2019 VCE Further Mathematics 1 examination report

General comments

Students generally found questions accessible in the Further Mathematics examination 1 in 2019. They found some questions involving the application of the key skills and key knowledge from the study design challenging, such as Questions 10, 11, 13, 15 and 23.

Specific information

In the multiple-choice tables throughout, the correct answers are indicated by shading.

Section A – Core

In 2019, the Core section comprised two components: Data analysis (Questions 1–16) and Recursion and financial modelling (Questions 17–24).

Question	% A	% B	% C	% D	% E	% No answer
1	3	92	2	1	2	0
2	2	86	0	1	11	0
3	7	6	7	71	9	0
4	0	1	1	96	1	0
5	6	4	87	2	1	0
6	3	6	6	79	6	0
7	6	72	8	10	3	0
8	11	63	1	14	11	0
9	14	3	9	7	67	0
10	41	3	3	6	46	0
11	37	31	26	4	2	0
12	17	57	4	14	8	0
13	18	10	15	12	45	0
14	3	12	10	6	69	0
15	13	28	11	37	10	0
16	7	11	74	5	2	0
17	16	2	7	68	6	0
18	8	5	66	17	3	0
19	76	2	13	3	5	0
20	10	12	58	11	10	1
21	19	9	54	11	7	1
22	3	11	15	17	53	1
23	19	8	38	25	9	1
24	18	51	15	8	7	1

Data analysis

Students generally answered the questions in this section well, particularly questions that required standard, routine calculations or interpretations (Questions 1, 2, 4, 5 and 6). Questions that required the use or analysis of graphical or tabular information were answered less well (Questions 11, 13, and 15).

Question 8

Students needed to recognise that for a two way-frequency table to be used, both variables had to be categorical variables.

Question 10

The value of the correlation coefficient r was required, given that the coefficient of determination was $r^2 = 0.893743$. Many students incorrectly took the positive square root value and chose option E.

The equation of the least squares line clearly showed that the direction of the association was negative and therefore -0.945 was required.

Question 11

Students needed to recognise that *sleep* was the response variable and *coffee* the explanatory variable, and be familiar with the rules connecting a least squares regression line equation to summary statistics.

The slope, b, representing the change in the amount of *sleep* for each additional cup of *coffee*, is

$$b = r \frac{s_y}{s_x}$$

$b = -0.770 \times \dfrac{1.12}{1.56} \approx -0.55$ correct to two decimal places.

Question 13

Students needed to be able to apply graphical smoothing of a time series plot using moving medians involving an odd number of points.

On day number 7 with three median smoothing the median of 60, 33 and 37 is 37.

On day number 7 with five median smoothing the median of 19, 60, 33, 37 and 46 is 37.

Question 15

With 12 monthly values the median is midway between the 6th and 7th rainfall values when values are placed in ascending order. These values can be read from the graph as approximately 96 and 110.

The median is therefore closest to 103.

Recursion and financial modelling

The performance of students was reasonable but not strong. Students had difficulty with questions that tested a variety of concepts, in particular Questions 20 to 24.

Question 20

This question required students to firstly calculate the interest rate per time period from the amortisation table, e.g. $\dfrac{900}{300\ 000} = 0.3\%$.

As the annual interest rate was 3.6% the repayments must be made monthly, since 0.3% per month equates to 3.6% per annum.

Question 21

Students needed to calculate the multiplying factor $\dfrac{580}{500} = 1.16$

The value of b is therefore $580 \times 1.16 = 672.80$.

Question 22

The amount of depreciation = \$30 000 – \$18 840 = \$11 520

The number of items produced in three years $= 3 \times 24\ 000 = 72\ 000$

The depreciation per item $\dfrac{11\ 520}{72\ 000} = 0.16$

Therefore $V_n = 30\ 000 - 0.16n$.

Question 23

For this question a finance solver could have been used with the following entries:

N = 1

I% = 7.5

PV = 995.49

PMT= solve

FV = 0

P/Y = 12

C/Y = 12

to result in a payment value of –\$1001.71

The negative indicates that this amount must still be paid back to the bank as the final payment.

Question 24

This question could have been solved by firstly determining the interest rate for the investment using the following entries:

N = 12

I% = solve

PV = −20 000

PMT = 0

FV = 20732

P/Y = 12

C/Y = 12

to result in an interest rate of $\approx 3.6\%$.

The second stage of the solution required the determination of the effective interest rate.

Using interest conversion from the calculator with a nominal rate of 3.6% and 12 compounding periods per year gives an effective rate of $\approx 3.66\%$.

Alternatively, the following formula could have been used:

$$r_{effective} = \left[\left(1 + \frac{r}{100 \times 12}\right)^{12} - 1\right] \times 100\% = 3.66\%$$

The difference is $3.66 - 3.6 = 0.06\%$.

Section B – Modules

Students were required to complete questions from two of the four modules.

The selection of modules by students in 2019 is shown in the table below.

Module	%
Matrices	45
Networks and decision mathematics	27
Geometry and measurement	14
Graphs and relations	15

Module 1 – Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	2	5	87	5	2	0
2	17	9	2	68	4	0
3	6	12	10	12	59	1
4	6	8	24	6	56	1
5	6	73	8	5	7	1
6	39	17	13	16	13	1
7	54	19	12	8	7	0
8	27	16	26	16	15	1

The questions students found challenging involved the application of a transition matrix or diagram (Questions 6 and 8) and the interpretation of a communication matrix (Question 7).

Question 4

Students were required to perform five transitions from Stella's answer on Question 1 to her answer on Question 6.

Students using technology would have been successful by calculating

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}^5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Alternatively, this could be done without technology following question to question, i.e.

$$E \rightarrow C \rightarrow A \rightarrow D \rightarrow B \rightarrow E$$

Question 6

Students needed to recognise that this question involved 'long term expectation'.

Although the number of half-hours that the group stays is not defined, it makes little difference whether this is taken to be 10, 15, 20 or higher. There is no initial state needed.

Raising the transition matrix to any of these powers will give the percentage at *(S)* to the nearest whole number to be 14%.

Question 7

W can only receive messages **directly** from U, V and Y.

For Y to get a message to W via one other person it would have to be directed to one of these.

The transition matrix shows none of these direct messages from Y are possible.

Question 8

12 planes move each night from Sydney to Melbourne

$$0.48s = 12 \Rightarrow s = 25$$

As the numbers stay the same at each location, 12 planes must also move from Melbourne to Sydney

$$0.2m = 12 \Rightarrow m = 60$$
$$m + s = 60 + 25 = 85$$

Module 2 – Networks and decision mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	8	5	1	1	85	0
2	6	11	64	12	7	0
3	2	3	85	4	7	0
4	71	2	5	4	18	0
5	4	75	11	5	3	1
6	39	31	14	11	4	1
7	21	42	12	14	9	1
8	16	21	22	31	9	1

The questions students found most challenging involved the forming of an adjacency matrix from a map (Question 6) and critical path analysis (Questions 7 and 8).

Question 6

It is possible to return to town *P* without passing through another town.

This loop is identified as a 1 in the first row and first column of the matrix. Options C and D do not show this loop so can be discarded.
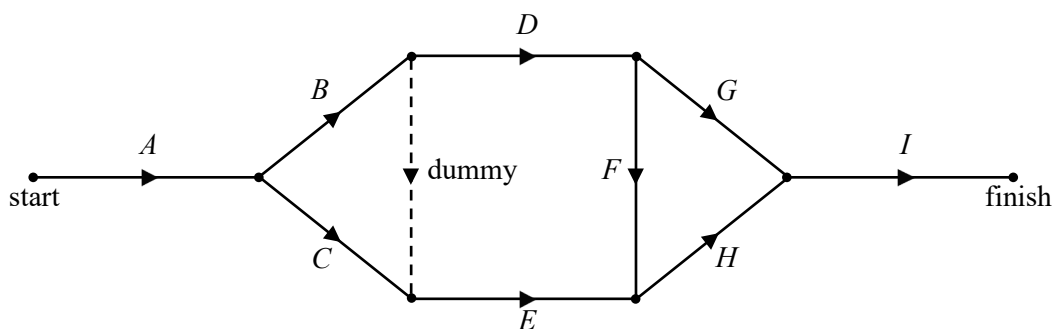
There are three different ways of travelling directly between *P* and *Q*. Option A is the only remaining matrix that shows this.

Question 7

Activity *E* has both *B* and *C* as immediate predecessors.

A sketch of the directed network shows where the dummy activity is required.

Students could have answered the question without sketching the entire network.

Question 8

The critical path for this directed network is *A-B-E-J-K-O-Q-S*, taking 51 hours.

There are three activities that have a float time of 10 hours:

Activity *G*   LST – EST = 16 – 6 = 10

Activity *I*   LST – EST = 22 – 12 = 10

Activity *N*   LST – EST = 29 – 19 = 10

Module 3 – Geometry and measurement

Question	% A	% B	% C	% D	% E	% No Answer
1	5	1	3	91	0	0
2	68	3	22	1	5	0
3	3	41	48	7	1	0
4	3	3	4	3	87	0
5	6	8	55	12	18	1
6	4	40	13	41	2	1
7	7	6	10	63	13	1
8	21	51	12	13	2	1

Questions involving elementary application of the key skills from this module were answered correctly by most students. Students had difficulty with time zones (Question 6) and Questions 3, 5 and 8 were also challenging for many students.

Question 3

As the top and the base of the dessert were to be covered in chocolate, the required calculation was:

Total Surface Area = area of hemisphere + area of circle

$$= 2\pi \times 5^2 + \pi \times 5^2 = 235.619...$$

A significant proportion of students chose option B, forgetting to include the circular base.

Question 5

Triangle *BDC* is isosceles therefore angle *BDC* = angle *ACB.*

Since the sum of angle *ADB* and angle *BDC* is $180°$, the sum of angle *ADB* and angle *ACB* must also be $180°$.

Question 6

The difference in longitude between Wuhan and Chengdu is $10°$.

Using the given information that $15°$ of longitude equates to a one-hour time difference,

$\dfrac{10}{15} \times 60$ = 40 minutes.

Wuhan is further east therefore the sun rises earlier in Wuhan.

The sun will rise 40 minutes later in Chengdu at 6.30 pm.

Question 8

Construct a square with its four vertices being the centres of the four circles.

Shaded area = area of square – the area of four quarter circles

$$= (2r \times 2r) - (4 \times \frac{1}{4}\pi r^2) = 4r^2 - \pi r^2$$

Module 4 – Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	1	90	8	0	0	0
2	1	1	2	94	2	0
3	1	8	5	4	80	1
4	8	11	12	66	2	1
5	5	57	19	13	5	1
6	16	15	37	30	2	1
7	7	17	12	17	47	1
8	13	14	32	30	10	1

Many students were challenged by Questions 6, 7 and 8, which required more in-depth analysis of the graphs and relations presented.

Question 5

Revenue = $35 \times 65 = \$2275$

Cost of making trains = $35 \times 40 = \$1400 \Rightarrow$ total costs = fixed cost + $\$1400$

Profit = $\$250$

Profit = Revenue – Costs

$250 = 2275 - (1400 + \text{fixed cost})$

Fixed cost = $\$625$

Question 6

Consider firstly a ratio of 200 mL of mango juice to 300 mL of pineapple juice:

$\dfrac{x:y}{3:2}$ and 'at least' is represented by $\geq$

An inequality could be $\dfrac{x}{y} \geq \dfrac{3}{2}$

Therefore $x \geq \dfrac{3y}{2}$ which is equivalent to $y \leq \dfrac{2x}{3}$

Question 7

The objective function has the same slope as the line from V to W.

The gradient of this line from $(9, 9)$ to $(11, 3)$ is $\dfrac{3-9}{11-9} = -3$.

The line joining S to T also has a slope of -3.

As m and n are both positive, slide the dotted line back so as to minimise x and y. The minimum is therefore at any point along the line segment ST.

Question 8

As seen from the graph below, Jenny and Alan meet when $2 < t \le 6$

Solving $100t + 40 = -80t + 900$ gives $t = \dfrac{43}{9}$

Alan's distance from the house will be $-80 \times \dfrac{43}{9} + 900 = 518$ m to the nearest metre.

The distance walked by Alan from the supermarket = 900 – 518 = 382 metres.