

2021 VCE

Further Mathematics Trial Examination 2 Suggested Solutions



Kilbaha Education

Quality educational content

Kilbaha Education
PO Box 2227
Kew Vic 3101
Australia

Tel: (03) 9018 5376

kilbaha@gmail.com
<https://kilbaha.com.au>

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Kilbaha Education (Est. 1978) (ABN 47 065 111 373) PO Box 2227 Kew Vic 3101 Australia	Tel: +613 9018 5376 Email: kilbaha@gmail.com Web: https://kilbaha.com.au
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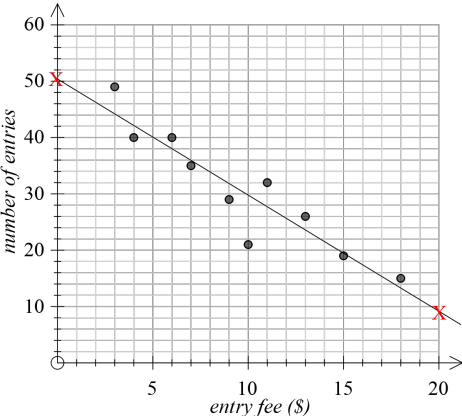
Data analysis**Question 1**

<p>a. Section.</p> <p><i>Section</i> is ordinal. Categorical data that has a natural order. Junior, adult and senior can be ordered by age group.</p> <p style="text-align: right;">(1 mark)</p>	<p>b.</p> <p>Two</p> <p>Competitors 18 and 24 are professional adults.</p> <p style="text-align: right;">(1 mark)</p>																		
<p>c.</p> <table border="1" data-bbox="153 831 700 1200"> <thead> <tr> <th></th> <th colspan="2">Status</th> </tr> <tr> <th>Section</th> <th>Amateur</th> <th>Professional</th> </tr> </thead> <tbody> <tr> <td>junior</td> <td>7</td> <td>0</td> </tr> <tr> <td>adult</td> <td>6</td> <td>2</td> </tr> <tr> <td>senior</td> <td>5</td> <td>5</td> </tr> <tr> <td>Total</td> <td>18</td> <td>7</td> </tr> </tbody> </table> <p style="text-align: right;">(2 marks)</p>		Status		Section	Amateur	Professional	junior	7	0	adult	6	2	senior	5	5	Total	18	7	<p>d.</p> $\frac{6}{18} \times 100 \approx 33.3 \%$ <p style="text-align: right;">(1 mark)</p>
	Status																		
Section	Amateur	Professional																	
junior	7	0																	
adult	6	2																	
senior	5	5																	
Total	18	7																	
<p>e.</p> <p>The median time of 39 minutes for juniors is greater than the median for seniors (30.5) which is greater than for adults (22.5).</p> <p style="text-align: right;">(1 mark)</p>	<p>f.</p> $IQR = 41 - 37 = 4$ $\begin{aligned} \text{Upper fence} &= Q_3 + 1.5 \times IQR \\ &= 41 + 1.5 \times 4 \\ &= 47 \end{aligned}$ <p style="text-align: right;">(1 mark)</p>																		

Data analysis**Question 2**

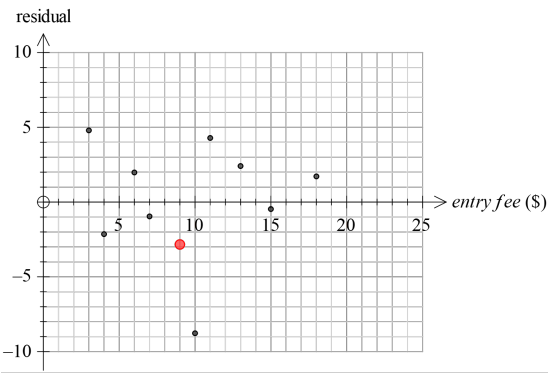
<p>a.</p> <p>2.5%</p> <p>19 is 2 standard deviations below the mean. 2.5% of data is expected to lie below this. (1 mark)</p>	<p>b.</p> <p>81</p> <p>40 and 47 are 1 and 2 standard deviations above the mean respectively. 13.5% of data is expected to lie between these values. $13.5\% \text{ of } 600 = 81$ (1 mark)</p>
<p>c.</p> <p>21.8 years.</p> $-1.6 = \frac{x - 33}{7}$ $x = 21.8$ <p>(1 mark)</p>	

Question 3

<p>a.</p> <p>Enter data into CAS with <i>number of volunteers</i> as the explanatory variable.</p> <p><i>number of campsite bookings</i> $= 2.42 + 0.568 \times \text{number of volunteers}$ (1 mark)</p>	<p>b. (i)</p> <p style="text-align: center;"><i>entry fee</i></p> <p>(1 mark)</p>
<p>b. (ii)</p>  <p>Use two points to accurately draw. Let <i>entry fee</i> = 0, then <i>number of entries</i> = 50.4 Let <i>entry fee</i> = 20, then <i>number of entries</i> = 8.4 (1 mark)</p>	<p>b. (iii)</p> <p>On average, for every extra one dollar in <i>entry fee</i>, the <i>number of entries</i> is expected to fall by 2.1 (1 mark)</p>

Data analysis

Question 3 (continued)

<p>b. (iv)</p> $(-0.927)^2 \times 100\% \approx \mathbf{85.9\%}$ <p style="text-align: right;">(1 mark)</p>	<p>b. (v)</p> <p>\$9.50</p> <p>The median value is between the 5th (\$9) and 6th (\$10) values.</p> <p style="text-align: right;">(1 mark)</p>
<p>b. (vi)</p> $\begin{aligned} \text{number of entries} &= 50.4 - 2.1 \times 14 \\ &= \mathbf{21} \end{aligned}$ <p style="text-align: right;">(1 mark)</p>	
<p>c.</p> <p>Predicted = $50.4 - 2.1 \times 9 = 31.50$ Residual = Actual – predicted = $29 - 31.50 = -2.5$</p>  <p style="text-align: right;">(1 mark)</p>	<p>d. (i)</p> <p>That the form is linear.</p> <p style="text-align: right;">(1 mark)</p>
<p>d. (ii)</p> <p>The residuals are distributed randomly around the horizontal axis.</p> <p style="text-align: right;">(1 mark)</p>	

Data analysis**Question 4****a.**

Average for 2018 = 480

Average for 2019 = 440

Average over the two years

$$= (480 + 440)/2 = 460$$

OR

2018 quarterly average

$$= \frac{436+540+340+604}{4} = 480$$

2018 Seasonal indices =

Summer	Autumn	Winter	Spring
0.908	1.125	0.708	1.258

Summer average = 411

Autumn average = 520

Winter average = 319

Spring average = 590

Dividing each season's average by 460 gives

Summer	Autumn	Winter	Spring
0.89	1.13	0.69	1.28

2019 quarterly average

$$= \frac{386+500+298+576}{4} = 440$$

2019 Seasonal indices =

Summer	Autumn	Winter	Spring
0.877	1.136	0.677	1.309

$$\text{S.I for Summer} = \frac{0.908+0.877}{2} \approx 0.89$$

$$\text{S.I for Winter} = \frac{0.708+0.677}{2} \approx 0.69$$

$$\text{S.I for Autumn} = \frac{1.125+1.136}{2} \approx 1.13$$

$$\text{S.I for Spring} = \frac{1.258+1.309}{2} \approx 1.28$$

(2 marks)

b.**251**

$$\text{deseasonalised figure} = 321 \div 1.28$$

$$\approx 250.781$$

$$\approx 251$$

(1 mark)

Recursion and financial modelling**Question 5**

<p>a.</p> <p>\$27000</p> <p>(1 mark)</p>	<p>b.</p> $Q_1 = 0.91 \times 27000 = 24570$ $Q_2 = 0.91 \times 24570 = 22358.70$ <p>(2 marks)</p>
<p>c.</p> <p>$100 - 91 = 9\%$</p> <p>(1 mark)</p>	<p>d. 9 years</p> <p>By continuing to multiply by 0.91, we get</p> $Q_8 = 12696.8 \dots$ $Q_9 = 11554.1 \dots$ <p>Alternatively, solving the equation</p> $27000 \times 0.91^n \leq 12000$ <p>gives $n \geq 8.5984 \dots$</p> <p>Now in this problem, n takes integral values, so $n = 9$.</p> <p>(1 mark)</p>
<p>e.</p> $Q_n = 27000 \times 0.91^n$ <p>(1 mark)</p>	<p>f. \$1831</p> $Q_3 = 20346.417 \dots$ <p>During the fourth year, the machine will depreciate by 9% of 20346.417... ie. \$1831 to the nearest dollar.</p> <p>(1 mark)</p>

Question 6

<p>a.</p> <p>\$9</p> <p>$340 \times 5 = 1700$ pieces quilted in 5 years</p> <p>Depreciation per piece</p> $= \frac{36000 - 20700}{1700} = 9$ <p>(1 mark)</p>	<p>b.</p> <p>Each year, depreciation = $9 \times 340 = 3060$</p> $V_0 = 36000, V_{n+1} = V_n - 3060$ <p>(1 mark)</p>
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Recursion and financial modelling**Question 6 (continued)**

<p>c.</p> <p>3782</p> <p>Depreciation = $36000 - 1962 = 34038$</p> <p>$34038 \div 9 = 3782$</p> <p style="text-align: right;">(1 mark)</p>	
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Question 7

<p>a.</p> <p>\$ 486322.40</p> <p>Use finance solver technology:</p> <p>$N = 108$ $I = 6.6$ $PV = -247000$ $PMT = -270$ $FV = ?$ $P/Y = 12$ $C/Y = 12$ This gives $FV = 486322.404487...$</p> <p style="text-align: right;">(1 mark)</p>	<p>b.</p> <p>\$435</p> <p>Find the balance after six years:</p> <p>$N = 72$ $I = 6.6$ $PV = -247000$ $PMT = -270$ $FV = ?$ $P/Y = 12$ $C/Y = 12$ This gives $FV = 390384.388166...$</p> <p>and make this the present value. He has three years, at 6.4%, to build his account to \$490000</p> <p>$N = 36$ $I = 6.4$ $PV = -390384.388166...$ $PMT = ?$ $FV = 490000$ $P/Y = 12$ $C/Y = 12$ This gives $PMT = -435.2537...$</p> <p>Jeff needs to make a monthly payment of \$435.</p> <p style="text-align: right;">(1 mark)</p>
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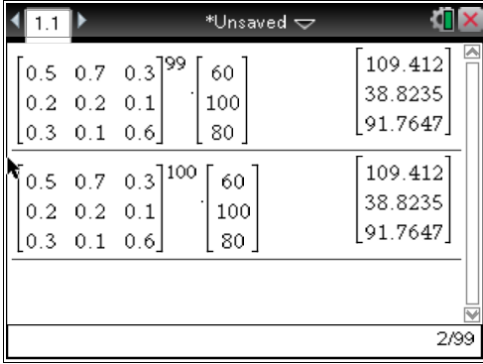
Module 1 – Matrices**Question 1**

<p>a.</p> <p>\$170 (1 mark)</p>	<p>b.</p> <p>3×1 (1 mark)</p>
<p>c.</p> <p>Q will be a 1×3 matrix giving the numbers of each type of quilt to be stitched.</p> <p>[2 3 0] (1 mark)</p>	

Question 2

<p>a.</p> $\begin{bmatrix} 23 & 16 & 2 \\ 18 & 22 & 1 \\ 32 & 10 & 2 \end{bmatrix}$ <p>(1 mark)</p>	<p>b.</p> $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 23 & 16 & 2 \\ 18 & 22 & 1 \\ 32 & 10 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 4615 \\ 5190 \\ 3910 \end{bmatrix} = \begin{bmatrix} 45 \\ 185 \\ 310 \end{bmatrix}$ <p>$b = \\$185$</p> <p>(1mark)</p>
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Module 1 – Matrices**Question 3**

<p>a.</p> $0.2 \times 60 + 0.2 \times 100 + 0.1 \times 80 = \mathbf{40}$ <p style="text-align: right;">(1 mark)</p>	<p>b.</p> $0.3 \times 60 = \mathbf{18}$ <p style="text-align: right;">(1 mark)</p>
<p>c.</p> $\mathbf{0.5} \times 60 + \mathbf{0.7} \times 100 + \mathbf{0.3} \times 80 = 124$ <p style="text-align: right;">(1 mark)</p>	<p>d.</p> <p>54.9%</p> $0.5 \times 124 = 62$ $\frac{62}{113} \times 100 = 54.8672 \dots$ <p style="text-align: right;">(1 mark)</p>
<p>e.</p>  <p>In the long term, the number of clients at location B settles at 38.8235</p> $\frac{38.8235}{240} \times 100 = 16.17646 \approx \mathbf{16.2\%}$ <p style="text-align: right;">(1 mark)</p>	

Module 1 – Matrices

Question 4

a.

$$300 + m - 500 + 100 = 0$$

$$m = \mathbf{100}$$

(1 mark)

b.

647

Use technology to find the state matrix for 2022, then substitute that state matrix into the recurrence relation to find the state matrix for 2023.

Lola is expected to stock 647 reels of Polycot in 2023.

				1.1	1.2	*Unsaved			
0.1	0.1	0.	0.2	600	300	490.			
0.2	0.1	0.	0.2	500	100	350.			
0.5	0.7	0.8	0.5	300	-500	590.			
0.2	0.1	0.2	0.1	400	100	370.			
				+					
0.1	0.1	0.	0.2	490	300	458.			
0.2	0.1	0.	0.2	350	100	307.			
0.5	0.7	0.8	0.5	590	-500	647.			
0.2	0.1	0.2	0.1	370	100	388.			
							2/99		

(1 mark)

Module 2: Networks and decision mathematics**Question 1**

<p>a.</p> $19 + 17 + 18 = 54$ <p style="text-align: right;">(1 mark)</p>	<p>b.</p> $19 + 21 + 18 = 58$ <p style="text-align: right;">(1 mark)</p>
<p>c.</p> <p>53</p> <p>Maximum flow = minimum cut Minimum cut = $18 + 17 + 18$ or $19 + 17 + 17$ Both make 53.</p> <p style="text-align: right;">(1 mark)</p>	

Question 2

<p>a.</p> <p>3 (vertices B, G and H)</p> <p style="text-align: right;">(1 mark)</p>	<p>b.</p> <p>65 km</p> <p>B-A-I-F-G</p> <p style="text-align: right;">(1 mark)</p>
<p>c.</p> <p>F</p> <p>The Eulerian trail must end at the other odd vertex. E and F are the only odd vertices.</p> <p style="text-align: right;">(1 mark)</p>	<p>d.</p> <p>D-C-B-A-I-H-G-F-E-D</p> <p style="text-align: right;">(1 mark)</p>
<p>e.</p> <p>Hamiltonian cycle</p> <p style="text-align: right;">(1 mark)</p>	

Module 2: Networks and decision mathematics

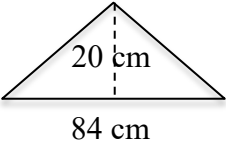
Question 3

<p>a.</p> <p style="text-align: right;">(1 mark)</p>	<p>b.</p> <p>26</p> $4 + 3 + 4 + 3 + 4 + 5 + 3 = 26$ <p style="text-align: right;">(1 mark)</p>
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Question 4

<p>a.</p> <p>B-D-C-E-H-J</p> <p style="text-align: right;">(1mark)</p>	<p>b.</p> <p>4 hours</p> <p>The current earliest completion time is 29 hours. Any one of the activities on the critical path can be reduced by 4 hours, making the completion time 25 hours. Reducing one of these activities by more than 4 hours creates a new critical path B-G-I-J taking 25 hours also. So the maximum possible time reduction for the job is 4 hours.</p> <p style="text-align: right;">(1 mark)</p>
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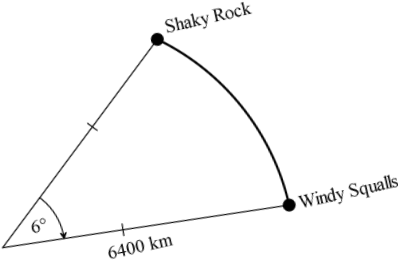
Module 3: Geometry and measurement**Question 1**

<p>a.</p> $d = \sqrt{125^2 + 84^2} = 150.6 \text{ cm}$ <p style="text-align: right;">(1 mark)</p>	<p>b.</p> $TSA = 2(125 \times 84 + 125 \times 95 + 95 \times 84)$ $= 60710 \text{ cm}^2$ <p style="text-align: right;">(1 mark)</p>
<p>c.</p>  <p>The height of the triangular end of the prism</p> $= \frac{125 - 85}{2} = 20$ <p>So, Volume = $\frac{84 \times 20}{2} \times 95 = 79800$</p> <p style="text-align: right;">(1 mark)</p>	<p>d.</p> <p>837900 cm³</p> <p>Volume of interlocking block = Volume of rectangular prism – 2 × 79800 = (125 × 84 × 95) – 159600 = 837900</p> <p style="text-align: right;">(1mark)</p>

Question 2

<p>a.</p> <p>ΔABC and ΔADE are similar For the volume of the upper section to be one quarter of the whole volume,</p> <p>area of ΔABC : area of $\Delta ADE = 1 : 4$ length AB : length $AD = \sqrt{1} : \sqrt{4} = 1 : 2$ length $AB = \frac{3.6}{2} = 1.8$ so length $BD = 3.6 - 1.8 = 1.8 \text{ m}$</p> <p>Hint: The prisms are not similar as not all three dimensions are in the same ratio.</p> <p style="text-align: right;">(1 mark)</p>	<p>b.</p> $DE = 2 \times 2.5 = 5\text{m}$ <p style="text-align: right;">(1 mark)</p>
<p>c.</p> $\frac{5 \times 3.6}{2} \times 6 = 54 \text{ m}^3$ <p style="text-align: right;">(1 mark)</p>	

Module 3: Geometry and measurement**Question 3**

<p>a.</p> <p>Latitude is taken from the equator as 0°. Windy Squalls is only 48° from the equator compared to Shaky Rock being 54° from the equator.</p> <p style="text-align: right;">(1 mark)</p>	<p>b.</p> <p>670 km</p>  $6400 \times \frac{\pi}{180} \times 6^\circ \approx 670.2$ <p style="text-align: right;">(1 mark)</p>
<p>c. (i)</p> $\begin{aligned} \text{radius}_{\text{small circle}} &= 6400 \times \cos 54^\circ \\ &= 3761.82 \dots \\ &\approx 3762 \end{aligned}$ <p>Could also use $\sin 36^\circ$</p> <p style="text-align: right;">(1 mark)</p>	<p>c. (ii)</p> <p>Difference in longitude = $51 - 6 = 45^\circ$ Shortest small circle distance</p> $\begin{aligned} &= 3762 \times \frac{\pi}{180} \times 45 = 2954.667 \dots \\ &\approx 2955 \text{ km} \end{aligned}$ <p style="text-align: right;">(1 mark)</p>
<p>d.</p> <p>Lonely Shoal is 45° further west than Shaky Rock, so is three hours behind.</p> <p>At 7:52 am, Tuesday 13th at Lonely Shoal, the time is 10:52 am on Tuesday 13th at Shaky Rock.</p> <p>11:15 pm to 10:52 am is 11 hours 37 minutes</p> <p style="text-align: right;">(1 mark)</p>	

Module 4: Graphs and relations

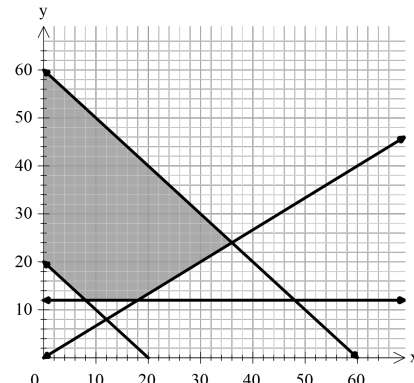
Question 1

<p>a.</p> <p style="text-align: center;">(1 mark)</p>	<p>b.</p> <p>\$50</p> <p>Note that the charge of \$60 is for more than but not including 20 pages as indicated by the open circle.</p> <p style="text-align: right;">(1 mark)</p>
<p>c.</p> <p>$40 + 50 + 70 = \mathbf{\\$160}$</p> <p style="text-align: right;">(1 mark)</p>	<p>d.</p> <p>\$8</p> <p>Magic Web: $Charge = 4.00 + 2.00 \times 19 = \\42 Wanda's Web: from step graph, \$50</p> <p>Saving of \$8</p> <p style="text-align: right;">(1 mark)</p>
<p>e.</p> <p>28 pages</p> <p>Wanda's Web Design charges \$60</p> <p>Solving the equation $60 = 4.00 + 2.00 \times n$ gives $n = 28$</p> <p style="text-align: right;">(1mark)</p>	

Module 4: Graphs and relations**Question 2**

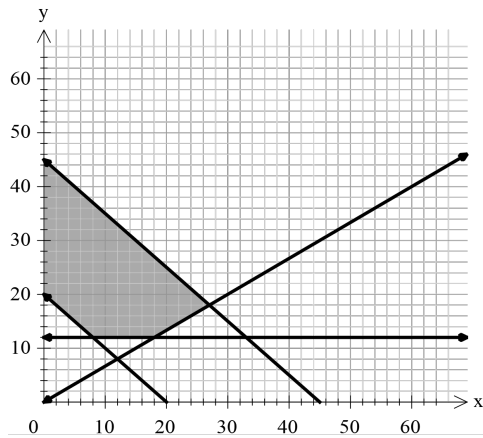
<p>a.</p> <p>1105 m</p> <p>When $t = 17$, $D = 65 \times 17 = 1105$</p> <p style="text-align: right;">(1 mark)</p>	<p>b.</p> <p>2762.5</p> <p>Substitute $(35, 1625)$ into the equation</p> $D = k - 32.5t$ $1625 = k - 32.5 \times 35$ $k = 2762.5$ <p style="text-align: right;">(1 mark)</p>
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Question 3

<p>a.</p> <p>She must make at least 12 blocks of Tuff each month.</p> <p style="text-align: right;">(1 mark)</p>	<p>b.</p>  <p style="text-align: right;">(1 mark)</p>
<p>c. 14</p> $y \geq \frac{2 \times 21}{3}$ $y \geq 14$ <p style="text-align: right;">(1 mark)</p>	<p>d.</p> <p>\$276</p> <p>All corner points of the feasible region are integral.</p> <p>The corner point $(36, 24)$ gives the maximum profit. (This can be determined by either substituting all corner points, or using the sliding line method)</p> $\text{Profit} = 5 \times 36 + 4 \times 24 = 276$ <p style="text-align: right;">(1 mark)</p>

Module 4: Graphs and relations**Question 3 (continued)**

e.

\$207

The intersection of $x + y = 45$ and $y = \frac{2x}{3}$
gives the new corner point at which maximum
profit is made $(27, 18)$

$$\text{Profit} = 5 \times 27 + 4 \times 18 = 207$$

(1 mark)

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Kilbaha Education
PO Box 2227
Kew Vic 3101
Australia

Tel: (03) 9018 5376

kilbaha@gmail.com
<https://kilbaha.com.au>