The Mathematical Association of Victoria

Trial Examination 2023

GENERAL MATHEMATICS

Trial Written Examination 1 - SOLUTIONS

Data Analysis

Question	Answer	Question	Answer
1	C	9	B
2	D	10	B
3	B	11	C
4	E	12	D
5	C	13	D
6	E	14	C
7	C	15	B
8	E	16	B

Recursion and financial modelling

Question	Answer
17	A
18	C
19	B
20	C
21	D
22	E
23	C
24	D

Matrices

Question	Answer
25	D
26	D
27	C
28	C
29	B
30	D
31	E
32	D

Networks and decision mathematics

Question	Answer
33	B
34	D
35	C
36	B
37	A
38	C
39	C
40	A

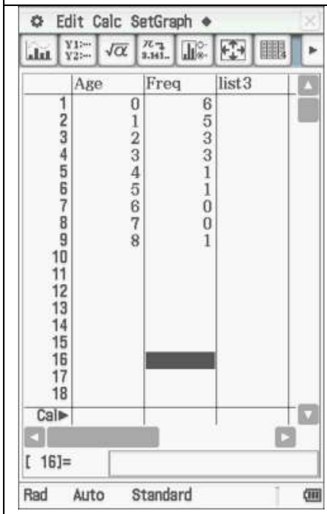
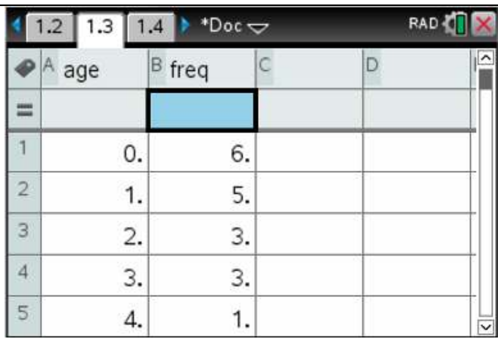
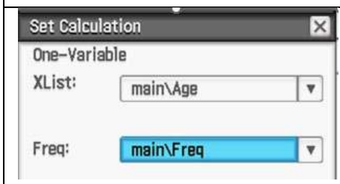
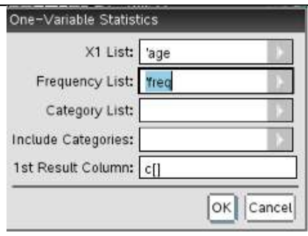
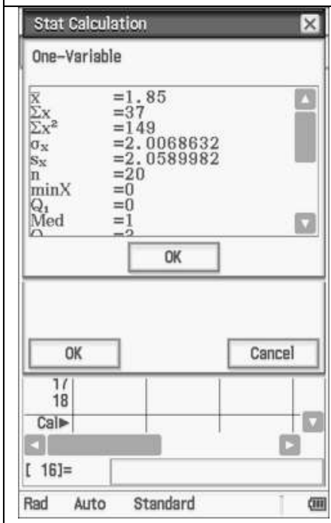
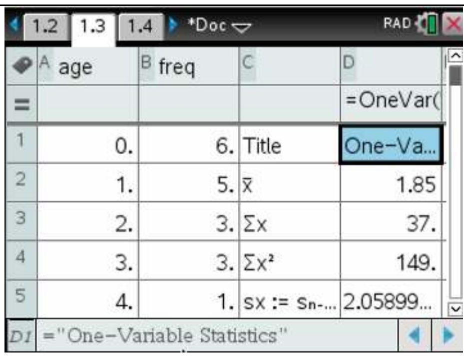
Question 1 **Answer C**

There were 20 sets of parents in the sample and two had age differences of more than 4 years.

$\frac{2}{20} \times 100 = 10\%$

Question 2 **Answer D**

Using the Statistics Application of the calculator;

Casio ClassPad	TI-Nspire
	
	
	

The mean is 1.85 and sample standard deviation, S_x, is 2.0589982… so correct to two decimal places, for this data set, the mean and sample standard deviation are 1.85 and 2.06 respectively.

Question 3 Answer B

The dot plot clearly shows that distribution of *Age Difference* tails off for the upper values, so the shape of the distribution is positively skewed eliminating the distractors A, C and D.

Drawing the box plot, the *Age Difference* of 8 is an outlier, so the answer is B.

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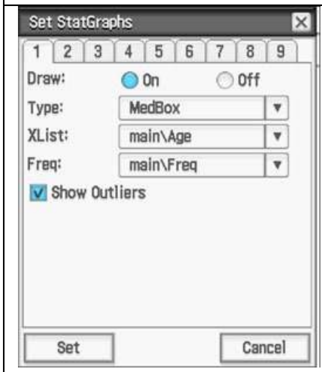
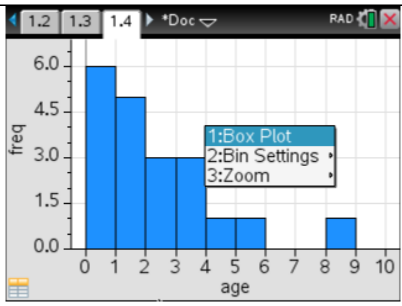
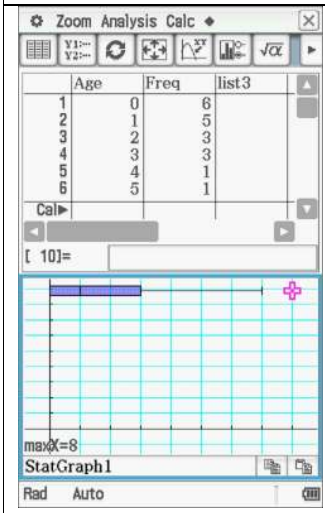
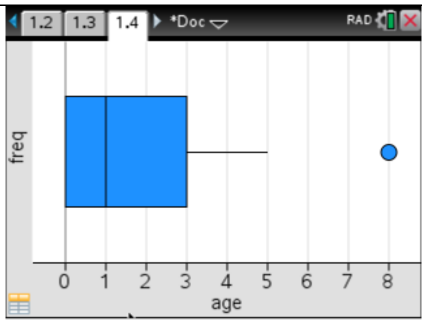
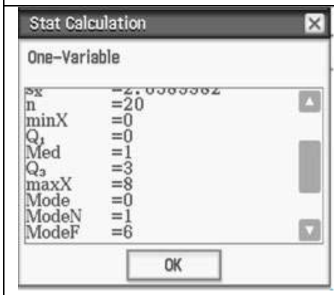
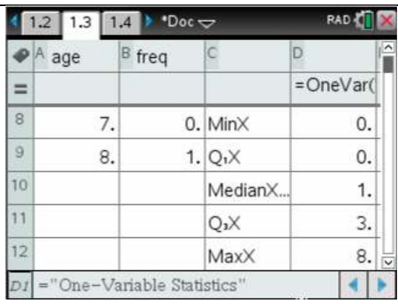
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
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Also, the upper fence is $Q_3 + 1.5 \times IQR = 3 + 1.5(3) = 7.5$ and $8 > 7.5$

Question 4 Answer E

$\log_{10}$ (100 000) is equal to 5.

The number of countries with $\log_{10}$(*population*) greater than 5 is 5+25+15+1 = 46

The percentage of countries with a population greater than 100 000 is $\frac{46}{55} \times 100 \approx 84\%$

Question 5 **Answer C**

The opening time of the school canteen is categorical with three options.

Although the year level is described using numbers, this could be classified as ordinal data as the number is just a label. The numbers do not count or measure.

Of the options, the display that could be used to show the survey's results, and the possible association, between two sets of categorical data is a set of percentaged segmented bar charts.

Question 6 **Answer E**

The median value for the 52 scores is between the 26^{th} and 27^{th} value and is equal to 207.5

The lower quartile is equivalent to the median of the bottom half so between the 13^{th} and 14^{th} value.

The upper quartile is equivalent to the median of the upper half so between the 13^{th} and 14^{th} value of the upper values.

$Q_1 = 198$ and $Q_3 = 222$ IQR = 222 – 198 = 24

The upper fence is calculated as $Q_3 + 1.5(1QR) = 222 + 36 = 258$ and the value of 270 is an outlier as it is above this upper fence.

Question 7 **Answer C**

Option A: The lowest *December maximum daily temperature* for Perth is 23°C.
The lowest *December maximum daily temperature* for Melbourne is 16°C and for Sydney is 19°C. Therefore, it is true that the lowest *December maximum daily temperature* for Perth in greater than the lowest *December maximum daily temperatures* for both Melbourne and Sydney.

Option B: Using the interquartile range, IQR, as a measure of variation. The IQR for the *December maximum daily temperature* for Perth is 32 – 26 = 6 which is less than the IQR for the *December maximum daily temperature* for Sydney, which is 28 – 23 = 5 so this is true. The range is also larger for Perth than Sydney.

Option C: The median of the Melbourne *December maximum daily temperatures* is 23°C which is lower than the median of 25°C for *December maximum daily temperatures* or Sydney. So, it is **not** true that the *December maximum daily temperatures* are, on average, lower in Sydney than in the other two cities.

Option D: The highest *December maximum daily temperatures* in Sydney is 30°C which is the same as the median for the *December maximum daily temperatures* in Perth.

Option E: the lower quartile for the *December maximum daily temperatures* for Perth is 26°C. The median *December maximum daily temperatures for* Sydney and Melbourne are 23°C and 25°C respectively. So, it is true that more than 75% of *December maximum daily temperatures* for Perth were hotter than the median *December maximum daily temperatures for* Sydney and Melbourne.

Question 8 **Answer E**

For the 31 days in December there are 16 days greater than or equal to median and 24 days greater than or equal to Q_1.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31.

Q_1 median Q_3

In Melbourne there were 16 days where the *December maximum daily temperature* 23°C or above.
In Perth all 31 days had the *December maximum daily temperature* 23°C or above.
In Sydney there were 24 days where the *December maximum daily temperature* 23°C or above

Total is $16 + 31 + 24 = 71$ days

Question 9 **Answer B**

The percentage of the variation in *life expectancy (2020)* that can be explained variation of *life expectancy (1960)* is r^2 as a percentage.

$r^2 = (0.745)^2 = 0.555025$ or 55.5%

The percentage of the variation in *life expectancy (2020)* that cannot be explained variation of *life expectancy (1960)* is r^2 as a percentage is therefore 100 – 55.5 = 44.5%

Question 10 **Answer B**

From the formula sheet, the equation of least squares line of best fit, $y = a + bx$ can be calculated using $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

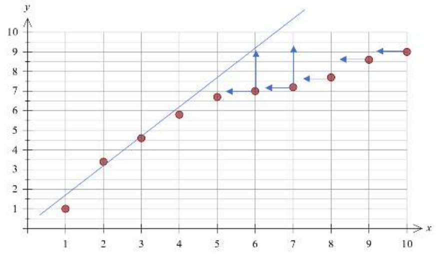
$$b = 0.745 \times \frac{7.22}{11.89} = 0.452 \ldots$$

This would eliminate answers A, C and E.

$$a = 72.27 - (0.452 \ldots)(51.93) = 48.777 \ldots$$

The closest answer values is $\textit{life expectancy(2020)} = 49 + 0.45 \times \textit{life expectancy(1960)}$.

Question 11 **Answer C**



To linearise this scatterplot we would need to

- compress the upper end of the scale on the x-axis with a $\log_{10}(x)$ transformation
- compress the upper end of the scale on the x-axis with a $\frac{1}{x}$ transformation
- stretch out the upper end of the scale on the y-axis with a y^2 transformation

The only option in the answers is plotting y against $log_{10}(x)$

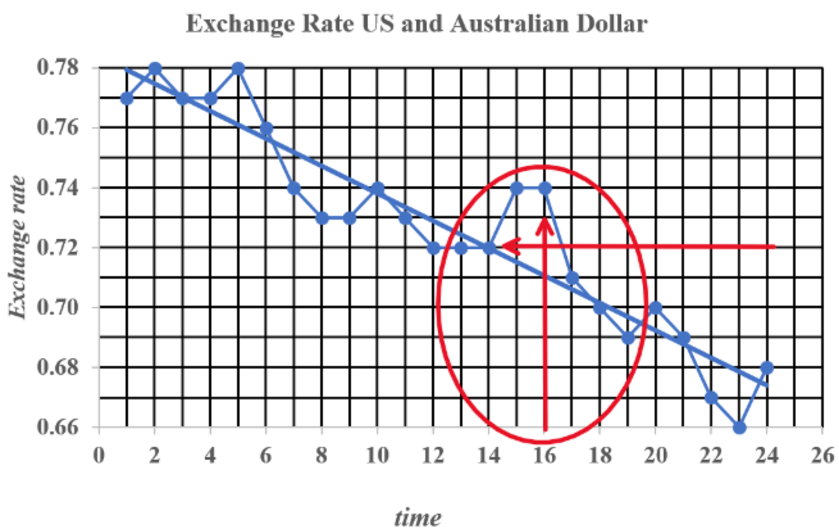
Question 12 **Answer D**

The least squares regression line for this data clearly has a negative gradient indicating a decreasing trend. This would eliminate answers B and C. There is no regular pattern in the fluctuations, so it is does not have seasonality. It would be best described as having a decreasing trend with irregular fluctuations.

Question 13 **Answer D**

The seven-median smoothed *Exchange Rate* for April 2022 would occur when $t = 16$.

For the seven values considered for the median smoothing, the *Exchange Rate for* $t = 13$ or $t = 14$ would be the median as both values are 0.72. The median is 0.72.



Question 14 **Answer C**

Exchange Rate is the response variable, so we eliminate answers A and E.

B uses the first and last data points, neither of which are on the line.

The *Exchange Rate* intercept for the least squares regression line for the time series data is closer to 0.784 than 0.0046, so we would eliminate answer D.

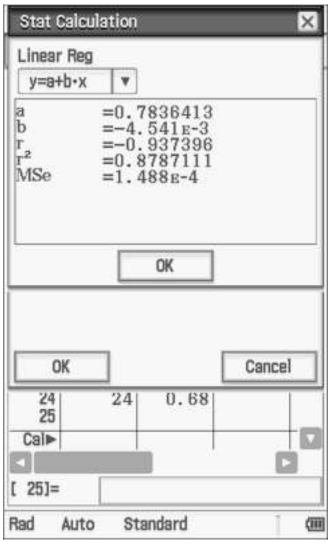
The gradient can be approximated by using two points on the line. Using the points (1, 0.78) and (24, 0.675) the gradient can be determined as follows:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0.675 - 0.78}{24 - 1}$$

$$m = -0.004565... \approx -0.0046$$

These values for the gradient and intercept are confirmed by putting all the points in the statistics mode of the calculator as shown below:



The equation of the least squares regression line for the time series data is closest to option C, $Exchange\ rate\ =\ 0.784\ -\ 0.0046 \times time$

Question 15 **Answer B**

For each year, the Q4 value is divided by the 'yearly average' to calculate the seasonal proportion:

	2015	2016	2017	2018
Q4	1.7÷1.75 = 0.971	1.5÷1.275 = 1.176	1.9÷1.925 = 0.987	1.8÷1.925 = 0.935

The seasonal index for these values is the average of the four seasonal proportions:

Seasonal Index = $\frac{(0.971+1.176+0.987+0.935)}{4} \approx 1.02$

Question 16 **Answer B**

Time	CPI	4 point moving mean	centred 4 point moving average
Q_1 2016	1.3		
Q_2 2016	1.0	$\frac{1.3+1.0+1.3+1.5}{4} = 1.275$	
Q_3 2016	1.3	$\frac{1.0+1.3+1.5+2.1}{4} = 1.475$	$\frac{1.275+1.475}{2} = 1.375$
Q_4 2016	1.5		
Q_1 2017	2.1		

Recursion and Financial Modelling

Question 17 Answer A

The printing press is depreciated at 6% per annum on a flat rate basis. Therefore, each year the printing press reduces in value by $\frac{6}{100} \times 43\,500 = \$2610$ per year.

Therefore, the value after three years is $43\,500 - 3 \times 2610 = \$35\,670$.

Question 18 Answer C

If the value of $A_{n+1} = A_n$ then $A_1 = A_0 = 5$. The value of k can be solved as shown below:

$$5 = -3 \times 5 + k$$
$$k = 5 + 15 = 20$$

Question 19 Answer B

If Peter uses the unit cost method, the value of his van after two years will be:

$Value = 50\,000 - 66\,000 \times 0.127 = \$41\,618$.

If Peter uses reducing balance depreciation, the rate that would achieve the same value after two years can be calculated using the equation below and solved using CAS:

$$V_2 = (1 - \frac{r}{100})^2 \times V_0$$
$$41\,618 = (1 - \frac{r}{100})^2 \times 50\,000$$
$$r = 8.7662...$$

The interest rate is closest to 8.8%.

Question 20 Answer C

The recurrence relation shows an initial balance of \$100 000. The balance is increased by a multiple of 1.0075, which is an increase of 0.75% per time period. Each time period the balance is decreased by \$800.

This information means that the recurrence relation could be either a reducing balance loan or an annuity.

The per annum interest rate could be $0.75 \times 12 = 9\%$ compounding monthly, $0.75 \times 4 = 3\%$ compounding quarterly or $0.75 \times 2 = 1.5\%$ compounding half-yearly.

Using this information, options A, B, D and E are all possible explanations for the recurrence relation.

Option C is not possible, and is therefore the correct answer, because the multiplier for option C would be $1 + \frac{7.5}{100} = 1.075$ rather than 1.0075.

Question 21 Answer D

Two different values have been given on the graph, \$9600 after one year and \$6144 after three years.

Using these two values, the multiplier over a two year period is $\frac{6144}{9600} = 0.64$ and therefore the multiplier per year is $\sqrt{0.64} = 0.8$. This means that the computer system is reducing by 20% each year, from a starting value of \$12 000.

The seventh year is from the end of six years to the end of seven years:

At the end of the sixth year the value is $0.8^6 \times 12\,000 = \$3145.73$.

At the end of the seventh year the value is $0.8^7 \times 12\,000 = \$2516.58$.

The depreciation in the seventh year is 3145.73 – 2516.58 = \$629.15, which is closest to \$629.

Question 22 Answer E

The interest rate for this loan is $\frac{480.15}{106\,700.00} \times 100 = 0.45\%$ per month.

The interest in month 22 would be $\frac{0.45}{100} \times 104133.64 = \$468.60$.

The balance after 22 months would be the balance at the end of 21 months plus the interest charged and less the payment made, so it is $104133.64 + 468.60 - 3046.51 = \$101555.73$.

Question 23 Answer C

At the end of 20 months, the balance of the loan is \$106 700 and the per annum interest rate is $0.45 \times 12 = 5.4\%$.

The original amount borrowed can now be calculated using CAS finance:

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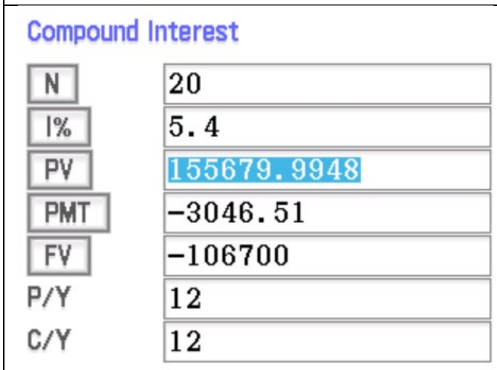
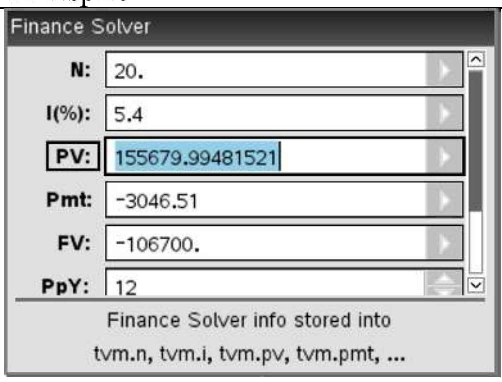
	

The original balance is therefore \$155 679.99 which is closest to \$155 680.

Question 24 **Answer D**

Given that Tony will retire in five years' time and he currently has \$250 000, will add \$1200 per month and receive interest at 3.2% per annum, his balance at retirement will be \$371 282.70 as shown below:

Casio ClassPad		TI-Nspire	
Compound Interest		Finance Solver	
N	60	N:	60.
I%	3.2	I(%):	3.2
PV	−250000	PV:	-250000.
PMT	−1200	Pmt:	-1200.
FV	371282.695	FV:	371282.69495297
P/Y	12	PpY:	12
C/Y	12		Finance Solver info stored into tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

He puts his money into a perpetuity for the next five years, and so the balance will not change during that period as he will take exactly the amount of interest added to the perpetuity out of the perpetuity.

Tony will then change his account to an annuity, taking payments for the next 30 years. The payment can be calculated using CAS:

Casio ClassPad		TI-Nspire	
Compound Interest		Finance Solver	
N	360	N:	360.
I%	4.8	I(%):	4.8
PV	−371282.69	PV:	-371282.69
PMT	1947.991641	Pmt:	1947.9916410965
FV	0	FV:	0.
P/Y	12	PpY:	12
C/Y	12		Finance Solver info stored into tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

This shows that his monthly payment from the annuity will be \$1947.99.

The recurrence relation must show a starting balance of \$371 282.70 and payments reducing the balance by \$1947.99. The multiplier must be $1+\dfrac{4.8/12}{100}=1.004$.

Therefore, the correct recurrence relation is $T_0 = 371282.70,\ T_{n+1} = 1.004 \times T_n - 1947.99$.

Matrices

Question 25 **Answer D**

The transpose of a matrix is made by swapping the rows and columns, so that row 1 becomes column 1, row 2 becomes column 2 and row 3 becomes column 3.

Therefore, $\begin{bmatrix} 3 & 6 \\ 1 & -8 \\ 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 & 0 \\ 6 & -8 & 2 \end{bmatrix}$.

Question 26 **Answer D**

The elements in A are in ascending order, so if the elements are required in descending order, the last element must be selected first, then the second last element, then the second element and finally the first element.

If a, b, c and d are numbers where $b > a$ and $c > b$ and $d > c$, then the permutation matrix must change $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ to $\begin{bmatrix} d \\ c \\ b \\ a \end{bmatrix}$.

This can be achieved using $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} d \\ c \\ b \\ a \end{bmatrix}$.

Question 27 **Answer C**

The matrix for the second Tuesday night is $S_2 = \begin{array}{c} \begin{bmatrix} 16 \\ 18 \\ 26 \end{bmatrix} \end{array} \begin{array}{l} F \\ C \\ G \end{array}$.

Each of the options is explored below:

Option	Statement	True or false?
A	In the long run half of the members will do general fitness every week.	True. The equilibrium state matrix is $T^{30} \times S_2 = T^{30} \times \begin{bmatrix} 16 \\ 18 \\ 26 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 30 \end{bmatrix} \begin{array}{l} F \\ C \\ G \end{array}$ so 30 members or half of the 60 members do general fitness.
B	In the long run the same number of members will do floor exercises and combat exercises every week.	True. As can be seen above, 15 members do floor exercises and 15 do combat exercises in the long run.
C	In the long run three members will change from general fitness to floor exercises from one week to the next.	False. The proportion of members who change from general fitness to floor exercises is 0.2, so the number of members who change is $0.2 \times 30 = 6$.
D	In the long run nine members will stay in combat exercises from one week to the next.	True. The proportion of members who stay in combat exercises is 0.6, so the number of members who stay in combat is $0.6 \times 15 = 9$.
E	In the long run 21 members will stay in general fitness from one week to the next.	True. The proportion of members who stay in general fitness is 0.7, so the number of members who stay in general fitness is $0.7 \times 30 = 21$.

Therefore, the option that is not true is C.

Question 28 **Answer C**

The matrix for the second Tuesday night is $S_2 = \begin{bmatrix} 16 \\ 18 \\ 26 \end{bmatrix} \begin{array}{l} F \\ C \\ G \end{array}$ so the matrix for the first Tuesday night is $T^{-1} \times S_2 = \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix} \begin{array}{l} F \\ C \\ G \end{array}$.

Using the leading diagonal of *T*, where the proportion of members who did not change is represented, the number of members who did not change is $0.5 \times 20 + 0.6 \times 20 + 0.7 \times 20 = 36$ members. Therefore, the number of members who did change is 60 – 36 = 24.

Question 29 **Answer B**

The elements in $L = M - N$ are determined by subtracting each of the elements in N from their corresponding elements in M, so the rule that determines each element is:

$$l_{ij} = (4i - 8j) - (5i + 2j) = -i - 10j$$

Question 30 **Answer D**

Matrix A is a 3×5 matrix. Matrix B is a 4×3 matrix. Matrix C is a 5×4 matrix.

Each of the options is explored below:

Option	Expression	Explanation
A	$(BAC)^2$	The expression is defined. Using the rules for matrix multiplication BAC is a $(4\times3)\times(3\times5)\times(5\times4) = 4\times4$ matrix. As BAC is a square matrix, it can be squared.
B	$(AC)^T - B$	The expression is defined. Using the rules for matrix multiplication AC is a $(3\times5)\times(5\times4) = 3\times4$ matrix. Therefore $(AC)^T$ is a 4×3 matrix which is of the same order as B. This means that the two matrices can be subtracted.
C	$(CB)^T + A$	The expression is defined. Using the rules for matrix multiplication CB is a $(5\times4)\times(4\times3) = 5\times3$ matrix. Therefore $(CB)^T$ is a 3×5 matrix which is of the same order as A. This means that the two matrices can be added.
D	$CBA + CA$	The expression is not defined. Using the rules for matrix multiplication CBA is a $(5\times4)\times(4\times3)\times(3\times5) = 5\times5$ matrix. But CA is not defined as the number of columns in C (4) is not the same as the number of rows in A (3). Therefore, the entire expression cannot be calculated.
E	$(CBA)^2$	The expression is defined. Using the rules for matrix multiplication CBA is a $(5\times4)\times(4\times3)\times(3\times5) = 5\times5$ matrix. As CBA is a square matrix, it can be squared.

The expression that is not defined is option D.

Question 31 **Answer E**

The final matrix must show the total cost to the school of all ordered pizzas, so it must have one element and be a 1×1 matrix. This eliminates option C as the expression would not be defined. All other options would result in a 1×1 matrix and so further exploration is required.

The discounted pizza cost is given by $\begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix} \times \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 5.4 \\ 6.4 \end{bmatrix} \end{array}\begin{array}{l} S \\ L \end{array}$.

The total number of small and large pizzas ordered is given by:

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times P^T$$
$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \times \begin{bmatrix} 23 & 43 \\ 16 & 5 \end{bmatrix}$$
$$= \begin{array}{c} \begin{array}{cc} S & L \end{array} \\ \begin{bmatrix} 39 & 48 \end{bmatrix} \end{array}$$

The total cost is given by $\begin{bmatrix} 39 & 48 \end{bmatrix} \times \begin{bmatrix} 5.4 \\ 6.4 \end{bmatrix} = \begin{bmatrix} 517.80 \end{bmatrix}$.

Therefore, the full expression would be $\begin{bmatrix} 1 & 1 \end{bmatrix} \times P^T \times \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix} \times \begin{bmatrix} 6 \\ 8 \end{bmatrix}$.

Question 32 **Answer D**

The initial state matrix for this breeding colony is $K_0 = \begin{array}{c} \begin{bmatrix} 200 \\ 100 \\ 100 \\ 100 \end{bmatrix} \end{array}\begin{array}{l} E \\ C \\ P \\ B \end{array}$.

The states after each of the first eight months, as well as the equilibrium state are shown below:

$K_1 = \begin{bmatrix} 1000 \\ 40 \\ 50 \\ 30 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$	$K_2 = \begin{bmatrix} 300 \\ 200 \\ 20 \\ 15 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$	$K_3 = \begin{bmatrix} 150 \\ 60 \\ 100 \\ 6 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$
$K_4 = \begin{bmatrix} 60 \\ 30 \\ 30 \\ 30 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$	$K_5 = \begin{bmatrix} 300 \\ 12 \\ 15 \\ 9 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$	$K_6 = \begin{bmatrix} 90 \\ 60 \\ 6 \\ 4.5 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$
$K_7 = \begin{bmatrix} 45 \\ 18 \\ 30 \\ 1.8 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$	$K_8 = \begin{bmatrix} 18 \\ 9 \\ 9 \\ 9 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$	$K_\infty = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{array}{l} E \\ C \\ P \\ B \end{array}$

Each of the options is explored below:

Option	Statement	True or false?
A	Only 30% of the female pupae survive to become butterflies.	True. The element in the fourth row and third column represents the transition from pupae to butterflies. As this element is 0.3, 30% of pupae survive to become butterflies.
B	After the first month there are never more than 30 female butterflies in the colony.	True. It can be seen that the number of butterflies after the first month is 30, but from then onwards the number of butterflies decrease.
C	Every fourth month, the predicted numbers of female caterpillars, pupae and butterflies are equal.	True. It can be seen by the matrices after 4 and 8 months that there are equal numbers of caterpillars, pupae and butterflies every 4 months. A similar result is seen after 12, 16, 20 months and so on.
D	The greatest number of female caterpillars at any time is 300.	False. The greatest number of caterpillars is 200 after two months, and from there onwards the numbers decrease.
E	Eventually the female colony is predicted to become extinct.	True. The equilibrium state has zero elements at each life stage and so the colony is extinct.

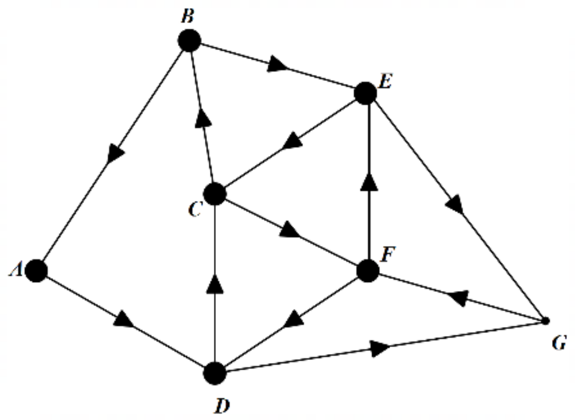
Networks and decision mathematics

Question 33 Answer B

Using the formula $v + f = e + 2$
$12 + 20 = e + 2$
$32 = e + 2$
$e = 30$

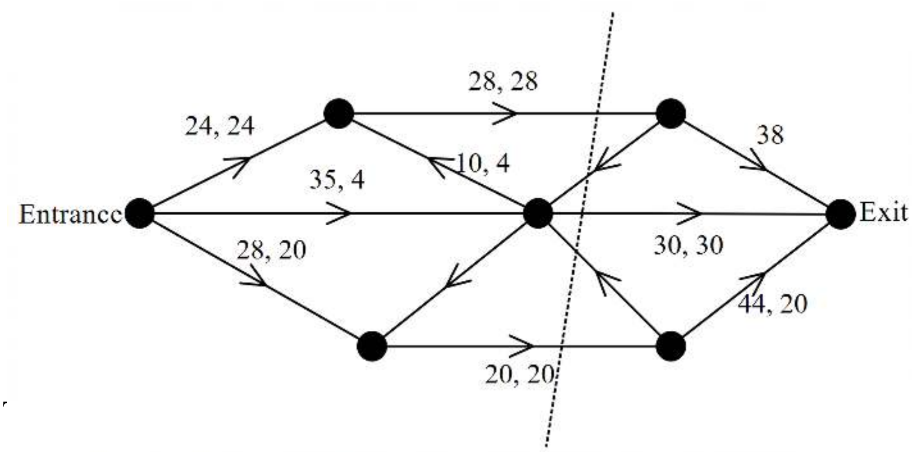
Question 34 Answer D

If a graph has an Eulerian trail, it must start and finish at vertices with odd degrees.
This will eliminate the options A, B and C.
Answer E has a repeated edge -C-F-E-F-D- so is not Eulerian, because by definition an Eulerian trail does not repeat edges.



Question 35 Answer C

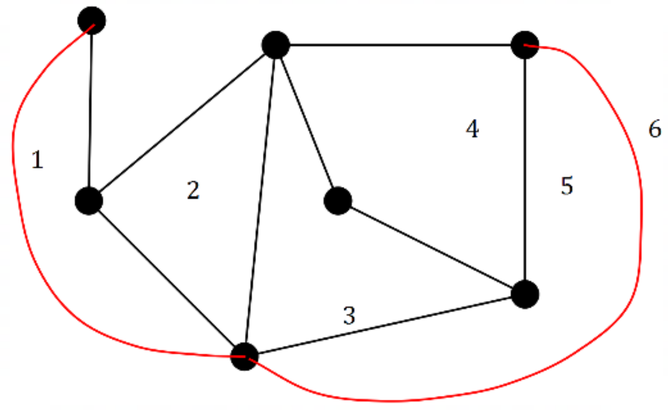
The maximum flow through a network is equal to the minimum cut. The minimum cut for this system is 20 + 30 + 28 = 78



The diagram shows one way in which 78 people can flow through the library.

Question 36 Answer B

The graph can be redrawn to show that it is planar graph, with no edges crossing.



The number of faces, including the outside face is 6.

Alternatively, there are 7 vertices and 11 edges.

Using the formula $v + f = e + 2$:
$7 + f = 11 + 2$
$7 + f = 31$
$f = 6$

Question 37 Answer A

The minimum total time can be calculated using Hungarian algorithm as follows:

	Activity 1	Activity 2	Activity 3	Activity 4
Allan	**6**	9	12	10
Brianna	**7**	10	14	8
Caroline	**6**	8	X	9
Daisy	9	**8**	16	8

Subtract the lowest value in each row from every number in the row.

	Activity 1	Activity 2	Activity 3	Activity 4
Allan	0	3	**6**	4
Brianna	0	3	7	1
Caroline	0	2	X	3
Daisy	1	0	8	0

Subtract the lowest value in each column from every number in the column.

	Activity 1	Activity 2	Activity 3	Activity 4
Allan	0	3	0	4
Brianna	0	3	1	1
Caroline	0	2	X	3
Daisy	1	0	2	0

We can cover all the zeros with less than four lines, so an additional step is required:

	Activity 1	Activity 2	Activity 3	Activity 4
Allan	0	3	0	4
Brianna	0	3	1	1
Caroline	0	2	X	3
Daisy	1	0	2	0

Subtract the lowest uncovered value from all the other uncovered values.
Add the lowest uncovered value to the values where two lines cross.

	Activity 1	Activity 2	Activity 3	Activity 4
Allan	1	3	0	4
Brianna	0	2	0	0
Caroline	0	1	X	2
Daisy	2	0	2	0

Tasks can now be allocated where there are zeros.
The total time is 6 + 8 + 12 + 8 = 34 minutes

If Daisy reduces her time for activity 3 to 12 minutes, the Hungarian algorithm gives

	Activity 1	Activity 2	Activity 3	Activity 4
Allan	0	3	2	4
Brianna	0	3	3	1
Caroline	0	2	X	3
Daisy	1	0	0	0

Subtract the lowest value in each column from every number in the column.

	Activity 1	Activity 2	Activity 3	Activity 4
Allan	0	2	1	3
Brianna	0	2	2	0
Caroline	0	1	X	2
Daisy	2	0	0	0

Repeat

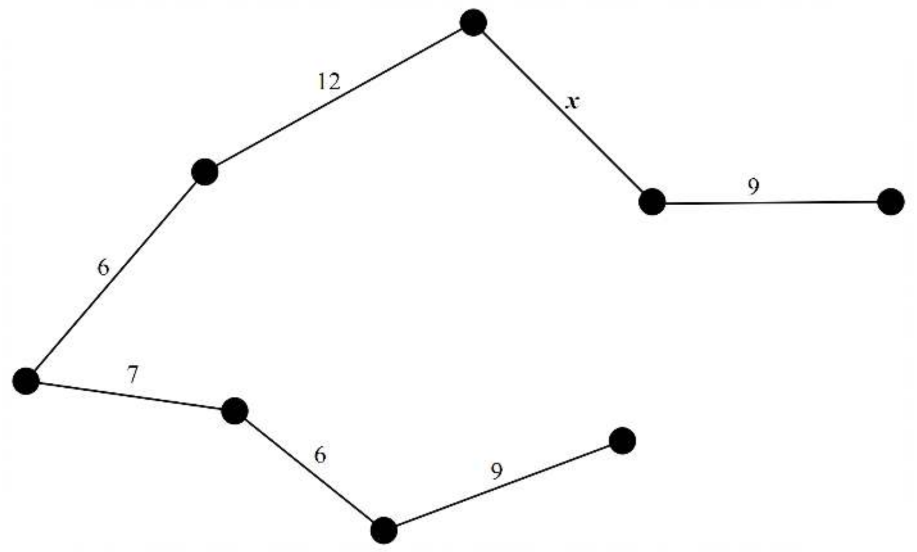
	Activity 1	Activity 2	Activity 3	Activity 4
Allan	0	1	0	2
Brianna	0	1	1	0
Caroline	0	0	X	2
Daisy	3	0	0	1

The allocation could stay the same.
Allocating Daisy to Activity 3 would give the same time.

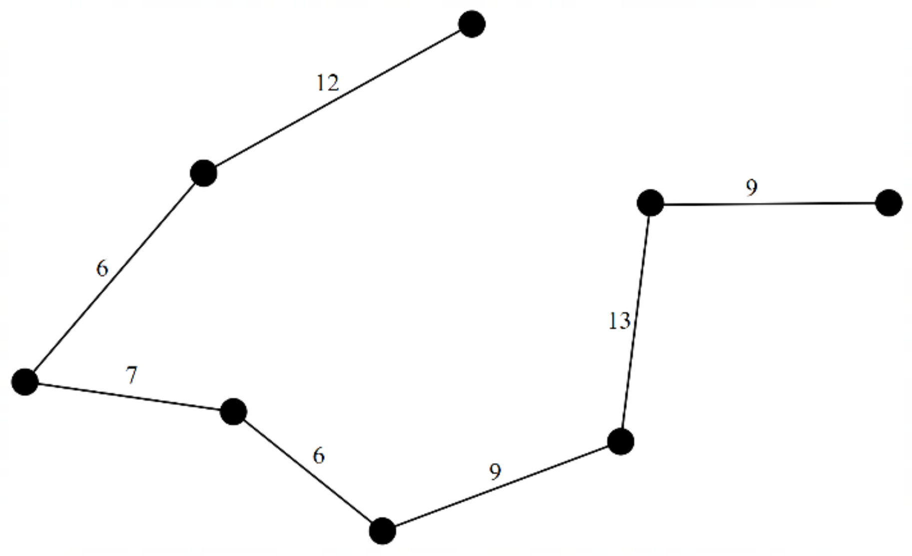
Question 38 **Answer C**

The weights on the edges of the network below show the cost, in millions of dollars, of building gas pipes, between eight towns.

A minimum spanning tree for this graph, including x is



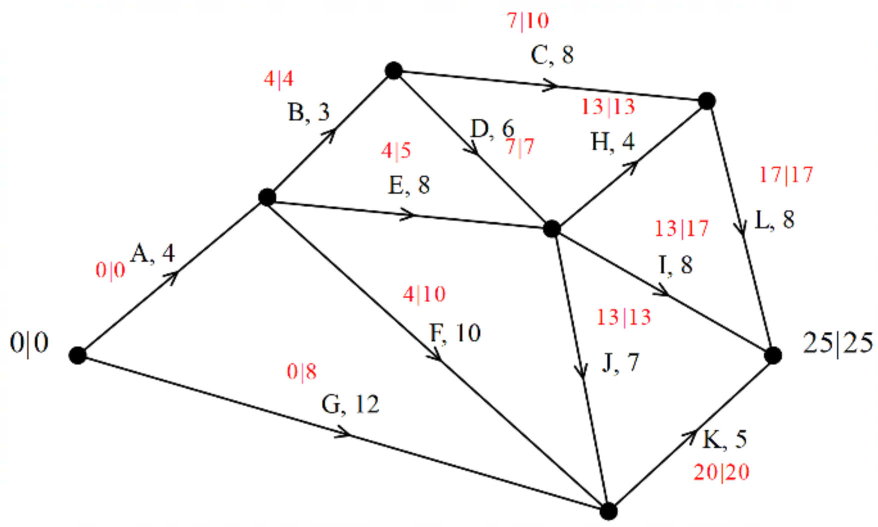
A minimum spanning that does not include the edge labelled x is



The maximum weight of the edge labelled ***x*** must be equal to the weight of the alternative edge.

Question 39 Answer C

The graph shows the forward and backtracking along the edges.



There are two critical paths
A – B – D – H – L
A – B – D – J – K

The float time for activity I is 17 – 13 = 4.

Question 40 Answer A

Option A: Reducing activity A by 2 days will reduce the overall time by 2 days because the float time of activity G is more than 2 days.

Option B: Reducing activity D by 3 days will reduce the overall time by only 1 day because the float time of activity E is only 1 day. The new critical paths would become A – E – H – L or A – E – J – K

Option C: Reducing activity E by 3 days would not change the critical path.

Option D: Reducing activity I by 3 days would not change the critical path.

Option E: Reducing activity L by 3 days would still leave the critical path A – B – D – J – K

END OF SOLUTIONS