Neap

Trial Examination 2020

VCE Further Mathematics Units 3&4

Written Examination 1

Suggested Solutions

SECTION A – CORE

Data analysis

Question 1 **C**

The mode is the most populous, which in this case is 15.

Question 2 **C**

The mode is the figure with the highest frequency, and a five-number summary does not provide information about frequency, only spread.

Question 3 **D**

The midpoint of each group must be found and multiplied by their frequencies. Adding the four numbers and dividing by the total frequency gives 3 + 12 + 10 + 4 = 29.

Question 4 **D**

There are sixteen pieces of data:.

8 8 8 9 | 9 9 9 9 | 9 9 10 10 | 10 10 11 11

The median is between the eight and ninth figure, which are both 9. The median of each half is 9 and 10.

The five-number summary is 8, 9, 9, 10, 11.

Question 5 **D**

A is incorrect as the IQR is 7 – 4 = 3; **B** is incorrect as the mean cannot be calculated; **C** is incorrect as the IQR is 3, the upper and lower fences are 7.5 + 1.5 × 3 = 12 and 4 – 1.5 × 3 = –0.5, and there are no outliers; and **E** is incorrect as the median is about 4.3 and no information is provided about the mode. Q_3 is 7 and Q_4 is 9, so 25% of the data lies between these two values; **D** is correct.

Question 6 **E**

The question asks for incorrect option. **A** is true as there are six figures that have a frequency of 2; **B** is true as 20 + 16 = 36 people interviewed; **C** is true as the IQR for under 30 is 19 – 4.5 =14.5 and for over 30 is 17.5 – 3 = 14.5; and **D** is true as the medians are respectively 13 and 11, with a difference of 2. **E** is false as the data is not symmetrical.

Question 7 **B**

The mode is 3 so the answer must be **A**, **B** or **D**. The median is 11 (calculated as part of **Question 4**) so the answer must be **B** or **D**. The mean must be between 0 and 30, so it cannot be 58.7; therefore, **B** is correct.

Question 8 **D**

The correlation is positive and moderate, so **D** is correct.

Question 9 E

A is not necessarily true, as one of the other quarters could have more sales as long as the four seasonal indices add to 4. **B** is false as $3 \times 0.7 = 2.1$, and $2.1 + 1.81$ does not add to 4. **C** is false as this suggests only 19% of sales occur in the other three quarters combined, and **D** is false as causation is not linked to seasonal indices. **E** is completely true as the four seasonal indices must add to 4, and $2.2 + 1.81 > 4$.

Question 10 D

Using $z = \frac{x - \mu}{\sigma}$ gives $-0.4 = \frac{x - 70}{22}$.

$$-0.4 = \frac{x - 70}{22}$$

$$x = -0.4 \times 22 + 70$$

$$= 61.2\%$$

Question 11 A

$8.14 - 7.86 = 0.28$. Since we have 95% confidence, then 0.28 is equivalent to four standard deviations (two either side of the mean). Therefore, $\frac{0.28}{4} = 0.07$.

Question 12 D

The vertical scale is a log scale. So the value that is read from the graph, approximately 5, means $10^5 = 100\ 000$.

Question 13 C

The x-axis needs to be stretched horizontally to the right. An x^2 transformation will do this.

Question 14 B

For a linear graph in the form $y = a + bx$, $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$.

$$b = 0.8 \times \frac{4.8}{1.6}$$

$$= 2.4$$

$$a = 22 - 2.4 \times 6$$

$$= 7.6$$

Therefore, **B** is correct.

Question 15 C

r^2 is 0.64. This gives the causation of 64%. The explanatory value causes the response value to change; hence, **C** is correct.

Question 16 B

The number of sports grounds and the number of hospital beds are unlikely to be directly related. The most likely reason is that they are both responding to a third variable – the population of the city. The higher the population of the city, the more sports grounds and hospitals are required.

Question 17 A

Substituting 20 into the equation gives $0.045 \times (20 + 273) + 3.7 = 16.89$.

residual = actual value – estimated value

Therefore, $16.5 - 16.89 = -0.39$.

Question 18 E

Substituting the values into $y = ax + b$ gives $y = 26x^2 - 13$. The variable is now x^2.

Question 19 D

The scale is a log scale, so a value of 3 on the horizontal scale means $10^3 = 1000$. So, fewer than 1000 mosquitoes recorded is $2 + 3 = 5$ months.

Question 20 B

Adding the total for each of the five opinions gives the following.

	Strongly agree	**Agree**	**No opinion**	**Disagree**	**Strongly disagree**
Group 1	12	25	0	14	10
Group 2	34	48	12	23	18
Group 3	32	23	15	24	12
Group 4	38	41	20	8	4
Total	116	137	47	69	44

Comparing the totals against the column heights in the graphs, in **A,** the ‘strongly agree’ column is too short; in **C**, the ‘no opinion’ column is not correct; in **D**, the ‘disagree’ column is incorrect; and in **E**, the ‘strongly agree’ column is too short. **B** is correct.

Recursion and financial modelling

Question 21 C

To find the next term, the previous term is multiplied by –2. The first term must also be given.

Question 22 D

$$t_1 = \frac{1000}{10} + 200 = 300$$

$$t_2 = \frac{300}{10} + 200 = 230$$

$$t_3 = \frac{230}{10} + 200 = 223$$

Question 23 D

An increase of 12% means multiplying by a factor of 1.12 from the previous day. Hence, $t_{n+1} = 1.12t_n$; $t_0 = 60$.

Question 24 A

The question asks about the number of new cases, so the total number of cases is irrelevant. Each day the number of new cases rises by six starting with three cases, so $t_{n+1} = t_n + 6$, $t_1 = 3$.

Question 25 B

The length of each step decreases by 10%, or is 0.9 times the length of the previous step. Since we want a recursive relationship that generates the next length from the previous, the answer is $t_{n+1} = 0.9t_n$, $t_1 = 1$.

Question 26 E

We are given the following information:

- $r = 1.3\%$ per annum, or $\frac{1.3}{4}\%$ per quarter $\left(R = 1 + \frac{r}{100}\right)$
- $P = \$40\ 000$
- $n = 2$ years = 8 quarters
- $A = ?$

Substituting into $T_8 = AR^n$:

$$T_8 = 40\ 000 \times \left(1 + \frac{\frac{1.3}{4}}{100}\right)^{2 \times 4}$$

Question 27 E

The printer loses value with every copy made. It loses $5700 over 10 million copies.

$$\frac{570\ 000 \text{ cents}}{10\ 000\ 000 \text{ copies}} = 0.057 \text{ cents per copy}$$

0.057 cents is $0.00057.

Therefore, **E** is correct.

Question 28 E

We are given the following information:

- $r = 2.1\%$ per annum or $\frac{2.1}{12}\%$ per month
- $P = \$?$
- $n = 1$ month

Since we are dealing with one month, the rate is changed to per month and the simple interest formula is used:

$$I = \frac{Prt}{100}$$

$$2000 = \frac{P \times \frac{2.1}{12} \times 1}{100}$$

$$P = \frac{2000 \times 100}{\frac{2.1}{12}}$$

$$= \$1\ 142\ 857.14$$

Therefore, **E** is correct.

Question 29 B

The lowest balance is found by going through the transactions.

Date	**Event**	**Balance**
January 1	–	\$4500
January 4	deposit of \$800	\$5300
January 8	withdrawal of \$2000	\$3300
January 18	deposit of \$800	\$4100

The lowest balance is \$3300.

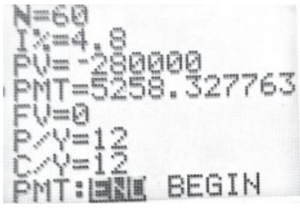
$$I = \frac{3300 \times 0.08 \times 1}{100}$$

$$= \$2.64$$

Therefore, **B** is correct.

Question 30 **A**

Using CAS to find the unknown (the monthly repayment) gives:



Therefore, **A** is correct.

Note that for these figures, since the money is borrowed and the balance is negative, the PV is set as negative. The PMT is therefore positive. It is equally correct to use +280000 for the PV, but the PMT will show as negative. **A** is correct regardless of which method you use.

SECTION B – MODULES

Module 1 – Matrices

Question 1 **B**

The sum of the row labelled C (Carrie) is 2, meaning that Carrie won two matches.

Question 2 **D**

$$a = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$$

$$b = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$c = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$3a - b = 3\begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 17 \\ 7 & 3 \end{bmatrix}$$

Question 3 **D**

The number of columns in B does not equal the number of rows in C, and so the product cannot be determined.

Question 4 **B**

$$b = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

Question 5 **A**

$$a = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$$

$$\det(a) = -10$$

Question 6 E

The matrix for the equation $G_{ij} = 3i - j$ must have columns that increase downwards in 3 (i) and rows that decrease to the right by 1 (j). The only matrix shown that fulfils these rules is **E**.

Question 7 C

$ab = c$

$\therefore b = a^{-1}c$

$a = \begin{bmatrix} 7 & -4 \\ 5 & -3 \end{bmatrix}$

$a^{-1} = \begin{bmatrix} 3 & -4 \\ 5 & -7 \end{bmatrix}$

Question 8 E

There is only one route from *D* to *D*, which means **D** is incorrect. There are two routes from *C* to *D*, which **A**, **B** and **C** are incorrect, so **E** is correct.

Question 9 A

There can only be one 1 in every column and every row. Looking at letter A, it is first in both the start and end combinations of the code, and so will be in the first column and first row of the matrix. J is second in the start combination and fourth in the end combination, so the 1 should be in the second row and fourth column to represent that change. F is third in both combinations and so will be in row 3 and column 3. By this point, through process of elimination, all matrices except **A** have been eliminated.

Question 10 A

When multiplying matrices, the number of columns in the first matrix must equal the number of rows in the second matrix for the solution to be defined. The question asks for a matrix calculation to get to the total rather than the actual total, so **E** is incorrect and **A** is correct.

Module 2 – Networks and decision mathematics

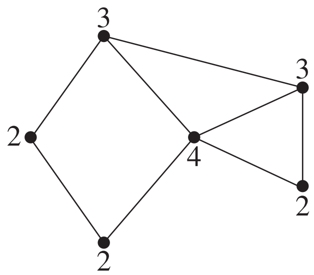
Question 1 B

Euler's formula is vertices – edges + faces = 2

There are six vertices and eight edges.

Substituting in with faces missing gives 6 – 8 + f = 2.

Question 2 D



2 + 2 + 2 + 3 + 3 + 4 = 16

Question 3 A

The maximum flow is equal to the minimum cut, which is 15.

Question 4 E

The critical path is *A–C–E–I–J* (5 + 3 + 4 + 10 + 3 = 25).

Question 5 A

The difference between the earliest start time and the latest start time is 1.

Question 6 A

Stage *B* takes 3, so stage *F* can start after 3.

Question 7 A

Belle is the only person who has a zero for project 1, and so she should complete project 1 and not project 2 to minimise completion time.

Question 8 D

6 + 5 + 3 + 4 + 3 + 5 = 26

Question 9 A

An Eulerian trail must start and end on an odd vertex. Currently there are 4 odd vertices, so 2 of these must be joined to make them even vertices. Of the options given, only *B* and *E* are both odd vertices.

Question 10 C

6 + 4 + 3 = 13, which is the shortest route, so the answer is *A–B–D–F*.

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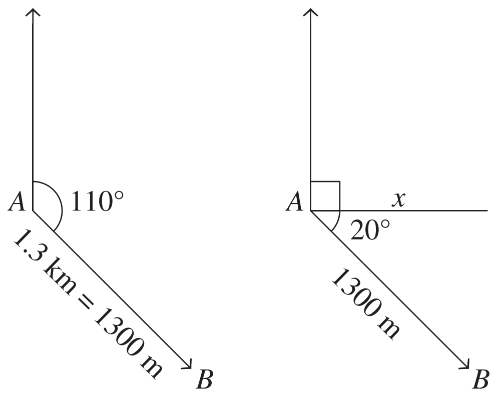
Module 3 – Geometry and measurement

Question 1 **D**



$$\cos(20) = \frac{x}{1300}$$

$$x = 1300\cos(20)$$

$$= 1221.6 \text{ m}$$

Question 2 **B**

Using the cosine rule to determine the distance (and note that a negative answer does not make sense in the context of this question) gives:

$$\text{solve}\left(a^2=157^2+74^2-2\cdot 157\cdot 74\cdot \cos(92),a\right)$$

$$a=-175.886 \text{ or } a=175.886$$

Question 3 **D**

384 000 000 m^2 is 384 km^2.

$$\text{solve}\left(\frac{1}{2}\cdot 45\cdot 19\cdot \sin(c)=384,c\right)|0<c<180$$

$$c=63.9284 \text{ or } c=116.072$$

OR

$$\sin^{-1}\left(\frac{384\cdot 2}{45\cdot 19}\right) \quad 63.9284$$

Question 4 **A**

The triangle removed is a similar shape to the original. Calculating the difference in height between the original shape and the truncated shape means it is possible to find the volume of what has been removed.

$$\frac{10 \times 15 \times 40}{2} = 3000$$

$$\frac{2.5 \times \frac{15}{4} \times 40}{2} = 187.5$$

$187.5 \text{ cm}^3 = 0.1875 \text{ L}$

Question 5 **C**

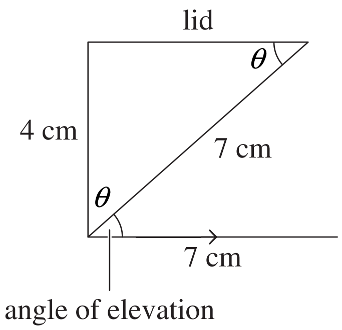
$$\sqrt{(7.5)^2 + 10^2} \times 40 \times 2 + 15 \times 10 + 40 \times 15 = 1750$$

Question 6 **D**

Pythagoras' theorem gives the radius of the lid, which can be used to find the circumference.

$$\sqrt{7^2 - 4^2} \times 2\pi = 36.0942$$

Question 7 **B**



$$\sin\theta = \frac{o}{h}$$
$$= \frac{4}{7}$$
$$\theta = \sin^{-1}\left(\frac{4}{7}\right)$$

Question 8 **D**

$$2 = \frac{\pi \times 7x}{180}$$

$$x = \frac{180 \times 2}{7\pi}$$
$$= 16.37$$

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Question 9 D

Seattle has the highest degree to the north or south and so is furthest from the equator.

Question 10 B

Calculating the volume of the cake and dividing it by the volume of one slice gives the total number of slices of cake.

$$\frac{12^2 \times 5\pi}{226} = 10.0086$$

Module 4 – Graphs and relations

Question 1 E

The equation is linear, so it is in the form $y = a + bx$. The x-intercept is 5, so we have the point (5, 0), and the line passes through the point (1, 3).

The gradient is $\frac{0-3}{5-1} = \frac{-3}{4}$.

$y = ax + b$ becomes $y = a - \frac{3x}{4}$.

Substituting in the point (1, 3) gives:

$$3 = a - \frac{3 \times 1}{4}$$

$$a = 3 + \frac{3}{4}$$

$$= \frac{15}{4}$$

$$\therefore y = \frac{15}{4} - \frac{3x}{4}$$

$$4y = 15 - 3x$$

$$4y + 3x = 15$$

OR

Using the two points (5, 0) and (1, 3) to create the a pair of simultaneous equations gives:

$0 = a + 5b$ (1)

$3 = a + b$ (2)

Then solve for a and b.

Question 2 D

The number of shoes and boots is physical, so neither can be negative. Both x and y are ≥ 0. The number of shoes, x, is at least ten times the number of boots, so $x \geq 10y$.

Question 3 B

The \$400 booking fee means that the y-intercept is 400.

Determining the gradient:

$$\frac{1400-400}{20-0} = \frac{1000}{20}$$

$$= 50$$

The charge per guest is \$50.

Question 4 B

The y-intercept is 3, so the answer must be **B** or **C**. Since the line is dotted and shaded below the graph, it is a < graph. Therefore, **B** is correct.

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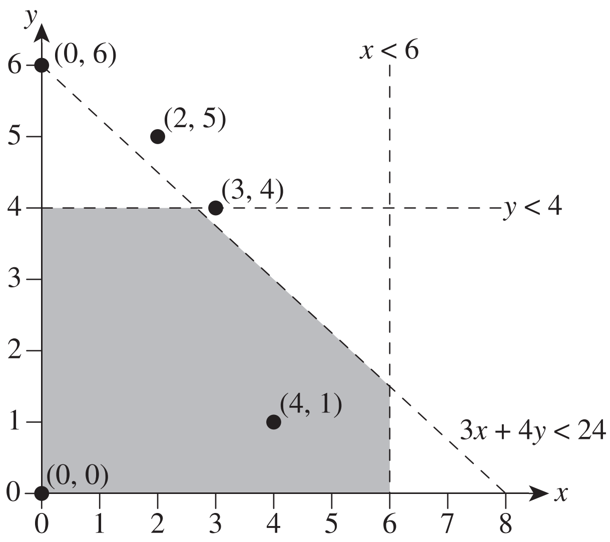
Question 5 **E**

A possible method of solution is to sketch the graphs and plot the points. Only the point (4, 1) is inside and not on the edge of the feasible area (note that (0, 0) is on the edge of the feasible region).



Question 6 **A**

We are only interested in the equation of the cycling section, which occurs between $1.5 \leq d \leq 41.5$. Therefore, **D** and **E** are incorrect.

The gradient of this section is found using $\frac{\text{rise}}{\text{run}} = \frac{80}{40} = 2$.

Since the gradient is 2, **B** is incorrect. Extending the line back to the *y*-axis we can see the intercept is less, than 40, so **A** is correct.

OR

The *y*-intercept can be calculated as follows.

The graph is linear and so is in the form $y = a + bx$.

Substituting in for *b*, the gradient, and using the point (1.5, 40):

$$40 = a + 2 \times 1.5$$
$$a = 37$$

Question 7 **C**

Substituting the point (3, 3600) into the equation to find the value of *k* gives:

$$y = kx^2$$
$$3600 = k(3)^2$$
$$k = \frac{3600}{9}$$
$$= 400$$

Question 8 E

Let x = the cost of an adult ticket and y = the cost of a child ticket.

$2x + 3y = 285 \quad (1)$

$9x + 0y = 675 \quad (2)$

From (2), the cost of an adult ticket is $\frac{675}{9}$ = \$75.

Substituting into (1) gives:

$$\begin{aligned} 2 \times (75) + 3y &= 285 \\ 150 + 3y &= 285 \\ 3y &= 135 \\ y &= \$45 \end{aligned}$$

$$\begin{aligned} 4x + 12y &= 4 \times (75) + 12 \times (45) \\ &= \$840 \end{aligned}$$

Question 9 B

The equation has a gradient of 0.57, which represents a charge of 57 cents for every kL of water used. The fixed charge is 28, which represents \$28. Therefore, **A** is correct.

Question 10 C

Let x = the number of bunches of six flowers sold and y = the number of bunches of 12 flowers sold.

$x + y = 9 \quad (1)$

$8x + 14y = 96 \quad (2)$

From (1), $x = 9 - y$.

Substituting into (2) gives:

$$\begin{aligned} 8(9 - y) + 14y &= 96 \\ 72 - 8y + 14y &= 96 \\ 6y &= 24 \\ y &= 4 \end{aligned}$$

Therefore y = 4 and x = 5.

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