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# GENERAL MATHS UNITS 3&4 TRIAL EXAMINATION 2 SOLUTIONS 2023

## Data analysis

**Question 1** (5 marks)

**a.** **i.** The median is the middle value. The 15th value, counting in from the left hand side of the dot plot is 23.
The 15th value, counting in from the right hand side of the dot plot is also 23.
The median is therefore 23.
Alternatively, the median data point is at location $\frac{n+1}{2}=\frac{30+1}{2}=15.5$.

**(1 mark)**

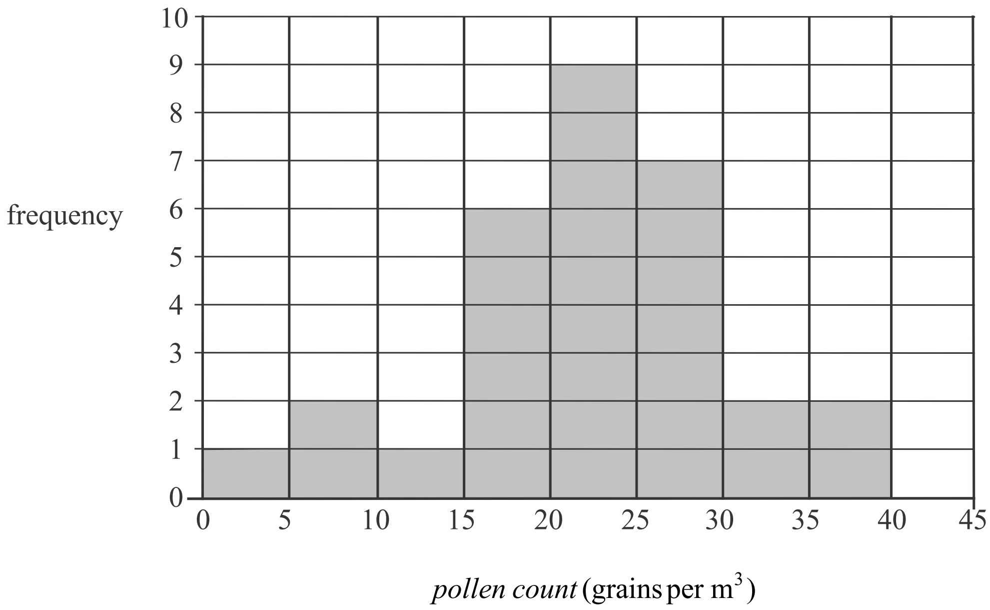
**ii.** $Q_1$ is the middle value of the lower 15 values of the distribution, i.e., it is the 8th value counting in from the left hand side of the dot plot. So $Q_1=18$.

**(1 mark)**

**iii.** There are 3 days in June with a grass pollen count greater than 30.
$\left(\frac{3}{30}\times\frac{100}{1}\right)\%=10\%$

**(1 mark)**

**b.** The question tells us that the first interval starts at 0. So the first interval includes 0 and goes up to 5 but does not include 5. The second interval starts at 5 and goes up to 10 but does not include 10. The third interval starts at 10 and goes up to 15 but does not include 15 and so on.




**(1 mark)** – four correct columns
**(1 mark)** – remaining four columns correct

**Question 2** (3 marks)

**a.** In 2021, 12% of days were classed as high pollen count days and 28% of days were classified as extreme. So 40% of 365 days $= 146$ days.

**(1 mark)**

**b.** The percentaged segmented bar chart **does** support the contention that the pollen count is associated with year because there is a difference in the percentage of days when the pollen count was **low** across the years. **(1 mark)**
The percentage of days in 2020 when the grass pollen count was low was 16% which was less than 2021 (24%) which in turn was less than 2022 (32%). **(1 mark)**

An alternative answer could state that there was a difference when the pollen count was moderate, high or extreme. Mentioning any <u>one</u> classification where there was a change in statistics, and comparing relevant, quoted statistics across the years for that classification would then be needed.

For example, the percentage of days in 2020 when the pollen count was **moderate** was 48% which was higher than 2021 (36%) which in turn was higher than 2022 (26%).

Alternatively, the percentage of days in 2020 when the pollen count was **high** was 16% which was higher than 2021 (12%) which was less than 2022 (34%).

Alternatively, the percentage of days in 2020 when the pollen count was **extreme** was 20% which was less than 2021 (28%) which was higher than 2022 (8%).

**Question 3** (5 marks)

**a.** There are two categorical variables, *day* and *rainfall.*

**(1 mark)**

**b.** Enter the *temperature* data into your CAS and use the one-variable statistics option to calculate the mean $(\bar{x})$.
$\bar{x} = 29$

**(1 mark)**

**c.** **i.** The response variable is *pollen count*.

**(1 mark)**

**ii.** Enter the data for *relative humidity* and *pollen count* into your CAS.
Using the Linear Regression $(a + bx)$ option, with *relative humidity* as the *x*-variable (explanatory variable) and *pollen count* as the *y*-variable (response variable), calculate the least squares equation line.
The slope is given by *b*. i.e. $b = -0.5500336....$
So slope is – 0.550 (correct to 3 significant figures).

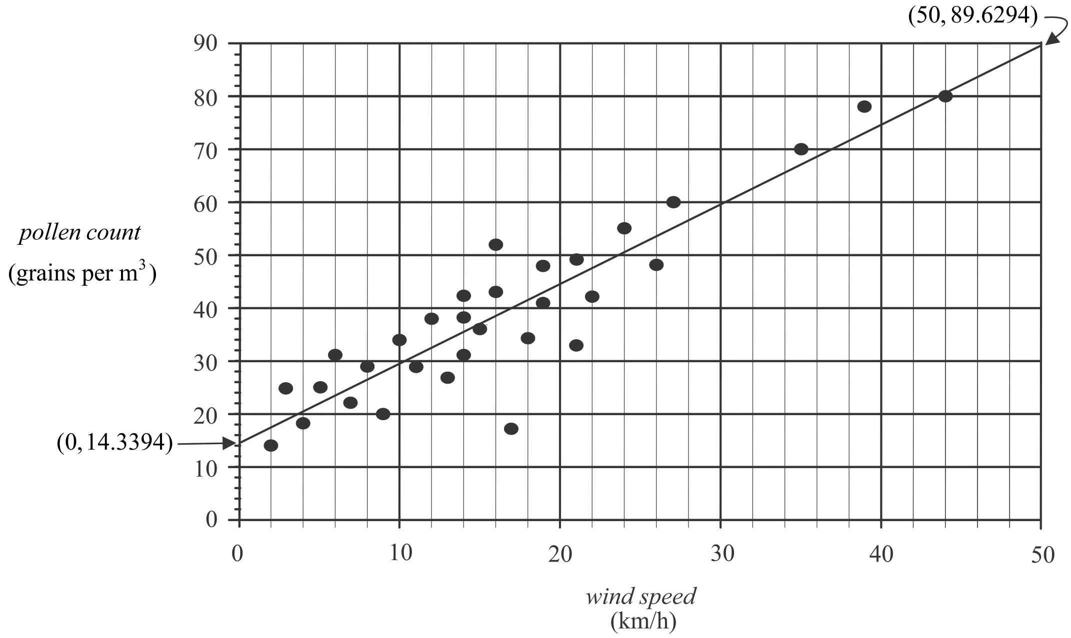
**(1 mark)**

**iii.** The correlation coefficient is given by *r*.
$r = -0.839942...$
So $r = -0.840$ (correct to 3 significant figures).

**(1 mark)**

**Question 4** (7 marks)

**a.**




Whilst not necessary to gain the mark, it is very useful to show the first and last points on the line i.e. $(0, 14.3394)$ and $(50, 89.6294)$, and then just use a ruler to connect them.

**(1 mark)**

**b.** The association between *pollen count* and *wind speed* is linear and positive.

**(1 mark)**

**c.** The intercept occurs at the point $(0, 14.3394)$. This means that when there is no wind (i.e. wind speed is zero), the expected pollen count is 14.3394 grains per m$^3$.

**(1 mark)**

**d.**
$$\begin{aligned} \textit{pollen count} &= 14.3394 + 1.5058 \times 30 \\ &= 59.5134 \end{aligned}$$

The predicted pollen count is 59.51 grains per m$^3$ (correct to 2 decimal places).

**(1 mark)**

**e.**
$$\begin{aligned} \text{residual value} &= \text{actual value} - \text{predicted value} \quad \text{(formula sheet)} \\ &= 20 - 27.8916 \\ &= -7.8916 \end{aligned}$$

**(1 mark)**

**f.** The percentage of variation in *pollen count* that can be explained by the variation in *wind speed* is given by the coefficient of determination i.e. $r^2$.
We don't have the raw data to calculate this but we do have the statistics in the table.

$$y = a + bx$$

and $b = r\dfrac{s_y}{s_x}$ (formula sheet)

Solve $1.5058 = r \times \dfrac{16.8030}{10.1386}$ for $r$ **(1 mark)**

$$r = 0.90857...$$

So $r^2 = 0.82549...$

The percentage required is 83% (to the nearest percentage).

**(1 mark)**

**Question 5** (4 marks)

**a.** The trend in the time series plot is decreasing. **(1 mark)**

**b.** Looking at the data value for quarter number 6 as well as the four data values to either side, the middle value for the number of contractors is 23.
So the smoothed number of contractors for quarter number 6 is 23. **(1 mark)**

**c.** The last quarter of 2019 is represented by the 4th data value from the left. We need 4 data values to the left and 4 data values to the right of the data value we are trying to smooth when using nine-median smoothing. Since there are not 4 data values to the left of the data value for the last quarter of 2019, we cannot smooth it using this smoothing method. **(1 mark)**

**d.** The assumption being made is that the same decreasing trend will continue into the future. **(1 mark)**

## Recursion and financial modelling

**Question 6** (4 marks)

**a.** Given that $V_{12} = 89\,682.35$,
then $V_{13} = 1.0035 \times V_{12}$
$= 1.0035 \times 89\,682.35$ **(1 mark)**

**b.** Using the formula $V_n = 86\,000 \times 1.0035^n$, where $n$ represents the number of months,
solve $91000 = 86\,000 \times 1.0035^n$ for $n$
$n = 16.1745...$
So the balance of the account will first exceed \$91 000 after 17 months. **(1 mark)**
Alternatively you can use trial and error, ie
$86\,000 \times 1.0035^5 = 87\,515.57$ too low
$86\,000 \times 1.0035^{10} = 89\,057.85$ too low
$86\,000 \times 1.0035^{15} = 90\,627.31$ too low but getting close
$86\,000 \times 1.0035^{16} = 90\,944.51$ very close
$86\,000 \times 1.0035^{17} = 91\,262.81$ which just exceeds 91000
If you have time, you can check your answer by generating the sequence on your CAS, ie 86 000, 86 301, 86 603.05, 86 906.16, ... 90 627.31, 90 944.51, 91 262.81
After 17 months, the balance first exceeds 91 000.
Note that $V_0 = 86\,000$ and $V_1 = 86\,301$ etc. In other words, $V_1 \neq 86\,000$.

**c.** **i.** $V_n = 86\,000 \times 1.0035^n$. **(1 mark)**

**ii.** $n = 5 \times 12 = 60$ **(1 mark)**

**Question 7** (3 marks)

**a.** purchase price = \$42 000
annual depreciation = \$3570

$$\text{annual flat rate of depreciation} = \left(\frac{3570}{42\,000} \times \frac{100}{1}\right)\% = 8.5\%$$ **(1 mark)**

**b.** $C_0 = 42\,000, \quad C_{n+1} = C_n - 3570$ **(1 mark)**

**c.** The car is depreciated by \$3570 each year.
The car travels 7000 km each year.
The car depreciates by $\frac{\$3570}{7000\,\text{km}} = \$0.51$ per km. **(1 mark)**

**Question 8** (5 marks)

**a.** **i.** For a perpetuity, the interest earned equals the payment made over the same timeframe. Jenita receives fortnightly payments of \$736 so interest earned fortnightly is also \$736.
So annual interest earned is $\$736 \times 26 = \$19\,136$

**(1 mark)**

**ii.** Method 1 – using solution to part **a.i.**

$$\left(\frac{19\,136}{460\,000} \times \frac{100}{1}\right)\% = 4.16\%$$

**(1 mark)**

Method 2 – using formula

$$736 = \frac{r}{100 \times 26} \times 460\,000$$

Solve for $r$.

$r = 4.16$

annual interest rate is 4.16%

**(1 mark)**

Method 3 – using TVM

$$\begin{aligned} N &: 26 \\ I(\%) &: ? \\ PV &: -460\,000 \\ Pmt &: 736 \\ FV &: 460\,000 \\ PpY &: 26 \\ CpY &: 26 \end{aligned}$$

$I(\%) = 4.16$

annual interest rate is 4.16%

**(1 mark)**

**b.** Using TVM,

$$\begin{aligned} N &: 12 \\ I(\%) &: 4.2 \\ PV &: -460\,000 \\ Pmt &: ? \\ FV &: 438\,101.64 \\ PpY &: 12 \\ CpY &: 12 \end{aligned}$$

$Pmt = 3400.0006...$

So Jenita's monthly payment is \$3400.00.

**(1 mark)**

**c.** Balance of annuity at start of year 2 is \$438 101.64.
Use TVM to find the balance of the annuity at the end of year 2

$N: 12$
$I(\%): 4.2$
$PV: -438\,101.64$
$Pmt: 3750$
$FV: ?$
$PpY: 12$
$CpY: 12$
$FV = 410\,983.8428...$

So balance at the end of year 2 is \$410 983.84. **(1 mark)**

Reduction in balance
$= \$438\,101.64 - \$410\,983.84$
$= \$27\,117.80$
Payments made during year 2
$= 12 \times \$3750$
$= \$45\,000$
Interest earned $= \$45\,000 - \$27\,117.80$
$= \$17\,882.20$

**(1 mark)**

## Matrices

**Question 9** (4 marks)

**a.** $4 \times 1$ **(1 mark)**

**b.** $G = \begin{bmatrix} 0 & 3 & 0 & 4 \end{bmatrix}$ **(1 mark)**

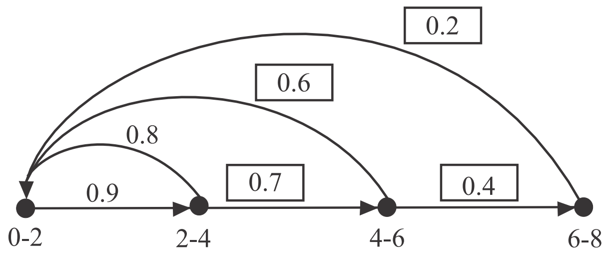
**c.** A discount of 15% means that last week's cost of \$13 for medium bags needs to be multiplied by 0.85.
An increase of 10% means that last week's cost of \$25 for large bags needs to be multiplied by 1.10 (or just 1.1).
Costs that remain the same need only be multiplied by 1.

$$N = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 \\ 0 & 0 & 1.1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**(1 mark)** – for 2 correct entries
**(1 mark)** for the remaining 2 correct entries

**Question 10** (4 marks)

**a.**




**(1 mark)**

**b.** $S_1 = LS_0$

$$= \begin{bmatrix} 220 \\ 45 \\ 105 \\ 40 \end{bmatrix}$$

After one two-year period, there are 105 rodents aged 4–6 years in the population.

**(1 mark)**

**c.** $S_3 = L^3 S_0$

$$= \begin{bmatrix} 185.7 \\ 96.3 \\ 138.6 \\ 12.6 \end{bmatrix}$$

$$\left(\frac{96.3}{(185.7 + 96.3 + 138.6 + 12.6} \times \frac{100}{1}\right)\%$$
$$= 22.2299...\%$$
$$= 22.2\% \text{ (correct to 1 decimal place)}$$

**(1 mark)**

**d.** Because the four age groups are showing approximately the same percentage increase in numbers from one two-year period to the next, we can choose any **one** of the four age groups. The same result, to the nearest whole percent, will be obtained.
The solutions below use the 0-2 year age group.

Method 1

Using the 0–2 year age group, we have $\frac{257.38}{243.16} = 1.0584...$ .

The percentage increase required is $6\%$ (to the nearest whole percent).

**(1 mark)**

Method 2

Using the 0–2 year age group, we have $257.38 - 243.16 = 14.22$ .

$$\text{percentage increase } \left(\frac{14.22}{243.16} \times 100\right)\% = 5.84...\%$$

The percentage increase required is $6\%$ (to the nearest whole percent).

**(1 mark)**

**Question 11** (2 marks)

**a.** 70% of young rodents change stages each month, as do 30% of breeding age rodents and 60% of elderly rodents.

In all, $0.7 \times 50 + 0.3 \times 120 + 0.6 \times 40 = 95$ rodents are at a different stage. **(1 mark)**

**b.** Method 1 - intuitively

We are told in the question that 'no new young rodents are born'. This tells us that over the long term the whole population will die. So over the long term $50 + 120 + 40 = 210$ rodents are expected to die. **(1 mark)**

Method 2 – using the recurrence relation

$$S_{100} = T^{100} \times S_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 210 \end{bmatrix} \qquad S_{101} = T^{101} \times S_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 210 \end{bmatrix}$$

No change exists between one state and the next so steady state has been reached. So 210 rodents are expected to die over the long term. **(1 mark)**

**Question 12** (2 marks)

**a.**

$$R_2 = T \times R_1 + E = \begin{bmatrix} 22 \\ 149 \\ 46 \\ 93 \end{bmatrix} \begin{matrix} Y \\ B \\ E \\ D \end{matrix}$$

There are 149 breeding age rodents expected to be in the population two months after the changed management program begins. **(1 mark)**

**b.**

$$\begin{aligned} R_1 &= T \times R_0 + E \\ R_1 - E &= T \times R_0 \\ T \times R_0 &= R_1 - E \\ T^{-1} \times T \times R_0 &= T^{-1}(R_1 - E) \\ R_0 &= T^{-1}(R_1 - E) \quad \text{since } T^{-1} \times T = I \text{ and } I \times R_0 = R_0 \\ &= \begin{bmatrix} 100 \\ 100 \\ 50 \\ 0 \end{bmatrix} \end{aligned}$$

The number of elderly rodents in the population initially is 50. **(1 mark)**

## Networks and decision mathematics

**Question 13** (4 marks)

**a.** 4 **(1 mark)**

**b.** The route is a Hamiltonian path. **(1 mark)**

**c.** Because the degree of each of the vertices is even, the sweeper must finish at ride C (thus completing an Eulerian circuit). **(1 mark)**

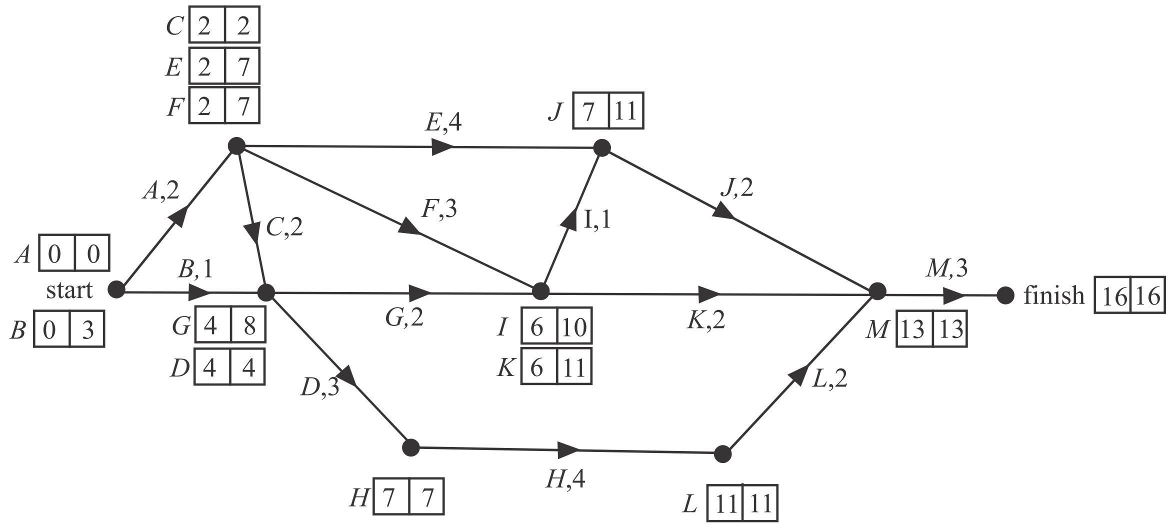
**d.** [8] + [6] = [12] + [2] **(1 mark)**

**Question 14** (5 marks)

**a.** Only one activity has three immediate predecessors. (It is activity *M*). **(1 mark)**

**b.** By observing the network, the earliest start time for activity *K* is 6 days. **(1 mark)**
(Note that this utilizes the path *A, C, G.*)

**c.** Do a forward and backward scan of the network. It will be useful in later parts.




The minimum completion time is 16 days. **(1 mark)**

**d.** Look at the float times of the activities from the forward and backward scan done in part **c**. There are 6 activities on the critical path (*A, C, D, H, L, M*).
All the other activities have float times of at least three days. So 7 activities can have their completion times increased by at least 3 days without affecting the minimum completion time. **1 mark)**

**e.** Again, looking at the forward and backward scan done in part **c**., the critical path has 6 activities on it. Of these, activity *H* has the longest completion time of 4 days. This could be reduced to 1 day.
The minimum float time of the activities **not** on the critical path is 3 days (activity *B*).
So reducing activity *H* by 3 days, means that the minimum completion time for the crashed project will take $16-3=13$ days. **(1 mark)**

**Question 15** (3 marks)

**a.** Nora can only service ride *C* which means Petra can only service ride *A* so Oliver can only service ride *B*. Min is the only employee who can service ride *D*.

| Employee | Ride |
|---|---|
| Min | *D* |
| Nora | *C* |
| Oliver | *B* |
| Petra | *A* |

**(1 mark)**

**b.** We use the Hungarian algorithm

| | *W* | *X* | *Y* | *Z* |
|---|---|---|---|---|
| *E* | 4 | 5 | 3 | 4 |
| *F* | 8 | 7 | 8 | 7 |
| *G* | 6 | 5 | 4 | 6 |
| *H* | 7 | 7 | 5 | 7 |

Subtract the minimum value in each row from each other element in that row.

| | *W* | *X* | *Y* | *Z* |
|---|---|---|---|---|
| *E* | 1 | 2 | 0 | 1 |
| *F* | 1 | 0 | 1 | 0 |
| *G* | 2 | 1 | 0 | 2 |
| *H* | 2 | 2 | 0 | 2 |

The first column contains no zeros so subtract the minimum element in that column, ie 1, from each of the other elements in the column.

| | *W* | *X* | *Y* | *Z* |
|---|---|---|---|---|
| *E* | 0 | 2 | 0 | 1 |
| *F* | 0 | 0 | 1 | 0 |
| *G* | 1 | 1 | 0 | 2 |
| *H* | 1 | 2 | 0 | 2 |

Try and cover all the zeros with a minimum number of straight lines.

| | *W* | *X* | *Y* | *Z* |
|---|---|---|---|---|
| *E* | 0 | 2 | 0 | 1 |
| *F* | 0 | 0 | 1 | 0 |
| *G* | 1 | 1 | 0 | 2 |
| *H* | 1 | 2 | 0 | 2 |

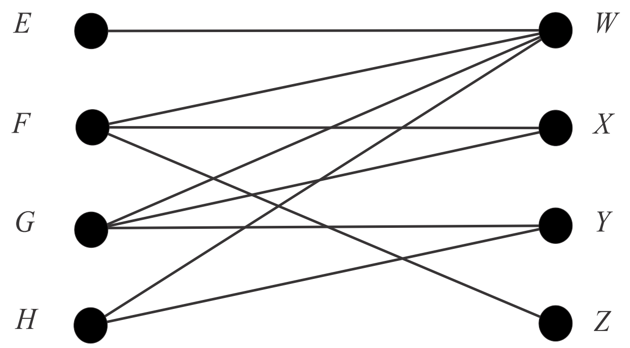
The zeros can be covered with 3 lines. In order to be able to make an allocation, this needs to be 4.
The smallest of the uncovered numbers is 1.
Add this to the numbers that are covered by two lines and subtract it from all the uncovered values.

| | *W* | *X* | *Y* | *Z* |
|---|---|---|---|---|
| *E* | 0 | 2 | 1 | 1 |
| *F* | 0 | 0 | 2 | 0 |
| *G* | 0 | 0 | 0 | 1 |
| *H* | 0 | 1 | 0 | 1 |

The minimum number of lines required to cover all the zeros is now 4.

| | *W* | *X* | *Y* | *Z* |
|---|---|---|---|---|
| ~~*E*~~ | ~~0~~ | ~~2~~ | ~~1~~ | ~~1~~ |
| ~~*F*~~ | ~~0~~ | ~~0~~ | ~~2~~ | ~~0~~ |
| ~~*G*~~ | ~~0~~ | ~~0~~ | ~~0~~ | ~~1~~ |
| ~~*H*~~ | ~~0~~ | ~~1~~ | ~~0~~ | ~~1~~ |

Draw a bipartite graph.




We are now ready to allocate.

| **Ride** | **Contractor** |
|---|---|
| *E* | *W* |
| *F* | *Z* |
| *G* | *X* |
| *H* | *Y* |

**(1 mark)** correct allocation

The minimum total time, in hours, for the four rides to be serviced is
$4+7+5+5=21$ hours. **(1 mark)**