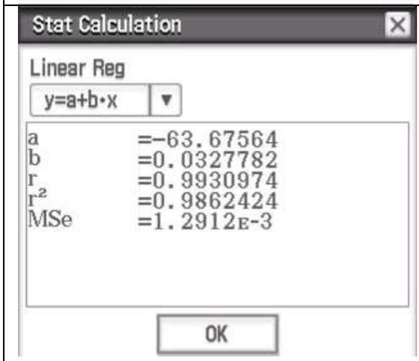
As stated a clear pattern in the residual plot is evidence that the original association is non-linear. The pattern seen is sometimes described as a valley pattern.

c. $\log(\textit{cruise passengers}) = -63.7 + 0.0328 \times \textit{year}$ 2M

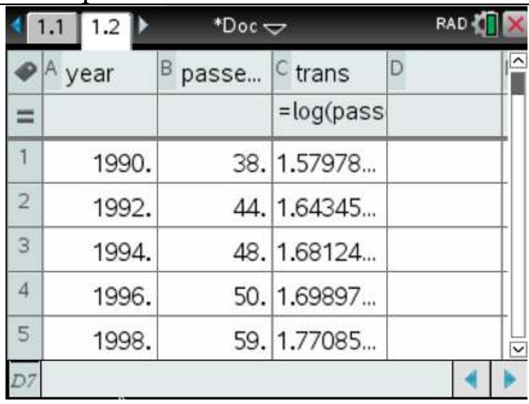
The log transformation of the variable *cruise passengers* is performed using CAS technology as shown below:

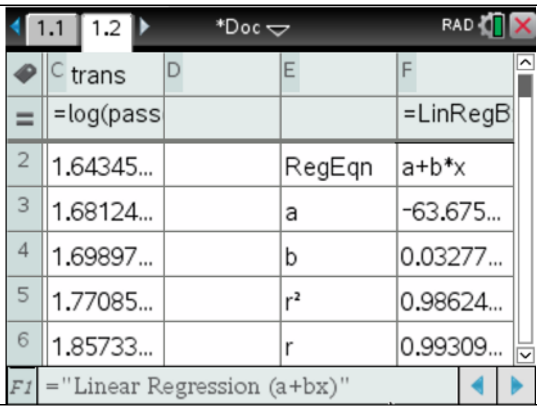
Casio ClassPad





TI-Nspire





Thus, the transformed equation can be written as
$\log(\textit{cruise passengers}) = -63.7 + 0.0328 \times \textit{year}$ correct to three significant figures.

d. 2026 1M

The value *cruise passengers* = 565 must be substituted into the transformed equation as shown below:

$$\log(565) = -63.7 + 0.0328 \times \textit{year}$$

Solving this equation, *year* = 2025.9770… so the number of cruise passengers will first exceed 565 million during the year 2026.

Recursion and Financial Modelling

Question 6

a. The principal reduction is the difference between the payment and interest added.

P = 1700 – 694.57 = **$1005.43**

Alternatively, it is the change in balance 171 498.63 - 170 493.20 = $1005.43 A1

b. Annual interest rate = $\frac{698.63}{172500} \times 100 \times 12 \approx 4.86\%$ M1

c. Monthly interest rate is $\frac{4.86}{12} \approx 0.405\%$

$R = 1 + \frac{0.405}{100} = 1.00405$ M1

$A_{n+1} = 1.00405A_n - 1700$ where $A_0 = 172\,500$ H1

d. The number of payments required can be found using the Finance Solver of the calculator. It will require 131 payments. A final payment of $1700 will overpay the loan by $94.17.

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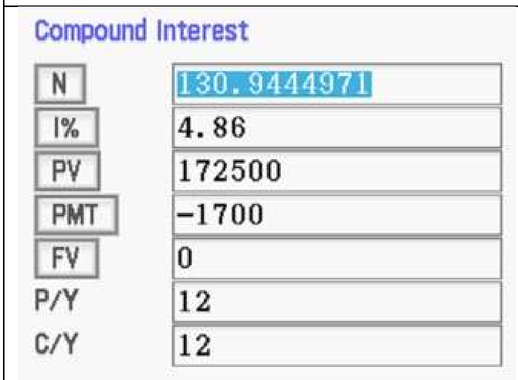
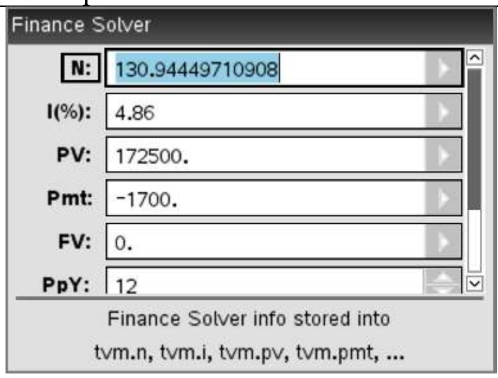
<tr><td>
</td><td>
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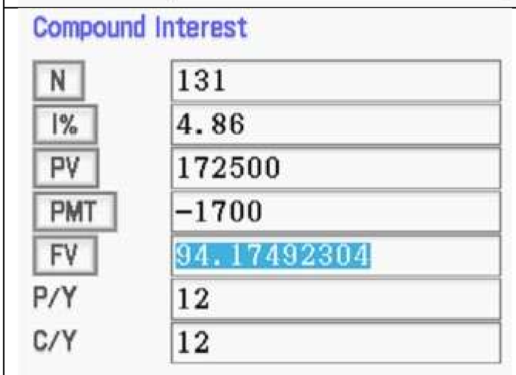
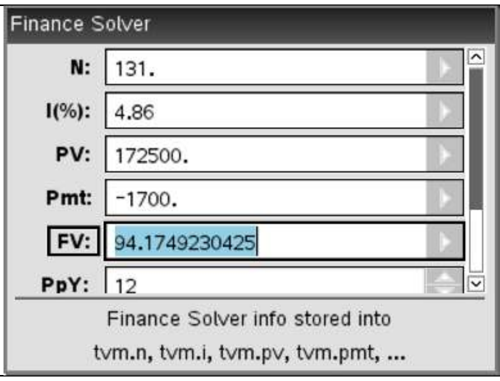
<tr><td>
</td><td>
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The final payment would be 1700 – 94.17 = $1605.83 ≈ $1606. A1

Question 7

a. $B_0 = 5000$

$B_1 = 1.0015(5000) + 375 = \$5382.50$

$B_2 = 1.0015(5382.50) + 375 = \$5765.57$ M1

b. Using the Financial Solver of the calculator:

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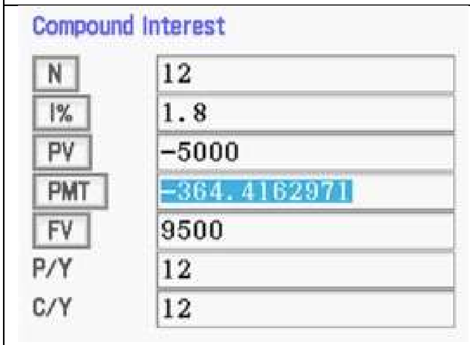

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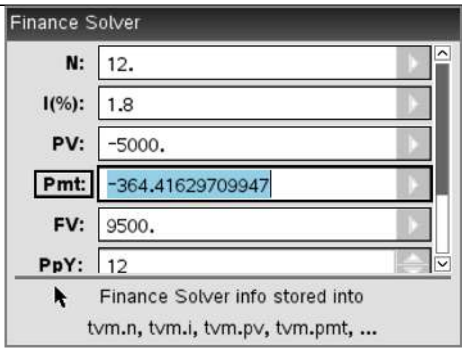

The payment needed is \$364.42 A1

c. i. Interest = $\frac{5000 \times 1 \times 3.35}{100} = \$167.50$

Balance after 1 year is 5000 + 167.50 = \$5167.50 M1

ii. Mary would need the balance of the savings account to be 9500 – 5167.50 = \$4332.50.

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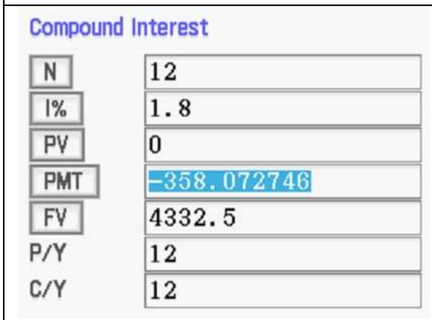

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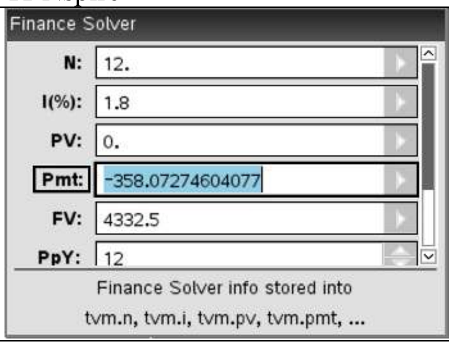

The payment needed is \$358.07

Question 8

a. After the first 12 months the balance of the loan is \$165 275.93
The loan has reduced by 182 000 – 165 275.93= **\$16 724.07** A1

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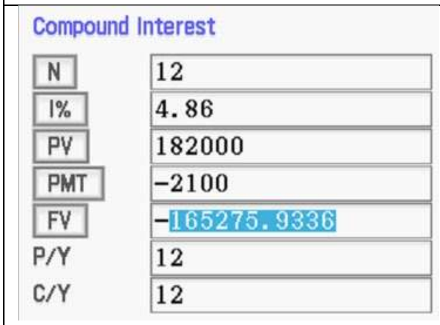

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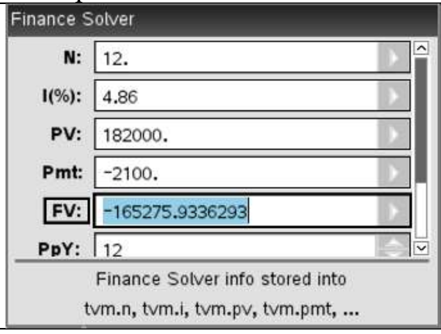
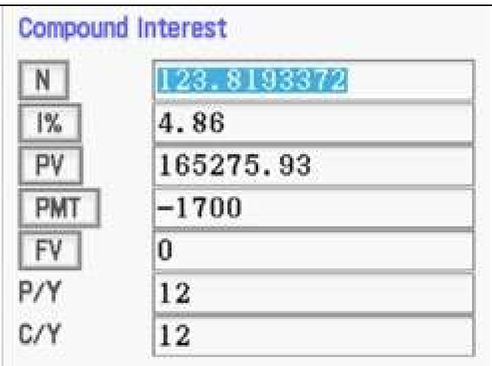


Mary's total payments for the 12 months are 12 × 2100 = \$25 200.
The total interest is difference between the payments and the reduction 25 200 −16 724.07= **\$8475.93** A1

b. Consider this to be a new loan with an initial balance of \$165 275.93.

Casio ClassPad



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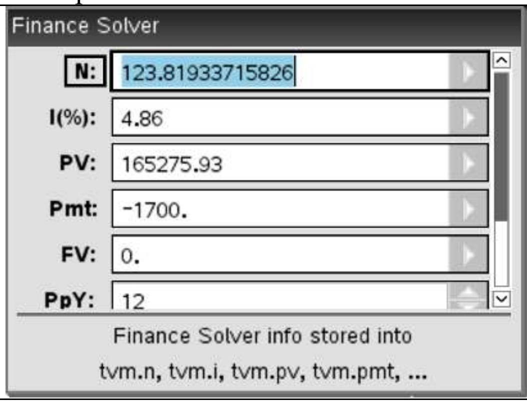


It will take 124 months to repay the rest of the loan.
The total time to repay the \$182 000 is 136 months. A1

Matrices
Question 9

a. 3×1 1M

Matrix P has three rows and one column, therefore it is a 3×1 matrix.

b. $\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}$ 1M

The matrix equation $C = N \times P$ must result in a 1×1 matrix, C.

As matrix P is a 3×1 matrix, matrix N must be a 1×3 matrix.

N must contain the numbers of adults (2), the number of children (3) and the number of seniors (1).

Therefore $N = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}$ and is used in the following to find the cost:

$$C = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 30 \\ 15 \\ 20 \end{bmatrix} = \begin{bmatrix} 125 \end{bmatrix}$$

c. $R = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 1M

The adult price is to have a 10% discount, so should be multiplied by $1 - \frac{10}{100} = 0.9$.

The children's price is to have a 5% discount, so should be multiplied by $1 - \frac{5}{100} = 0.95$.

The senior price does not change and so should be multiplied by 1.

Using the matrix R: $D = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 30 \\ 15 \\ 20 \end{bmatrix} = \begin{bmatrix} 27 \\ 14.25 \\ 20 \end{bmatrix}$

Question 10

a. Bandicoots, lizards and crickets. 1M

The creatures that eat crickets are represented by a "1" in the third row of the matrix, *W*. Bandicoots, lizards and crickets all have a "1" in the third row.

b. Bandicoots eat two creatures, lizards and crickets, that both eat seeds. 1M

The matrix W^2 represents the two step pathways. The element in row 4 column 1 represents that there are two two-step pathways from bandicoots to seeds. Looking at the matrix *W,* it can be seen that bandicoots eat both lizards and crickets (as well as seeds) and that both lizards and crickets eat seeds.

Question 11

a. $\begin{bmatrix} 120 \\ 1510 \\ 1520 \\ 1850 \end{bmatrix}$ 1M

The matrix, B_{2020}, is obtained by calculating the matrix equation below. 2020 is two transitions after 2018, so the transition matrix must be squared to obtain B_{2020}:

$$B_{2020} = T^2 \times B_{2018}$$

$$B_{2020} = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.6 & 0.5 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 \\ 0.2 & 0.1 & 0.4 & 1 \end{bmatrix}^2 \times \begin{bmatrix} 3000 \\ 1000 \\ 1000 \\ 0 \end{bmatrix}$$

$$B_{2020} = \begin{bmatrix} 120 \\ 1510 \\ 1520 \\ 1850 \end{bmatrix}$$

b. 61% 1M

The number of bandicoots at each stage during 2019 can be determined as shown below:

$$B_{2019} = T \times B_{2018}$$

$$B_{2019} = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.6 & 0.5 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 \\ 0.2 & 0.1 & 0.4 & 1 \end{bmatrix} \times \begin{bmatrix} 3000 \\ 1000 \\ 1000 \\ 0 \end{bmatrix}$$

$$B_{2019} = \begin{bmatrix} 600 \\ 2300 \\ 1000 \\ 1100 \end{bmatrix}$$

The number of young adults at the start of 2019 who were mature adults at the start of 2020 is calculated using $0.4 \times 2300 = 920$.

As there are 1520 mature adults in 2020, the percentage of these that were young adults in 2019 is $\frac{920}{1520} \times 100 = 60.5263... \approx 61\%$.

c. 96 joeys, 683 young adults and 4 mature adults. 2M

In order to preserve the numbers from 2020 to 2021, any change between the two state matrices must be reversed by adding bandicoots.

At the start of 2020 it has already been seen that the state matrix was $B_{2020} = \begin{bmatrix} 120 \\ 1510 \\ 1520 \\ 1850 \end{bmatrix} \begin{matrix} J \\ Y \\ M \\ D \end{matrix}$.

The number of bandicoots that would be at each life stage in 2021, if there was no intervention would be:

$$B_{2021} = T^3 \times B_{2018}$$

$$B_{2021} = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.6 & 0.5 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 \\ 0.2 & 0.1 & 0.4 & 1 \end{bmatrix}^3 \times \begin{bmatrix} 3000 \\ 1000 \\ 1000 \\ 0 \end{bmatrix}$$

$$B_{2021} = \begin{bmatrix} 24 \\ 827 \\ 1516 \\ 2633 \end{bmatrix} \begin{matrix} J \\ Y \\ M \\ D \end{matrix}$$

The change therefore is:

- from 120 down to 24 joeys, so 96 must be replaced
- from 1510 down to 827 young adults, so 683 young adults must be replaced
- from 1520 down to 1516 mature adults, so 4 mature adults must be replaced

Question 12

a. Each senior female adult bandicoot will have an average of 2.5 female pre-breeding bandicoots on average each year. 1M

The top row in a Leslie matrix represents the average birth rate for each life stage of a female animal. The 2.5 in the matrix is positioned from *S* this year to *P* next year, so it represents the average number of births per female senior adult. As Leslie matrices only consider female populations, the average number of female births is 2.5 per female senior adult.

b. 1665 1M

The matrix showing the population in 2023 can be calculated as shown below:

$$F_{2023} = L^2 \times F_{2021}$$

$$F_{2023} = \begin{bmatrix} 0 & 1.3 & 2.5 \\ 0.6 & 0 & 0 \\ 0 & 0.4 & 0 \end{bmatrix}^2 \times \begin{bmatrix} 10 \\ 50 \\ 850 \end{bmatrix}$$

$$F_{2023} = \begin{bmatrix} 507.8 \\ 1665 \\ 2.4 \end{bmatrix} \begin{matrix} P \\ I \\ S \end{matrix}$$

c. 10 years 1M

The population of bandicoots increases each year <u>overall</u> according to this Leslie matrix, but one of the age groups decreases each year until 10 transitions have occurred. At this time, the population becomes self-sustaining and all age groups increase every year.

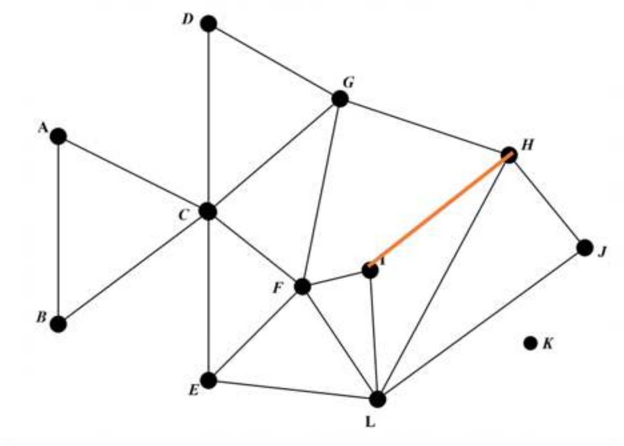
The series of state matrices for this population are seen below:

Year	State matrix	Explanation
2022	$F_{2022}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}2775\\6\\200\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have increased. Independant adults have decreased. Senior adults have decreased.
2023	$F_{2023}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^2\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}507.8\\1665\\2.4\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have decreased. Independent adults have increased. Senior adults have decreased.
2024	$F_{2024}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^3\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}2170.5\\304.68\\666\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have increased. Independent adults have decreased. Senior adults have increased.
2025	$F_{2025}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^4\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}2061.084\\1302.3\\121.872\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have decreased. Independent adults have increased. Senior adults have decreased.
2026	$F_{2026}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^5\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}1997.67\\1236.65...\\520.92\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have decreased. Independent adults have decreased. Senior adults have increased.
2027	$F_{2027}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^6\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}2909.9...\\1198.6...\\494.6...\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have increased. Independent adults have decreased. Senior adults have decreased.
2028	$F_{2028}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^7\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}2794.8...\\1745.9...\\479.4...\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have decreased. Independent adults have increased. Senior adults have decreased.
2029	$F_{2029}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^8\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}3468.3...\\1676.8...\\698.3...\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have increased. Independent adults have decreased. Senior adults have increased.
2030	$F_{2030}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^9\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}3925.9...\\2081.0...\\670.7...\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have increased. Independent adults have increased. Senior adults have decreased.
2031	$F_{2031}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^{10}\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}4382.2...\\2355.5...\\832.4...\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have increased. Independent adults have increased. Senior adults have increased.
2032	$F_{2032}=\begin{bmatrix}0 & 1.3 & 2.5\\0.6 & 0 & 0\\0 & 0.4 & 0\end{bmatrix}^{11}\times\begin{bmatrix}10\\500\\850\end{bmatrix}=\begin{bmatrix}5143.2...\\2629.3...\\942.2...\end{bmatrix}\begin{matrix}P\\I\\S\end{matrix}$	Pre-breeding have increased. Independent adults have increased. Senior adults have increased.

Networks and Decision mathematics

Question 13

a. Edge added shown below: 1M



There is a border between Lake Eyre and the Northeast coast. An edge is needed between Vertex F and Vertex H.

b. 5 1M

There are 5 edges that meet at vertex L, the Murray Darling vertex. The degree of the Murray Darling vertex is 5.

c. X = 0, Y = 1 1M

There is no edge between D and H so X is 0.

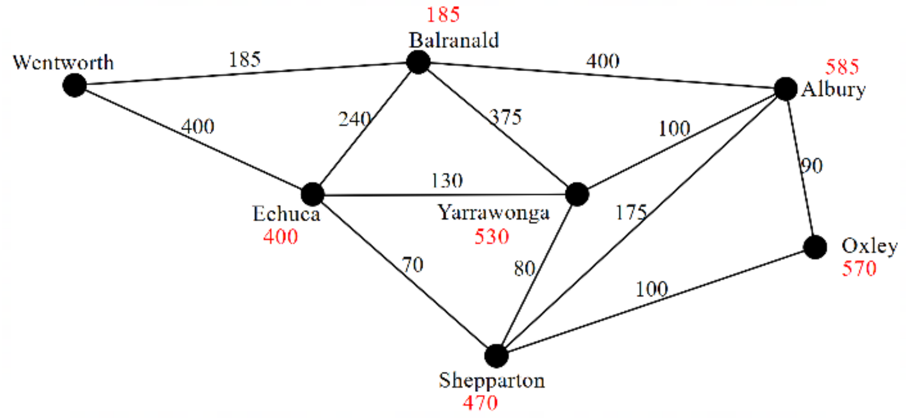
There is an edge between F and I so Y is 1.

The adjacency matrix for an undirected graph also has symmetry.

Question 14

a. 570 (km) 1M

The shortest distance between Wentworth and Oxley is 570km via Echuca and Shepparton.



b. i. Y – S – O – A – B – W – E – Y or the reverse. 1M

Other Hamiltonian cycles are possible answers.

ii. Hamiltonian Cycle 1M

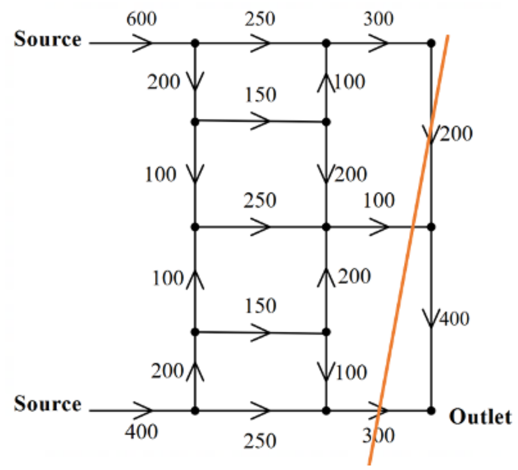
c. The manager can do this because all of the vertices have even degree. 1M

Question 15

a. 600 (kL per min) 1M

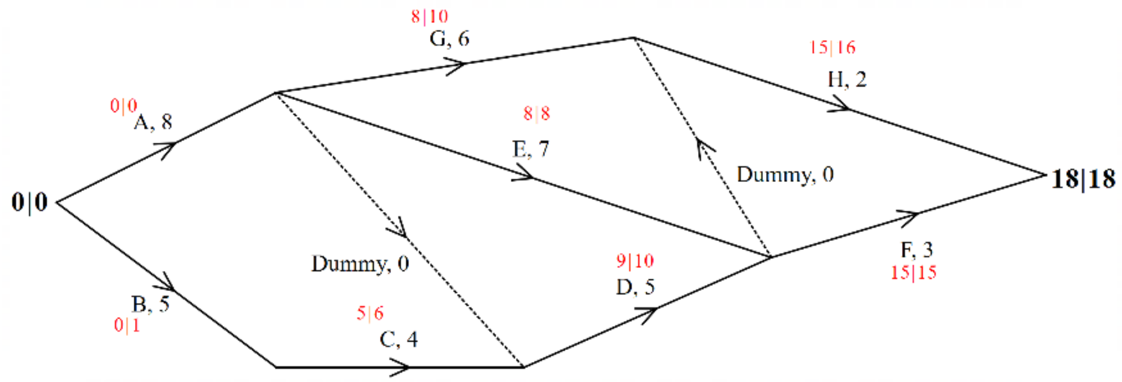
The maximum flow is 600 kilolitres per minute because the capacity of the minimum cut (shown in the next question) is 600 kilolitres per minute. The maximum flow through any network is equal to the minimum cut.

b. The minimum cut as shown below: 1M



Question 16

The scanned activity network for Question 16 is shown below:



Forwarding and backward scanning gives the earliest and latest starting times for the activities.

a. 15 (weeks) 1M

The earliest starting time for activity H is 15 weeks as activities A and E must be completed before activity H can begin.

b. 6 (weeks) 1M

The critical path is A- E- F, taking 18 weeks. The longest path from the start of C to the finish is 12 weeks (durations of C, D and F combined), so the latest starting time for activity C is 18 – 12 = 6 weeks**.**

c. A or E by 2 weeks and B or D by 1 week. 1M

A – E – F is the critical path.

Activity A or E can be reduced by 2 weeks, but any more than that would make B – C – D - F the critical path. Activity B or D would also need to be reduced by 1 week.

The project can only be reduced by two hours overall.

END OF SOLUTIONS