FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

TSSM™
Creating VCE Success

2018 Trial Examination

<u>SOLUTIONS</u>

SECTION A: Core

Question 1

a. 6 days

1 mark

b.

i. Median is 4.5 bush rats

1 mark

ii. Q1 = 2 and Q3 = 7

1 mark each quartile

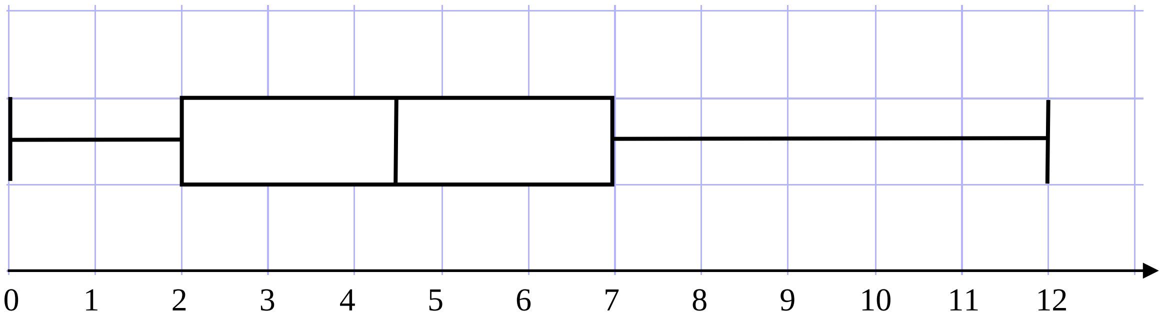
c. IQR $= 7 - 2 = 5$

1 mark

d. For more than 50% of the traps to contain a bush rat, there would need to be 8 or more rats caught. percentage $= \frac{6}{30} \times 100 = 20\%$

1 mark

e.



½ mark each for correct location of end points, median and quartiles and use of ruler

Question 2

a. The distribution of weights of bush rats at Parker River in 2017 is positively skewed with no outliers. The centre of the distribution is 223.5, the median value. The spread of the distribution, as measured by the IQR, is 143 and, as measured by the range, 332.

½ mark each for positively skewed, absence of outliers, median and either IQR and/or Range

b. Upper fence $= Q3 + 1.5 \times IQR = 295 + 1.5 \times 143 = 509.5$ Since 439 is lower than the upper fence, it is not an outlier.

1 mark for calculation, 1 mark for stating not an outlier

c. Both distributions of weights are positively skewed, although the 2015 study contained one outlier. The 2017 study found that bush rats were generally heavier than in 2015 with a median of 223.5 grams compared to just under 150 grams in 2015. The weights were also more variable in 2017 with an IQR of 143 grams compared to approximately 80 grams in 2015.

1 mark each for comparison of shape, centre and spread

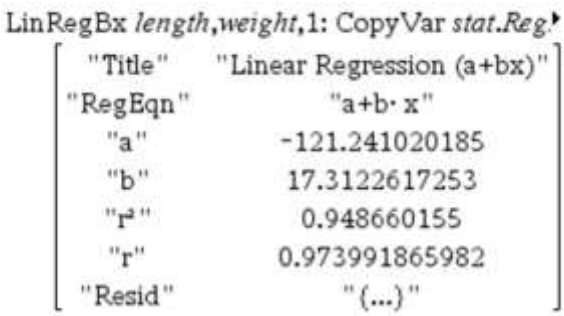
Question 3

a. The scatter plot shows the association between weight and length is a strong, positive and linear. (With the ends turning up, I would also accept a suggestion the form is non-linear!)

1 mark

b.

i. Regression equation calculated on CAS:
Thus the equation of the least squares regression line is
$weight = -121.2 + 17.31 \times length$



3 marks

ii. The slope of 17.31 suggests that for each additional centimetre in length a bush rat's weight increases by 17.31 grams.

1 mark

c. The coefficient of determination suggests that 94.9% of variation in weight can be explained by variation in length.

1 mark

d. The residual plot shows a clear pattern which does not support the assumption of linearity for the relationship between weight and length of bush rats.

Possible transformations include log(*y*), 1/*y* and x^2. On CAS:

LinRegBx *length*,*logweight*,1: CopyVar *stat.*▸

"Title"	"Linear Regression (a+bx)"
"RegEqn"	"a+b·x"
"a"	1.64207175049
"b"	0.0339748880479
"r²"	0.960486423286
"r"	0.980044092521
"Resid"	"{...}"

LinRegBx *length*,*recweight*,1: CopyVar *stat.*▸

"Title"	"Linear Regression (a+bx)"
"RegEqn"	"a+b·x"
"a"	0.012902189416
"b"	-3.89001403612E-4
"r²"	0.921000238166
"r"	-0.959687573206
"Resid"	"{...}"

LinRegBx *lengthsqr*,*weight*,1: CopyVar *stat.*▸

"Title"	"Linear Regression (a+bx)"
"RegEqn"	"a+b·x"
"a"	44.9359809054
"b"	0.426222801224
"r²"	0.966457574996
"r"	0.983085741426
"Resid"	"{...}"

The best transformation is the x^2 transformation because it has the highest coefficient of determination.

2 marks

Question 4

a. Over the period of time 2008 to 2015 this is a general increasing trend.

1 mark

b. 5 median smoothed value centred at 1990 is shown below:

1 mark

Recursion and financial modelling

Question 5

a. Annual depreciation rate $= \frac{5760}{48000} \times 100 = 12\%$ p.a.

1 mark

b. The rule is $V_n = 48000 - 5760n$

1 mark

c. $10000 = 48000 - 5760n \Rightarrow n = 6.597$ so after 7 years the cars value will first fall below \$10000

1 mark

d. First need to determine the value of $V_7 = 48000 - 5760 \times 7 = \$7680$

The rule for reducing balance value of the car is of the form $C_n = R^n \times 48000$

$C_{10} = R^{10} \times 48000 = V_7 + 1769.97$

$\therefore R^{10} \times 48000 = 7680 + 1769.97$

So R = 0.85, thus the depreciation rate is 15% p.a.

solve for R on CAS:

$\text{solve}\left(r^{10} = \frac{7680. + 1769.97}{48000}, r\right)$

$r = -0.849999987332$ or $r = 0.849999987332$

1 mark for calculating V_7 and 1 mark for correct depreciation rate

e. Recurrence relation is $C_0 = 48000, \quad C_{n+1} = 0.85C_n$

1 mark

Question 6

a. Recurrence relation is $S_0 = 20000, \quad S_{n+1} = \left(1 + \frac{5.1 \div 12}{100}\right) \times S_n = 1.00425 \times S_n$

1 mark

b. Use recurrence relation to find S_4:

$S_1 = \$20085$

$S_2 = \$20170.36$

$S_3 = \$20256.09$

$S_4 = \$20342.17$

20000	20000.
20000.· 1.00425	20085.
20085.· 1.00425	20170.36125
20170.36125· 1.00425	20256.0852853
20256.085285313· 1.00425	20342.1736478

1 mark

c. Either continue to use the recurrence relation for 120 iterations or use the rule as shown here:
$S_{120} = \$33269.85$ is the value of the investment after 10 years.

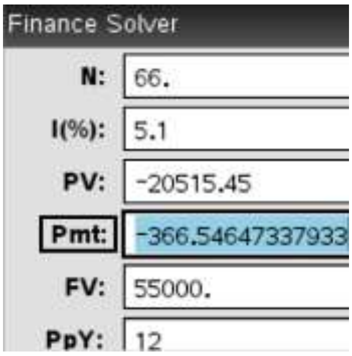
$(1.00425)^{120} \cdot 20000$ 33269.8500988

1 mark

d. First need to determine the amount in the account after 6 months. $S_6 = \$20515.45$ found on CAS as shown:

$(1.00425)^{6} \cdot 20000$ 20515.4495543

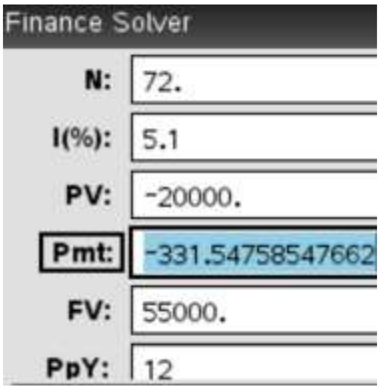
Now Finance Solver can be used to determine the required repayments:



The minimum repayment per month that Chris would need so that she will have \$55000 after 6 years is \$367 (to the nearest dollar)

1 mark for S_6, 1 mark for \$367

e. Use finance solve to find deposit amount if started at the commencement of investment:



Minimum whole dollar payment would be \$332 if commenced at the beginning of the investment, which is \$35 per month less.

1 mark

SECTION B: Modules

Module 1 – Matrices

Question 1

a. Order of matrix N is 3x1, and $N = \begin{bmatrix} 12 \\ 40 \\ 448 \end{bmatrix}$

2 marks

b. $C = [15000 \quad 10000 \quad 2200]$

1 mark

c. $C \times N = 1 \times 3 \times 3 \times 1 = 1 \times 1$ Matrix multiplication on CAS:

The total revenue if all seats are booked is $1,565,600

$$\begin{bmatrix} 15000 & 10000 & 2200 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 40 \\ 448 \end{bmatrix} \quad [1565600.]$$

1 mark for dimensions, 1 mark for matrix product

Question 2

a. Matrix A will be a 3x1 matrix which calculates the revenue from each of the three classes of seat sold for this flight.

1 mark

b. Matrix product $X \times A$ will be a 1x1 matrix containing the total cost of just first class and business tickets sold for this flight.

1 mark

Question 3

a. Complete matrix:

$$\begin{array}{c} \text{This week} \\ \begin{array}{ccc} A & B & C \end{array} \end{array}$$

$$T = \begin{bmatrix} 0.74 & 0.08 & 0.03 \\ 0.15 & 0.81 & 0.06 \\ 0.11 & 0.11 & 0.91 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \text{ next week}$$

1 mark

b. Initial state matrix:

$$S_0 = \begin{bmatrix} 654 \\ 421 \\ 396 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

1 mark

c. Total passengers who travel each week is 1471 (the sum of passengers from each airline)

1 mark

d. To find the long term numbers calculate $T^{50} x S_0$ on CAS:

$$t^{50}\cdot s \quad \begin{bmatrix} 227.141811593 \\ 434.814083654 \\ 809.044104753 \end{bmatrix}$$

In the long run, there would be 435 passengers expected on the Bragstar flight each week.

1 mark

e. Using CAS the weekly figures can be calculated. This shows that the number of Bragstar passengers rises during the next three weeks levels out briefly and then begins to decline. The maximum number of passengers is about 490.

$$t\cdot \begin{bmatrix} 654 \\ 421 \\ 396 \end{bmatrix} \quad \begin{bmatrix} 529.52 \\ 462.87 \\ 478.61 \end{bmatrix}$$

$$t\cdot \begin{bmatrix} 529.52 \\ 462.87 \\ 478.61 \end{bmatrix} \quad \begin{bmatrix} 443.2327 \\ 483.0693 \\ 544.698 \end{bmatrix}$$

$$t\cdot \begin{bmatrix} 443.2327 \\ 483.0693 \\ 544.698 \end{bmatrix} \quad \begin{bmatrix} 382.978682 \\ 490.452918 \\ 597.5684 \end{bmatrix}$$

$$t\cdot \begin{bmatrix} 382.978682 \\ 490.452918 \\ 597.5684 \end{bmatrix} \quad \begin{bmatrix} 340.56751012 \\ 490.56776988 \\ 639.86472 \end{bmatrix}$$

$$t\cdot \begin{bmatrix} 340.56751012 \\ 490.56776988 \\ 639.86472 \end{bmatrix} \quad \begin{bmatrix} 310.461320679 \\ 486.836903321 \\ 673.701776 \end{bmatrix}$$

$$t\cdot \begin{bmatrix} 310.4613206792 \\ 486.8369033208 \\ 673.701776 \end{bmatrix} \quad \begin{bmatrix} 288.899382848 \\ 481.329196352 \\ 700.7714208 \end{bmatrix}$$

1 mark

Module 2 – Networks and decision mathematics

Question 1

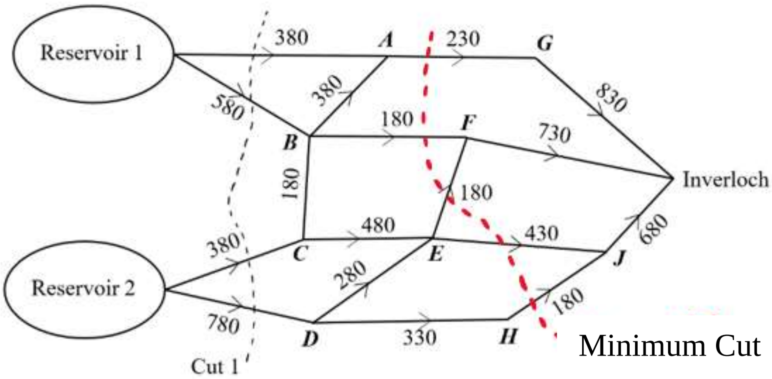
a. Capacity of Cut 1 $= 380 + 580 + 380 + 780 = 2120$

1 mark

b. The pipes leading from Reservoir 1 can take 380 units to A, but only 230 units can flow from A, and 580 units to B, but the pipe which can carry 380 units away from B towards A is unable to deliver that quantity because of the over supply to A, and the 180 units diverted to F is only a fraction of the volume available. The pipes from Reservoir 2 can take 380 units to C, but 480 can flow away from C. The undersupply would be provided through pipe BC. Thus water is more likely to flow towards C.

2 marks

c. Maximum flow through to Inverloch is 1200 units. The minimum cut is show below:



1 mark

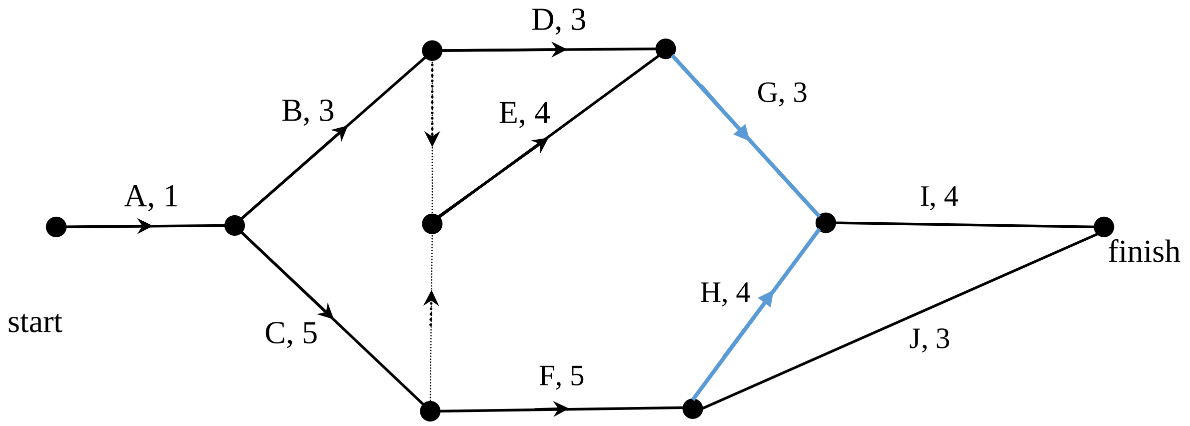
d.

e. The section of pipe from A to G is the first section that should be upgraded. The maximum flow through AG is limited by the inflow to A which is $380 + 380 = 760$ units, so the flow through AG could be increased from 230 units to 760 units. This is a larger increase than for any other section of pipe. AG would still be on the minimum cut so the increase through AG would increase the flow through the whole system by the same amount.

1 mark

Question 2

a. The completed activity network is shown here:



½ mark for each line correctly labelled

b. The minimum set up time is 19 hours and the critical path is $A \to C \to F \to H \to I$

2 marks

c. The latest starting time for activity B is 5 hours.

1 mark

d. The new minimum completion time is 15.5 hours
The activities to crash are activity C by 1.5 hours and activity F by 2 hours
The cost of crashing these tasks is $3.5 \times \$50 = \$175$

3 marks

Module 3 – Geometry and measurement

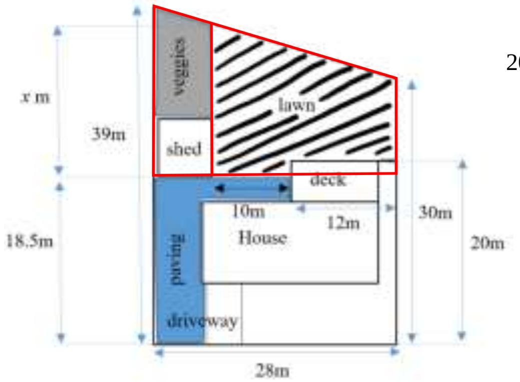
Question 1

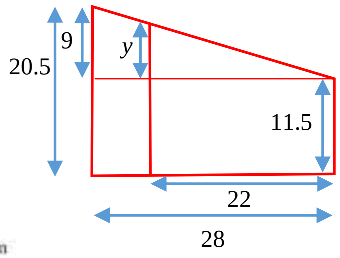
a. Area of house block is 966 m^2. Area of a trapezium:

$$Area = \frac{(a+b)}{2} \times h = \frac{30+39}{2} \times 28 = 966$$

1 mark

b. Similar shapes can be used to determine the required length:





$$\text{Scale factor } = \frac{9}{28} \quad \therefore y = 22 \times \frac{9}{28} = 7.07 \;\approx 7.1\text{m}$$

$$x = y + 11.5 = 7.1 + 11.5 = 18.6 \text{ m}$$

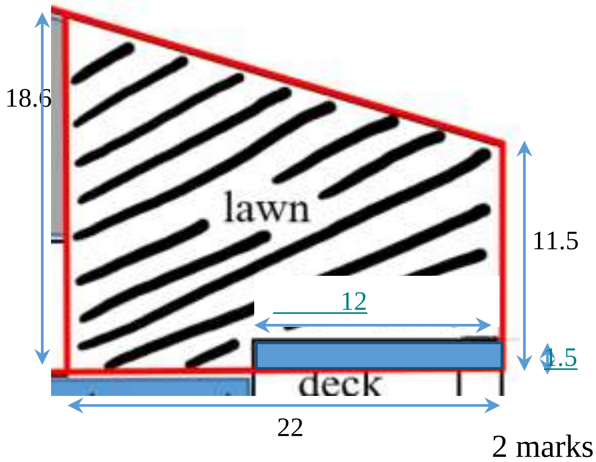
2 marks

c. Area to be sown with lawn seed is trapezium minus a rectangle.

$$\text{Area of trapezium } = \frac{(11.5+18.6)}{2} \times 22 = 331.1\text{m}^2$$

$$\text{Area of rectangle } = 12 \times 1.5 = 18$$

$$\text{Area of lawn } = 331.1 - 18 = 313.1\text{m}^2$$



2 marks

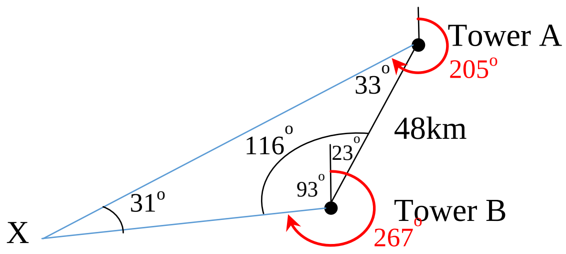
d. Number of packets of lawn seed required $\frac{313.1}{50} = 6.262$

7 packets of lawn seed will be required.

1 mark

Question 2

a. Diagram shown here:



1 mark

b. Angle $\angle AXB = 31^0$

1 mark

c. Distance XA: $AX = \frac{48}{\sin 31^0} \times \sin 116^0 = 83.76\,\text{km}$

Distance XB: $BX = \frac{48}{\sin 31^0} \times \sin 33^0 = 50.7587 \approx 50.76\,\text{km}$

2 marks

Question 3

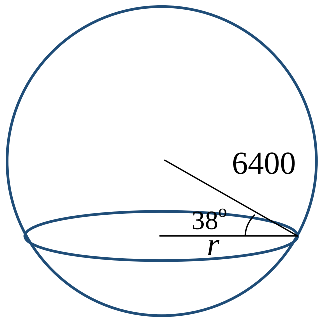
If the plane flies due west from Melbourne it will travel along the line of latitude.

The radius of the circle $r = 6400 \times \cos\left(38^0\right) = 5043.27\,\text{km}$

Circumference of this circle is $2 \times \pi \times 5043.27 = 31687.79$

Angle from centre to the west is $\frac{2700}{31687.79} \times 360^0 = 30.67^0 \approx 31^0$

So coordinates of beacon is $38^0\text{S}\ 114^0\text{W}$



2 marks

Module 4 – Graphs and relations

Question 1

a. The lowest temperature was 8^0C at 3 am

2 marks

b. The temperature exceeded 30^0 from 12pm to 3pm, 3 hours

1 mark

c. The temperature increased from 10^0 to 30^0 in 5 hours, rate of change of temperature is 4^0 per hour.

1 mark

Question 2

a. The fastest boat will complete the course in 5 minutes 33 seconds
$2.5\,km \div 27\,km/hr = 0.0925\,hr = 5.55\,\text{min} = 5:33\,\text{min}$

1 mark

b. The slowest boat will take 10 minutes to complete the 2.5km course, so the fastest boat would need to start 4 minutes 27 seconds after the first boat leaves.

1 mark

c. The second boat will take 6 minutes 15 seconds to complete the course and will need to leave 3 minutes 45 seconds after the first boat. The time then will be 2:03:45 pm

1 mark

d. The boats are all due to complete the race at 2:10 pm

1 mark

Question 3

a. The inequalities for this feasible region are:

$x \geq 40$

$y \leq 100$

$2y - 3x \leq 20$

$2y + 5x \leq 800$

½ mark for each equation

b. Use the corner points to determine which values of *x* and *y* gives the greatest profit:

$(40, 0)$ $P = 2\times 40 + 3\times 0 = 80$

$(40, 70)$ $P = 2\times 40 + 3\times 70 = 290$

$(60,100)$ $P = 2\times 60 + 3\times 100 = 420$

$(120,100)$ $P = 2\times 120 + 3\times 100 = 540$

$(160, 0)$ $P = 2\times 160 + 3\times 0 = 320$

Maximum profit when $x = 120$ and $y = 100$.