



Trial Examination 2022

VCE Further Mathematics Units 3&4

Written Examination 1

Suggested Solutions

SECTION A – CORE**Data analysis****Question 1 B**

Since this is grouped data, the centre of each group is taken as representative of the group. Therefore, 17.7 is the mean of the set.

Question 2 D

D is incorrect and is therefore the required response. The median is $\frac{32 + 33}{2} = 32.5$, not 33.

A is incorrect. The range is $56 - 6 = 50$.

B is incorrect. The IQR is $42.5 - 21 = 21.5$.

C is incorrect. The shape of the stem plot is vertically symmetrical.

E is incorrect. The most frequently occurring number (the mode) is 33.

Question 3 C

C is correct. The y -intercept is (0, 5000).

The gradient is found using the y -intercept and the point (5, 4750), which uses the information that 250 L of water is used for a 5-minute car wash.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4750 - 5000}{5 - 0} \\ &= -50 \end{aligned}$$

Therefore, the equation is $w = 5000 - 50t$.

A and **B** are incorrect. The gradient is -50 .

D is incorrect. The volume of water in the tank (w) is the response variable, and the time taken to wash the car (t) is the explanatory variable.

E is incorrect. The gradient is -50 ; -0.8 is the correlation coefficient, which applies when w is the subject, not t .

Question 4 A

A is correct and **C** is incorrect. The amount of variation in the response variable (w) that is due to variation in the explanatory variable (t) is given by: $r^2 = (-0.8)^2 = 0.64 = 64\%$.

B and **D** are incorrect. The response variable is the volume of water in the tank (w) and the explanatory variable is the time taken to wash the car (t).

E is incorrect. A negative correlation coefficient does not affect causation.

Question 5 B

B is correct. Before calculating the five-number summary, the figures in the data set must be placed in ascending order.

A is incorrect. This is the mode, which is only one of the values in a five-number summary.

C is incorrect. The numbers in the data set were not placed in ascending order.

D is incorrect. Outliers do not matter for five-number summaries, and 6 is not an outlier.

E is incorrect. Having two identical pieces of data does not affect the calculation of a five-number summary.

Question 6 E

E is correct. The horizontal axis represents time, so it is a time series line graph.

A is incorrect. A scatterplot has no line connecting the points.

B and **D** are incorrect. These options are not time series graphs.

C is incorrect. More than one year's data is required to establish a seasonal pattern.

Question 7 B

B is correct. Using a three-point moving median is a simple graphical tool that would be very efficient and achieve the purpose of smoothing the curve.

A is incorrect. A three-point moving mean would smooth the curve, but each calculation would take some time.

C is incorrect. There are 12 data points in the graph, so the number of points in the moving median should be a factor of 12.

D is incorrect. Lines of best fit are used to make predictions, not smooth curves.

E is incorrect. The data is bivariate and the IQR is a univariate measure.

Question 8 D

D is correct. Given the $\frac{1}{x}$ transformation, substituting the values for the gradient and y-intercept into $y = a + bx$ gives the equation of the least squares line as $y = 25 - \frac{3.2}{x}$.

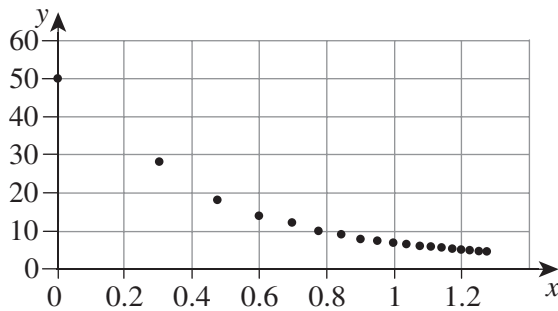
A and **B** are incorrect. The transformation of $\frac{1}{x}$ has not been applied.

C is incorrect. The correlation coefficient (0.9) has been used as the y-intercept instead of 25.

E is incorrect. The gradient is negative.

Question 9 E

E is correct. A $\log(x)$ transformation would decrease the values of $x > 1$. This will bring the points closer to the y-axis and straighten the curve, as shown in the graph below.



A and **B** are incorrect. Performing a moving median or mean will smooth the curve but not transform it.

C is incorrect. Lines of best fit are used to make predictions, not transform data.

D is incorrect. Applying an x^2 transformation would stretch the graph to the right but not linearise it.

Question 10 C

$$\frac{\text{rating of 1} + \text{rating of 2} + \text{rating of 3}}{\text{total number of ratings}} \times 100 = \frac{20 + 25 + 10}{200} \times 100$$

$$= 27.5\%$$

Question 11 E

The y value of 5 is on a log scale, so the number of mosquitos is $10^5 = 100000$.

Question 12 B

Substituting $v = 15$ into the equation gives $E = 2.6(15) + 0.2 = 39.2$. Subtracting this predicted value from the actual value gives $40 - 39.2 = 0.8$.

Question 13 D

D is correct and **A** is incorrect. Since the actual point is closer to the line than the incorrect point, the line becomes a better fit; therefore, the value of r will increase slightly.

B is incorrect. The residual for the point $(10, 27)$ will decrease slightly.

C is incorrect. The gradient will decrease slightly but the y-intercept will increase slightly.

E is incorrect. The five-number summary is used for univariate, not bivariate, data.

Question 14 B

95% of sultana snack packs will be within two standard deviations on either side of the mean. Hence,

$$9.8 - 9.4 = 0.4 \text{ and the standard deviation will be } \frac{0.4}{4} = 0.1.$$

Question 15 C

C is correct. A box plot shows the full range of each student's previous results. The most recent result can be placed as a point of comparison in the plot.



A is incorrect. A scatterplot would compare each student's most recent result to the results of the whole class.

B is incorrect. A back-to-back stem plot would place the results in ascending order, so it would not be possible to compare a single student's previous and most recent results.

D is incorrect. A time series graph is used for data involving time.

E is incorrect. A residual plot is used for bivariate data; the students' results are univariate data.

Question 16 C

$$\frac{\frac{12\,000}{14\,250} + \frac{11\,500}{13\,750} + \frac{12\,000}{13\,875}}{3} = 0.85$$

Recursion and financial modelling**Question 17 C**

Each subsequent term is generated by dividing the previous term by -7 or multiplying it by $-\frac{1}{7}$. The initial term, t_1 , is 42. Thus, the rule that generates the sequence is $t_{n+1} = -\frac{1}{7}t_n$, $t_1 = 42$.

Question 18 E

On day 1, t_1 , there were 4 views and the number increased by a factor of 1.65 each day for the first 20 days. Thus, the rule that generates the sequence is $t_{n+1} = 1.65t_n$, $t_1 = 4$.

Question 19 A

After five years, the value is $45\,000 \times 0.8 \times 0.9^4 = \$23\,619.60$.

This is a depreciation of \$21 380.40

Finding the depreciation as a percentage of the initial value of the car gives:

$$\frac{21\,380.40}{45\,000} \times 100 = 47.5\%$$

Therefore, the average percentage depreciation is $\frac{47.5}{5} = 9.5\%$.

Question 20 A

The compound interest formula is given by $T_n = P \times \left(1 + \frac{r}{100}\right)^n$, where $P = 50\,000$, $r = \frac{3.4}{12}$ (as the interest rate should be converted to a monthly interest rate) and $n = 5 \times 12$ (the number of compounding periods).

Therefore, the expression is $T_5 = 50\,000 \times \left(1 + \frac{\frac{3.4}{12}}{100}\right)^{(5 \times 12)}$.

Question 21 A

The deposit is 20% of \$25 000 = \$5000. Hence, Sara borrows and needs to repay \$20 000.

The interest to be paid is $20\,000 \times 0.06 \times 3 = \3600 .

The total to be paid in repayments is $\$20\,000 + \$3600 = \$23\,600$.

The monthly repayment is $\frac{23\,600}{36} = \$655.56$.

Question 22 C

The cost of the printer over 40 million prints is \$255 000.

$$\frac{255\,000}{40\,000\,000} \times 100 = 0.6375 \text{ cents}$$

Question 23 D

D is correct. After 10 years, the investment is valued at $V_{10} = 1.04210 \times 6000 = \9053.75 (after nine years, the value was \$8688.82).

A, **C** and **E** are incorrect. The interest compounds annually.

B is incorrect. After five years, the value of the investment is \$7370.

Question 24 B

Using the financial solver on a CAS calculator with the following values gives:

$$N = 120$$

$$PV = -280\,000$$

$$PMT = 1509$$

$$P/Y = 12$$

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N=120
I% = 4.199812546
PV = -280000
PMT = 1509
FV = 201272.13
P/Y = 12
C/Y = 12
PNT: BEGIN
  
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SECTION B – MODULES**Module 1 – Matrices****Question 1 D**

D is correct. The formula for calculating the determinant is given by $\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$.

Therefore, $\det(\text{option D}) = 1 \times 1 - 1 \times 0 = 1$.

A is incorrect. $\text{Det}(\text{option A}) = 1 \times 1 - 1 \times 1 = 0$.

B is incorrect. $\text{Det}(\text{option B}) = 0 \times 0 - 1 \times 1 = -1$.

C is incorrect. $\text{Det}(\text{option C}) = 2 \times 2 - 1 \times 1 = 3$.

E is incorrect. $\text{Det}(\text{option E}) = 4 \times 2 - 3 \times 3 = -1$.

Question 2 C

C is correct.

$$\begin{aligned} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} &= \begin{bmatrix} 2 \times 3 + 0 \times 1 & 2 \times 0 + 0 \times 1 & 2 \times 2 + 0 \times 0 \\ 1 \times 3 + 1 \times 1 & 1 \times 0 + 1 \times 1 & 1 \times 2 + 1 \times 0 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 0 & 4 \\ 4 & 1 & 2 \end{bmatrix} \end{aligned}$$

A and **B** are incorrect. These options contain mistakes in the calculations.

D and **E** are incorrect. The size of the matrix product is the number of rows in the first matrix by the numbers of columns in the second; therefore, the matrix product needs to be 2×3 .

Question 3 C

$$\begin{aligned} B^2 &= \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}^2 \times \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 19 & 18 \\ 6 & 7 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} AB^2 &= \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 19 & 18 \\ 6 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 38 & 36 \\ 25 & 25 \end{bmatrix} \end{aligned}$$

$$3AB^2 = \begin{bmatrix} 114 & 108 \\ 75 & 75 \end{bmatrix}$$

Question 4 D

For each element of a 2×2 matrix, substituting the row and column number into the equation $a_{i,j} = 2i - 3j$ and evaluating gives:

$$a_{1,1} = 2 \times 1 - 3 \times 1 = -1$$

$$a_{1,2} = 2 \times 1 - 3 \times 2 = -4$$

$$a_{2,1} = 2 \times 2 - 3 \times 1 = 1$$

$$a_{2,2} = 2 \times 2 - 3 \times 2 = -2$$

Hence, the correct matrix is $\begin{bmatrix} -1 & -4 \\ 1 & -2 \end{bmatrix}$.

Question 5 E

Defining two matrices to represent the information in the table gives $A = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ 5 & 0 & 6 & 0 \\ 0 & 10 & 5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 50 \\ 44 \\ 31 \\ 51 \end{bmatrix}$.

Solving the matrix equation using a CAS calculator to determine the cost matrix gives:

$$A \times \begin{bmatrix} p \\ s \\ r \\ l \end{bmatrix} = B$$

$$A^{-1}B = \begin{bmatrix} 5 \\ 4 \\ 1 \\ 3 \end{bmatrix}$$

Question 6 D

$$T = \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix}$$

$$S_0 = \begin{bmatrix} 50 \\ 50 \end{bmatrix}$$

The number of teenagers that chose each genre is given by $\begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} \begin{bmatrix} 50 \\ 50 \end{bmatrix} = \begin{bmatrix} 35 \\ 65 \end{bmatrix}$.

Question 7 B

Defining the matrices gives $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ 5.7 \end{bmatrix}$. Using a CAS calculator to solve

the equation gives:

$$\begin{aligned} A \times \begin{bmatrix} x \\ y \end{bmatrix} &= B \\ \begin{bmatrix} x \\ y \end{bmatrix} &= A^{-1}B \\ &= \begin{bmatrix} 4.4 \\ 3.75 \end{bmatrix} \end{aligned}$$

Question 8 D

D is correct. A column matrix is a matrix with a single column and any number of rows.

A is incorrect. A symmetrical matrix is symmetrical around the leading diagonal.

B is incorrect. In a unit (or square) matrix, the elements on the leading diagonal have a value of one and all the other elements have a value of zero.

C is incorrect. In a triangular matrix, all elements above or below the main diagonal have a value of zero and the remaining elements are unspecified.

E is incorrect. A binary matrix has a single element with a value of one on each row and all the other elements have a value of zero.

Module 2 – Networks and decision mathematics**Question 1 E**

E is correct. An Euler trail can be found because there are two odd-numbered vertices.

A is incorrect. Castlemaine is connected to two other towns.

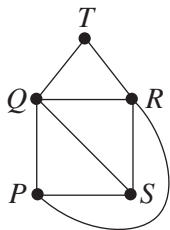
B is incorrect. Daylesford has the highest value vertex with $n = 5$.

C is incorrect. A minimum spanning tree uses the minimum number of edges to connect the vertices. There are two edges that connect both Ararat and Daylesford and Ballarat and Daylesford.

D is incorrect. There is no direct link between Ararat and Castlemaine.

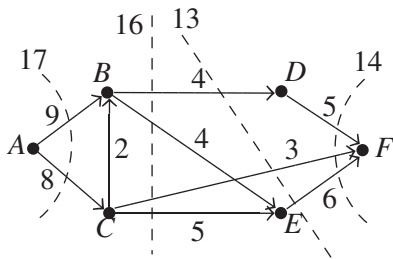
Question 2 B

A planar graph has no edges that cross each other. By removing $P-R$ as shown in the original diagram and relocating the line as shown in the diagram below, the graph becomes planar.



Question 3 B

The vertical cuts shown below produce a maximum flow of 13.



Question 4 D

D is correct. This is the correct adjacency matrix.

A is incorrect. This matrix lists the value of n for each vertex.

B is incorrect. This matrix is missing the route from C to A .

C is incorrect. This matrix only counts each route once; that is, it includes A to B but not B to A .

E is incorrect. This option is a circuit starting and finishing at A .

Question 5 C

Applying Euler's rule gives $F + V = E + 2$ where $F = 4$.

Therefore:

$$4 + V = E + 2$$

$$V = E - 2$$

The combination of four vertices and six edges is the only option that satisfies the rule.

Question 6 A

A is correct. The minimum spanning tree is found by identifying one of the shortest paths; in this case, either $D-E$ or $F-G$. It is expanded by connecting the shortest path from one of the connected nodes to an unconnected node, which is repeated until all nodes are connected.

B and **C** are incorrect. These minimum spanning trees include path $A-B$, which is a longer path.

D is incorrect. There is no path from D to C .

E is incorrect. Path $B-C$ is longer than path $B-D$.

Question 7 E

E is correct. The critical path is $A-D-J$, which results in a time of $9 + 7 + 5 = 21$.

A is incorrect. This option refers to the shortest path from X to Y , $B-I$.

B is incorrect. This option is the minimum flow measured by a cut through 5, 2 and 1 to the left of G .

C is incorrect. This option uses the pathway $C-G-H$, which results in a time of $2 + 9 + 1 = 12$.

D is incorrect. This option uses the pathway $A-E-I$, which results in a time of $9 + 5 + 2 = 16$.

Question 8 D

D is correct. The critical path is already the shortest time to complete all activities. Any increase to the path will increase the time.

A is incorrect. A critical path may contain all the longest activities and will likely contain at least one of them, but it is not required for the critical path to have all the longest activities.

B is incorrect. Reducing the time of the critical path may reduce the overall time, but this may also change the critical path itself.

C and **E** are incorrect. These options show an incorrect understanding of the purpose of a dummy activity.

Module 3 – Geometry and measurement**Question 1 C**

The capsule volume is given by:

$$\begin{aligned} V &= \pi r^2 L + \frac{4\pi r^3}{3} \\ &= \pi(3)^2 \times 14 + \frac{4\pi(3)^3}{3} \\ &= 509 \text{ mm}^3 \end{aligned}$$

Question 2 A

$$\begin{aligned} 50 \times \frac{30}{100} \times 450^2 &= \frac{3\,037\,500 \text{ cm}^2}{10\,000} \\ &= 303.75 \text{ m}^2 \\ &\approx 304 \text{ m}^2 \end{aligned}$$

Question 3 D

Determining the angle of each slice:

$$\begin{aligned} V &= \frac{\theta}{360} \pi r^2 h \\ 176.7 &= \frac{\theta}{360} \times \pi \times 15^2 \times 2.5 \\ \theta &= 36^\circ \end{aligned}$$

Determining the number of slices:

$$\frac{360}{36} = 10 \text{ slices}$$

Question 4 C

Every 15° of longitude represents 1 hour of time difference.

$$116 - 11 = 105^\circ$$

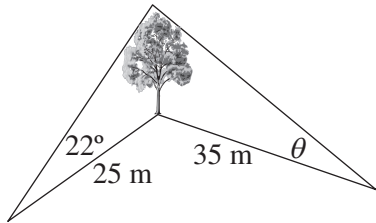
$$\text{Therefore, } \frac{105}{15} = 7.$$

Question 5 B

$$\begin{aligned} \text{distance} &= \frac{14}{360} \times \pi \times 6400 \\ &= 781.9 \text{ km} \\ &\approx 782 \text{ km} \end{aligned}$$

Question 6 E

The following diagram illustrates the scenario.



Height of the tree:

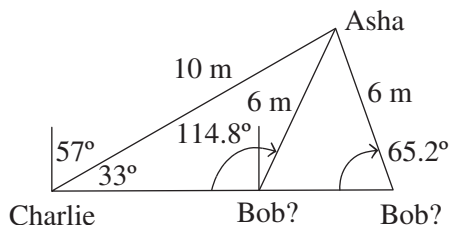
$$\begin{aligned} h &= 25 \tan(22) \\ &= 10.1 \text{ m} \end{aligned}$$

Angle of elevation from Bree:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{10.1}{35}\right) \\ &= 16.1^\circ \end{aligned}$$

Question 7 C

This is a case of the ambiguous sine rule, where there are two possible answers but only one is included as an option. The following diagram illustrates the scenario.



$$\frac{10}{\sin B} = \frac{6}{\sin 33}$$

$$B = 65.2^\circ \text{ or } 114.8^\circ$$

$$114.8 - 90 = 24.8^\circ$$

Therefore, the bearing is N 24.8° E.

Question 8 C

As each 15° of longitude represents 1 hour of time difference, the 30° between Dakar and Vienna represents 2 hours of time difference. As the plane landed 1 hour later, the total flight is 3 hours long.

Module 4 – Graphs and relations**Question 1 E**

The x -intercept is $(2, 0)$, so the coefficient of x is $\frac{10}{2} = 5$.

The y -intercept is $(0, 5)$, so the coefficient of y is $\frac{10}{5} = 2$.

Since the line is solid and the area is below the graph, the equation of the area is $2y + 5x \leq 10$.

Question 2 E

E is correct. At point Z , Travis is moving towards home and the gradient is the steepest.

A and **D** are incorrect. At points V and Y , Travis is moving away from his house.

B is incorrect. At point W , Travis is not moving.

C is incorrect. At point X , Travis is moving towards home; however, the gradient is not the steepest.

Question 3 C

The equation of graph **A** is $y = x^{-2}$. The equation of graph **B** is $y = 3x^2$. Graph **A** is a reciprocal graph $\frac{1}{x^n}$ as it cannot equal 0, so it must be $\frac{1}{x^2}$. Graphs **B** and **C** are both x^2 in shape and substituting the point $(2, 12)$ gives the equation of graph **B** as $y = 3x^2$.

Question 4 E

E is correct. The end points of each step have been used to create the graph correctly.

A is incorrect. This graph shows the price for each individual stage of water use but does not show the linear relationship between the stages.

B is incorrect. This graph shows the end points for each step.

C is incorrect. The end points of each step have not been added to this graph.

D is incorrect. This graph shows a constant increase in cost for all volumes.

Question 5 B

$$\begin{aligned} \text{average speed} &= \frac{\text{total distance travelled}}{\text{time elapsed}} \\ &= \frac{23 - 2}{10 - 0} \\ &= 2.1 \text{ km/hr} \end{aligned}$$

Question 6 E

E is correct. The gradient of the trend line is the value of k .

A is incorrect. Plotting the data to observe a x^2 shape will not calculate the value of k .

B and **D** are incorrect. Depending on which coordinate is used, a different value for k will be obtained.

C is incorrect. Plotting the graph of $y = x^2$ to see if the graph is linear does not calculate the value of k .

Question 7 D

After three days, there was an increase of 116.1 mosquitoes. Therefore, the average increase

was $\frac{116.1}{3} = 38.7$ mosquitoes per day.

Question 8 B

Using the point (3, 116.1) to solve for k gives:

$$y = kx^3$$

$$116.1 = k \times 3^3$$

$$k = \frac{116.1}{27}$$

$$= 4.3$$