2018 TRIAL EXAMINATION 2

UNITS 3&4

STUDENT NAME

First Name	
Last Name	

FURTHER MATHEMATICS
Written examination 2

2018

Reading time: 15 minutes
Writing time: 1 hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	7	7	36
Section B – Modules	*Number of Modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
		Total	60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 38 pages.
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

THIS PAGE IS BLANK

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

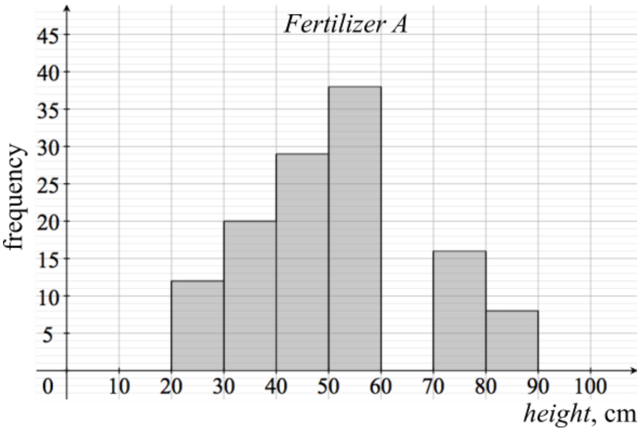
Data analysis

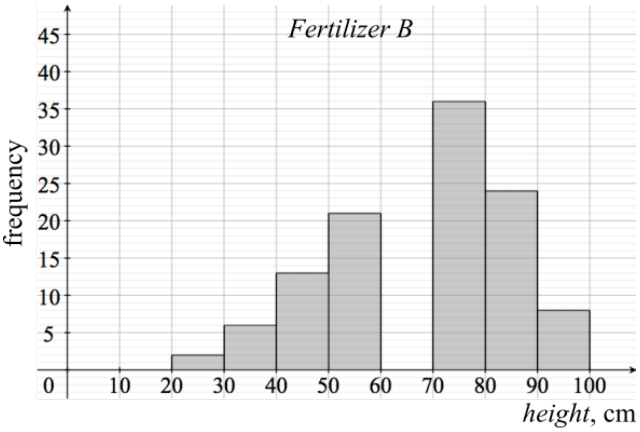
Question 1 (9 marks)

Aaden grows tomatoes on his farm. He always experiments with new fertilizers in order to improve the quality of the tomatoes. He has two experimental batches where he used *Ferilizer A* for one batch and *Fertilizer B* for the other batch. The table below shows the heights of the tomato plants in the two experimental batches.

height, cm	*Fertilizer A*	*Fertilizer B*
20 –< 30	12	2
30 –< 40	20	6
40 –< 50	29	13
50 –< 60	38	21
60 –< 70	27	40
70 –< 80	16	36
80 –< 90	8	24
90 –< 100	0	8

The two histograms below show the distributions of the height of the tomato plants for the two batches. Both histograms are incomplete.





a. Complete the two histograms above. 2 marks

b. State the modal class for the distribution of the height of the tomato plants when *Fertilizer A* was used. 1 mark

c. What is the shape of the distribution of the height of the tomato plants when *Fertilizer B* was used? 1 mark

d. Calculate the average height of the tomato plants when *Fertilizer B* was used.
Give your answer correct to the nearest centimetre. 2 marks

SECTION A – Question 1 – continued

Lydia, Aaden's daughter studies Further Mathematics at school and wants to help her father with this experiment. She calculates the mean and standard deviation of the height of all plants in the batch where *Fertilizer A* was used. Lydia assumes this data is approximately normally distributed with a mean of 53.5 cm and a standard deviation of 15.8 cm.

e. Using the rule 68–95–99.7% rule, calculate

i. the percentage of plants in this batch that are expected to have a height of at least 69.3 cm. 1 mark

ii. the percentage of plants in this batch that are expected to have a height of at most 21.9 cm. 1 mark

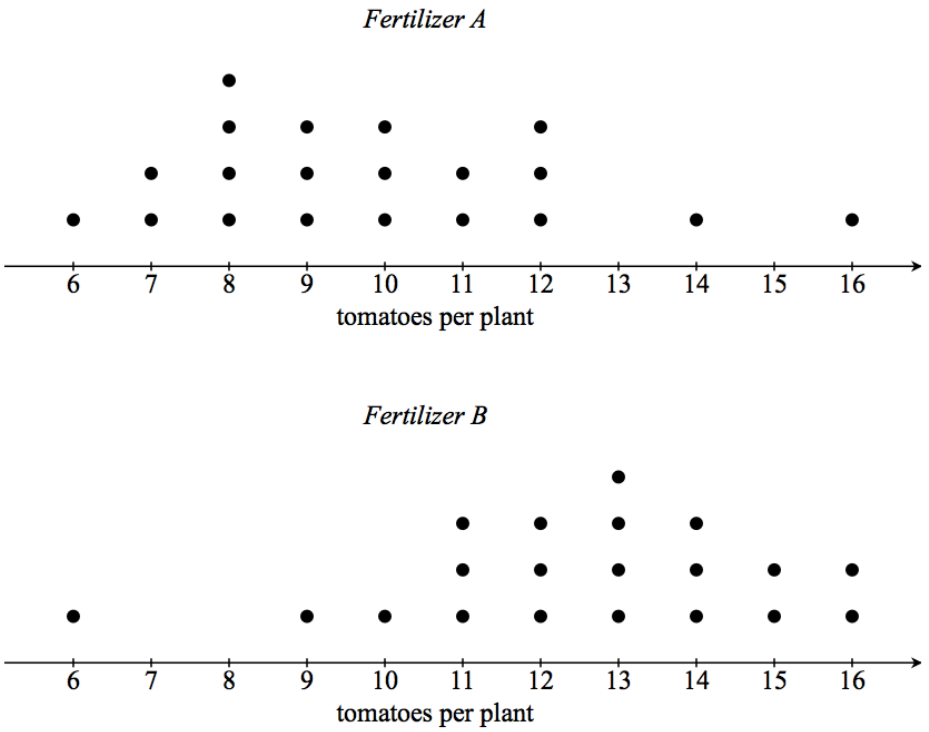
f. How many standard deviations from the mean is a height of 26 cm?
Give your answer correct to one decimal place. 1 mark

SECTION A – continued
TURN OVER

Working space

Question 2 (5 marks)

Aaden picked a random sample of 20 tomato plants from each batch and recorded the number of tomatoes on each plant as shown on the parallel dot plots below.



The two variables used in these dot plots are *Type of fertilizer* (*Fertilizer A* and *Fertilizer B*) and *Number of tomatoes per plant*.

a. Indicate whether the two variables are categorical (ordinal or nominal) or numerical (discrete or continuous). 2 marks

b. What is the median number of tomatoes per plant for the tomato plants where *Fertilizer A* was used? 1 mark

SECTION A – Question 2 – continued

TURN OVER

In the batch of tomato plants where *Fertilizer B* was used there is one plant that has six tomatoes.

c. Show that this plant is an outlier. 2 marks

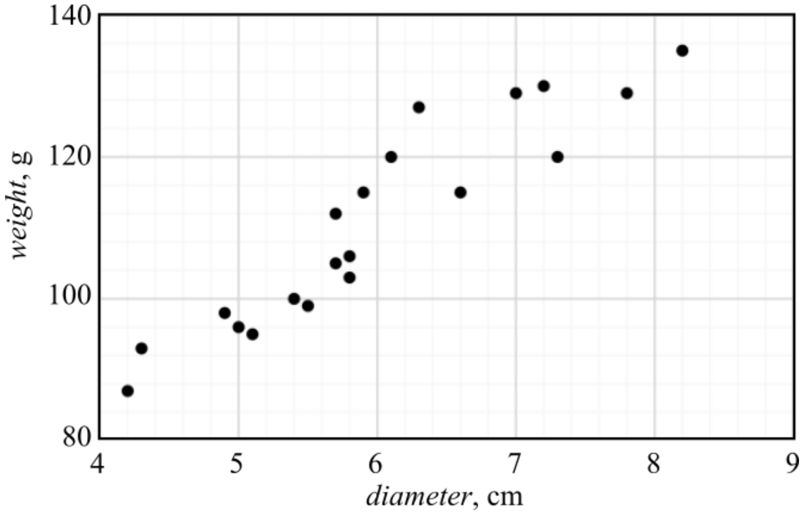
Question 3 (5 marks)

To further check the quality of the tomatoes Aaden weighed the 20 tomatoes from the batch that used *Fertilizer A* and then measured their diameters.

The weight of each tomato, W, in grams, and its corresponding diameter, d, in cm, are shown in the table below.

W, g	d, cm
95	5.1
100	5.4
98	4.9
120	7.3
115	6.6
129	7.8
87	4.2
112	5.7
135	8.2
93	4.3
106	5.8
99	5.5
130	7.2
120	6.1
105	5.7
96	5.0
115	5.9
129	7.0
127	6.3
103	5.8

This data is displayed on the scatterplot below.



a. State the equation of the least squares line that can be used to predict the weight of a tomato from the diameter of the tomato. Give your answers correct to one decimal place. 2 marks

SECTION A – Question 3 – continued

TURN OVER

b. The coefficient of determination for this relationship is $r^2 = 0.866$. Explain the meaning of this coefficient for this set of data. 1 mark

Aaden also weighed 10 tomatoes from the batch that used *Fertilizer B* and measured their diameters. He calculated the equation of the least squares line for this set of data, which is given below.

$$\textit{weight} = 42.6 + 11.1 \times \textit{diameter}$$

c. Use this least squares line to predict the weight of a tomato with a diameter of 5.4 cm. 1 mark

Aaden has an actual tomato in his sample that has a diameter of 5.4 cm and weighs 98 grams.

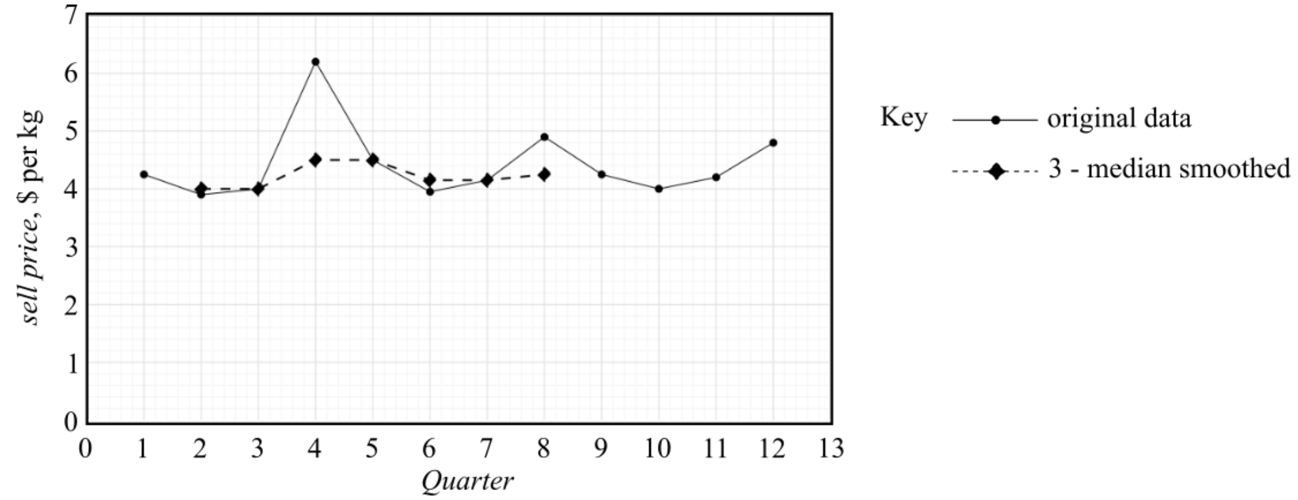
d. Calculate the residual value for this tomato. 1 mark

Question 4 (5 marks)

Aaden sells the tomatoes to fruit and veggie shops. The average quarterly sell prices per 1 kg of tomatoes over a period of three years are displayed in the table below.

Year	*Season*	*Quarter*	*Price*, $
2016	Spring	1	4.25
	Summer	2	3.90
	Autumn	3	4.00
	Winter	4	6.20
2017	Spring	5	4.50
	Summer	6	3.95
	Autumn	7	4.15
	Winter	8	4.90
2018	Spring	9	4.25
	Summer	10	4.00
	Autumn	11	4.20
	Winter	12	4.80

This data is displayed on the time series below.



Aaden started to smooth this data using a 3–median smoothing marked with **diamonds** on the time series plot above.

a. What is the smoothed *sell price*, in $ per kg, in Spring 2018? 1 mark

b. Complete the 3–median smoothing by clearly marking the smoothed values on the time series plot above. 1 mark

SECTION A – Question 4 – continued

TURN OVER

Lydia decided to smooth this data using a 4–mean smoothing with centring.

c. What is the smoothed *sell price*, in \$ per kg, in Autumn 2016, Lydia should obtain using this smoothing method? Give your answer correct to 2 decimal places. 1 mark

Lydia also calculated the seasonal indices for the sell price of tomatoes as shown in the table below.

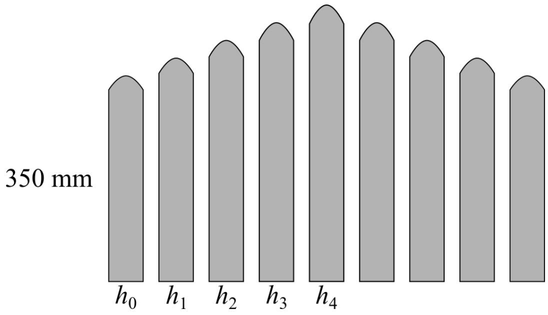
Season	Spring	Summer	Autumn	Winter
Seasonal indices	0.98	S	0.93	1.20

d. What is the seasonal index, S, for summer? 1 mark

e. If the predicted de-seasonalised *sell price* for spring 2019 is \$5.10, what would the actual *sell price* be? Give your answer correct to the nearest dollar. 1 mark

Recursion and financial modelling

Question 5 (6 marks)
Theo is designing two picket fences for two clients. The first picket fence design is in the shape shown in the diagram below, where $h_0 = 350$ mm.

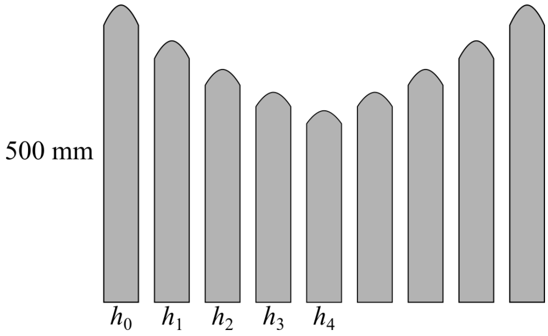


The heights of the first five pickets, h_0, h_1, h_2, h_3 and h_4, are the first five terms of an arithmetic sequence.

a. Determine the common difference, d, in height between two consecutive pickets if the height of the fifth picket is 470 mm. 1 mark

b. Write a first-order recurrence relation, which can be used to generate the height, h_n, of the n^{th} picket, for $n = \{0, 1, 2, 3, 4\}$. 1 mark

The second picket fence design is in the shape shown in the diagram below, where the height of the first picket is h_0 = 500 mm, the height of the second picket is h_1 = 440 mm and the height of the third picket is h_2 = 392 mm.



The height, h_n, in mm, of the n^{th} picket can be calculated using the recurrence relation $h_{n+1} = ah_n + b$, where a and b are real non-zero numbers.

c. Show that $h_{n+1} = 0.8h_n + 40$. 2 marks

d. Calculate the height of the 5th picket, correct to the nearest millimetre. 2 marks

SECTION A – continued

Question 6 (3 marks)

Theo has been using a van for his fencing business for the last six years, which he purchased for $26000. His accountant calculated the depreciation of the van to be $16250 over the five years using the **flat rate** depreciation method.

a. What is the current value of the van? 1 mark

Theo travels, on average, 15000 km per year for his business. The accountant used the depreciation calculated using the flat rate depreciation to calculate the depreciation of the van per kilometre.

b. What is the **unit cost** depreciation for this van? Give your answer correct to the nearest cent. 1 mark

c. If the accountant uses the **flat rate** depreciation method to calculate the value of the van, after how many more years will the van have no value? 1 mark

SECTION A – continued

TURN OVER

Working space

Question 7 (3 marks)

Theo deposits $14000 into an investment account to go on an overseas trip. He has had the account for 3 years and he also makes monthly deposits of $100. Currently, the balance of the account is $20800, correct to the nearest dollar.

a. Calculate the annual interest rate of this account if interest is compounded monthly.
Give your answer correct to one decimal place. 1 mark

At the end of the three years Theo is looking at re-investing the $20800 into another account that pays interest at a rate of 8.5% per annum compounded quarterly. He now deposits $300 per quarter.

b. Write a formula that could be used to calculate the balance of this account at the end of the first quarter immediately after the $300 was deposited into the account. 1 mark

c. Calculate the balance of this account at the end of the first quarter. 1 mark

END OF SECTION A
TURN OVER

THIS PAGE IS BLANK

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

THIS PAGE IS BLANK

Module 1 – Matrices

Question 1 (5 marks)
Three shop assistants, Trevor (T), Uyanah (U) and Vlad (V) have part time jobs at the same shop. The numbers of hours they work each day of the week are displayed in the matrices below.

$$P = \begin{array}{c} \begin{array}{ccccc} M & T & W & Th & F \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1.5 & 2.5 \\ 0 & 1 & 0 & 0 & 2 \\ 2 & 0 & 1.5 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ U \\ V \end{array} \quad \text{and} \quad R = \begin{array}{c} \begin{array}{cc} Sat & Sun \end{array} \\ \begin{bmatrix} 2 & 3.5 \\ 3.5 & 4 \\ 3 & 4 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ U \\ V \end{array}$$

Matrix P displays the hours worked by the three shop assistants from Monday to Friday, and matrix R displays the hours worked on weekends.
The shop assistants are paid as follows:

- From Monday to Friday Trevor is paid \$12.50 per hour, Uyanah is paid \$13.75 per hour and Vlad is paid \$17.20 per hour.
- On weekends Trevor is paid \$18.00 per hour, Uyanah is paid \$21.50 per hour and Vlad is paid \$25.60 per hour.

Let W_1 be the matrix that displays the weekdays hourly payments for the three shop assistants.
Let W_2 be the matrix that displays the weekend hourly payments for the three shop assistants.

a. Fill in the two row matrices W_1 and W_2. 2 marks

$$W_1 = \begin{array}{c} \begin{array}{ccc} T & U & V \end{array} \\ \begin{bmatrix} 12.50 & \_\_\_\_ & \_\_\_\_ \end{bmatrix} \end{array} \quad \text{and} \quad W_2 = \begin{array}{c} \begin{array}{ccc} T & U & V \end{array} \\ \begin{bmatrix} \_\_\_\_ & 21.50 & \_\_\_\_ \end{bmatrix} \end{array}$$

b. What is the order of the matrix product W_1P? 1 mark

____

____

c. Calculate the matrix product W_2R. 1 mark

____

____

d. What do the elements in the matrix product W_2R represent? 1 mark

____

____

____

____

Question 2 (4 marks)

The shop sells coats (C), shirts (S) and jumpers (J).

A new stock has just been brought in and it consists of 200 coats, 380 shirts and 144 jumpers.

The numbers of items sold by each of the three shop assistants in the first week are shown in the matrix M below.

$$\begin{array}{cc} & \begin{array}{ccc} C & S & J \end{array} & \\ M = & \begin{bmatrix} 22 & 106 & 30 \\ 86 & 154 & 81 \\ 28 & 101 & 33 \end{bmatrix} & \begin{array}{c} T \\ U \\ V \end{array} \end{array}$$

a. How many items did Trevor sell in the first week? 1 mark

b. Explain why matrix M is invertible. 1 mark

During one day Trevor sold \$1127 worth of items, Uyanah sold \$1517 worth of items and Vlad sold \$1287 worth of items.

Let c be the cost of one coat.

Let s be the cost of one shirt.

Let j be the cost of one jumper.

The values of c, s and j satisfy the system of linear equations given below.

$$\begin{cases} 2c + 9s + 4j = 1127 \\ 3c + 12s + 5j = 1517 \\ c + 11s + 6j = 1287 \end{cases}$$

c. Write the elements in the matrix equation below corresponding to the system of linear equations above. 1 mark

$$\begin{bmatrix} c \\ s \\ j \end{bmatrix} = \begin{bmatrix} \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} \\ \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} \\ \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} \end{bmatrix} \times \begin{bmatrix} \text{\_\_\_\_} \\ \text{\_\_\_\_} \\ \text{\_\_\_\_} \end{bmatrix}$$

d. **Hence** or otherwise determine the values of c, s and j. 1 mark

Question 3 (3 marks)

The initial state for the number of items in stock for this shop, S_0, can be written as

$$S_0 = \begin{array}{c|c} \begin{bmatrix} 200 \\ 380 \\ 160 \end{bmatrix} & \begin{array}{c} C \\ S \\ J \end{array} \end{array}$$

Vlad is in charge of the weekly stock. From market research he knows that the percentages of items **not** sold from one week to the next can be calculated using the matrix T below.

$$T = \begin{array}{cc} \text{this week} & \\ \begin{bmatrix} 0.32 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.10 \end{bmatrix} & \text{next week} \end{array}$$

a. How many shirts were **not** sold in the first week? 1 mark

The matrix equation $S_{n+1} = TS_n + B$ can be used to calculate the stock available at the end of any week n after the shop was restocked.

b. Write the elements of matrix B below such that the stock at the beginning of each week is equal to the initial stock. 1 mark

$$B = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

c. Write an equation for S_2 in terms of T, S_0 and B. 1 mark

End of Module 1 – SECTION B – continued

TURN OVER

THIS PAGE IS BLANK

Module 2 – Networks and decision mathematics

Question 1 (3 marks)

Four friends, Leona, Max, Natalie and Oliver, are participating in a bike race as a team. Each member of the team has to ride one of four routes, Route *A*, Route *B*, Route *C* and Route *D*. In order to choose the most efficient route for each team member, the four friends decided to time themselves on the four routes.
The table below displays the average time, in minutes, it took each team member to ride each route.

	Leona	**Max**	**Natalie**	**Oliver**
Route *A*	10	14	15	16
Route *B*	18	15	12	14
Route *C*	12	8	10	9
Route *D*	14	12	15	17

Natalie has just learnt how to use the Hungarian algorithm and applied it to this situation. The table below shows the results after Natalie applied the first two steps of the algorithm: row and column subtractions.

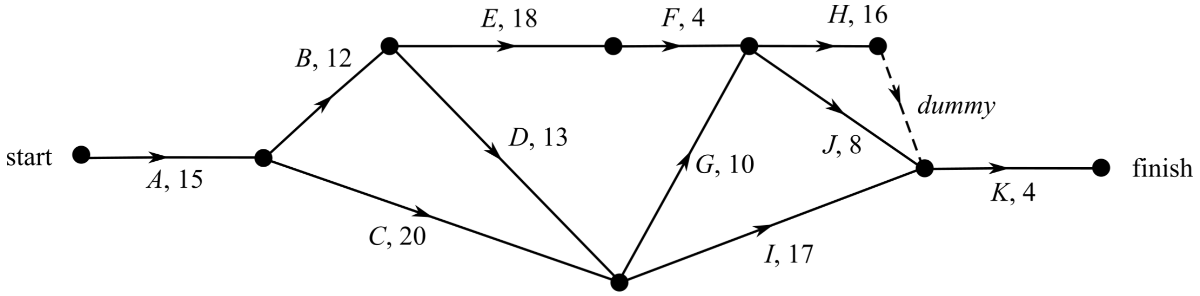
	Leona	**Max**	**Natalie**	**Oliver**
Route *A*	0	1	3	5
Route *B*	4	0	0	1
Route *C*	2	0	0	0
Route *D*	0	0	1	4

a. Explain why the four friends can now decide who is going to ride each route in this bike race. 1 mark

b. Who is the first team member that can be assigned a route and why? 1 mark

c. Calculate the expected average minimum time for this team. 1 mark

Question 2 (5 marks)
The organisers of the bike race have to plan the event for its smooth running. The various activities involved in setting up the race on the day and their respective completion times, in minutes, are shown on the directed network below.



a. Explain the reason for the introduction of the *dummy* activity between activities *H* and *K*. 1 mark

The organisers started to set up the race at 7:40 am and the race is planned to start at 9:00 am.

b. Explain, with calculations, why the organisers have enough time to start the race as planned. 2 marks

There are two activities with a float time of 1 minute each.

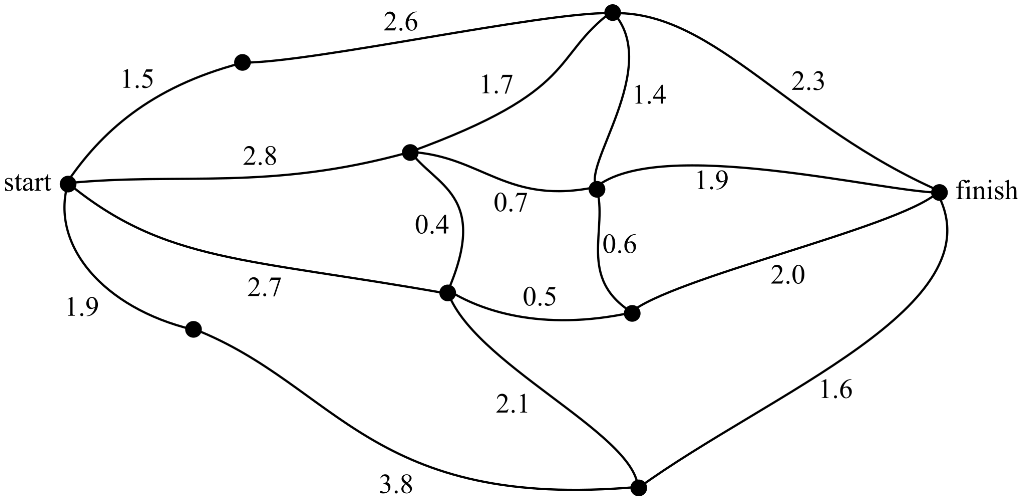
c. State the two activities. 1 mark

Extra volunteers turned up on the day of the race and the organisers managed to complete activity *G* in 8 minutes.

d. Explain the effect of the extra volunteers on the duration of the project and its critical path. 1 mark

Question 3 (4 marks)

Along the four routes there are ten checkpoints altogether. These points are the vertices shown in the network below, where the numbers on edges represent the distances between two checkpoints, in kilometres.



The first aid team must be placed at a checkpoint that can be reached from all the other checkpoints on the shortest possible route.

a. On the graph above mark the vertex, with a cross, where the first aid team should be placed and highlight the edges that satisfy this condition. 2 marks

b. Is this the minimum spanning tree for this graph? Explain. 1 mark

The water needed during the bike race is going to be given out by a sponsor. The sponsor wants to find a way to visit all the checkpoints once only starting and ending at the same point.

c. What is the mathematical name for such a circuit? 1 mark

End of Module 2 – SECTION B – continued
TURN OVER

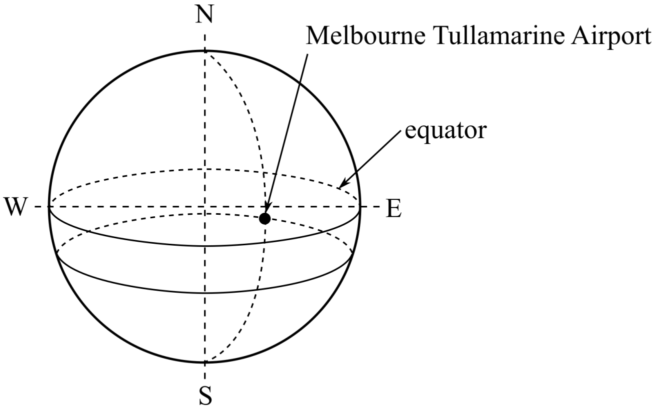
THIS PAGE IS BLANK

Module 3 – Geometry and trigonometry

Question 1 (5 marks)
Corrine is an Australian mathematician who lives in Melbourne. She is taking a trip to New York, US, to visit the National Museum of Mathematics.

Corrine is flying from Melbourne Tullamarine Airport (GPS coordinates 38° S and 145° E), Australia to JFK Airport, New York, US (GPS coordinates 41° N and 74° W).

The circle below is a sketch of Earth showing the equator and the position of the Melbourne Tullamarine Airport.



a. On the diagram above draw the small circle of latitude 41° N, the great circle of longitude 74° W and hence clearly mark the position of the JFK Airport, New York, US. 2 marks

b. Calculate how far from the equator Melbourne Tullamarine Airport is, on the great circle of 145° E. Assume the radius of Earth is 6400km. Give your answer correct to two decimal places. 1 mark

c. Explain, with appropriate calculations, why Melbourne is approximately 15 hours ahead of New York (no daylight saving being used). 1 mark

SECTION B – Module 3 – Question 1 – continued
TURN OVER

Corrine takes a flight that leaves Melbourne Tullamarine Airport on the 21st of December at 11:15 am and arrives at JFK Airport on the 21st of December at 4:30 pm.

c. How many hours will the flight take? 1 mark

Question 2 (4 marks)

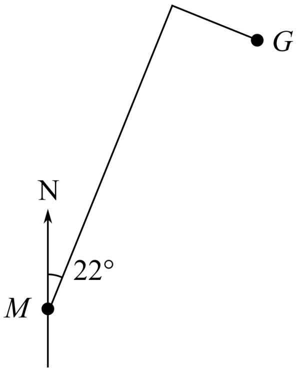
The National Museum of Mathematics is 26 km away from the JFK Airport, US, on a bearing of N65°W.

a. Write this compass bearing as a three-figure bearing. 1 mark

Corrine is going to walk from the National Museum of Mathematics, *M*, to the Grand Central Terminal, *G*, to catch a train to the Columbus Circle.

She walks 1.2 km on a bearing of N22°E from the museum and then turns right and walks for another 240 m.

The diagram below shows the position of the Grand Central Terminal relative to the National Museum of Mathematics.

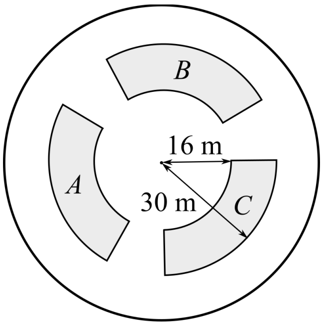


b. Calculate the direct distance *MG*. Give your answer correct to two decimal places. 1 mark

c. What is the bearing of *G* from *M*? Give your answer correct to the nearest degree. 2 marks

Question 3 (3 marks)

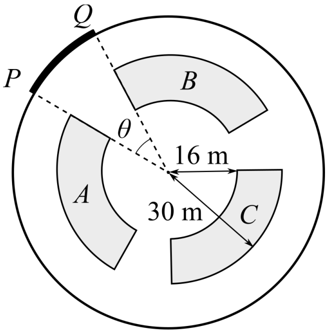
Corrine likes the mathematics involved in the diagram of the Columbus Circle. A sketch of this circle is shown in the diagram below. The shaded areas *A*, *B* and *C* are not equal.



The shaded areas are filled with water from a water fountain. The angle at the centre for the shaded area *C* is 104°.

a. Calculate the volume of water in the shaded area *C* if the depth of water is 100 mm.
Give your answer correct to the nearest cubic metre. 2 marks

b. Calculate the value of angle θ if the length of arc *PQ* is 9 m and the radius of the largest circle is 65 m.
Give your answer correct to the nearest degree. 1 mark



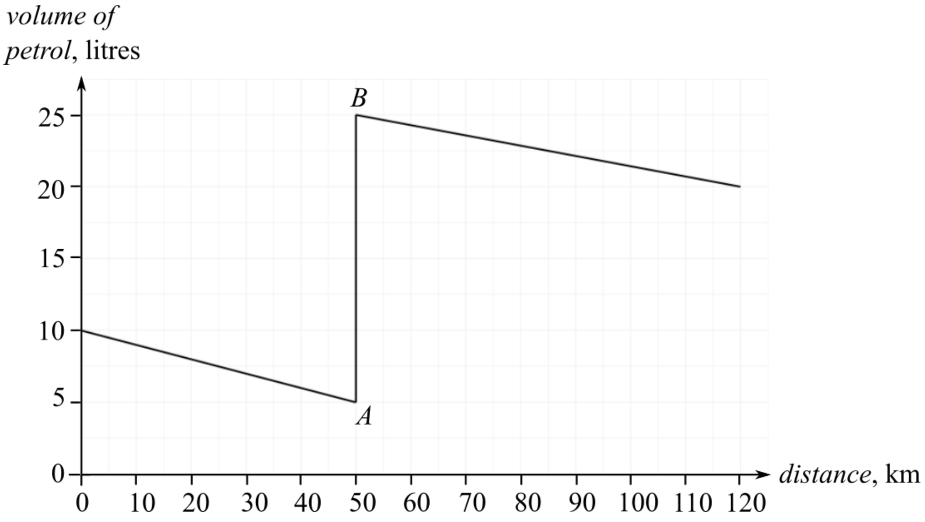
Module 4 – Graphs and relations

Question 1 (4 marks)

Madison and Ethan went to a business conference together. They travelled 120 km in 1.6 hours.

a. What was their average speed for this trip? 1 mark

The graph below displays the volume of petrol in the car's tank during this trip.



b. How many litres per kilometre did the car use for the first 50 kilometres of the trip? 1 mark

c. Using appropriate mathematical reasoning, determine the part of the trip, before or after 50 km, for which the car used more petrol per kilometre? 1 mark

SECTION B – Module 4 – Question 1 – continued
TURN OVER

d. What is the interpretation of line *AB* in the context of this situation? 1 mark

Question 2 (4 marks)

Madison and Ethan are business partners in a company that makes snowboards. While Madison was driving, Ethan looked at some data collected about the speed of a snowboarder using one of their snowboards. The data is shown in the table of values below, where time, *t*, is measured in seconds and distance travelled, *d*, in metres.

t, seconds	0	1	2	3	5
d, metres	0	3.5	14.0	31.5	87.5

The relationship between distance and time can be written as $d = kt^2$.

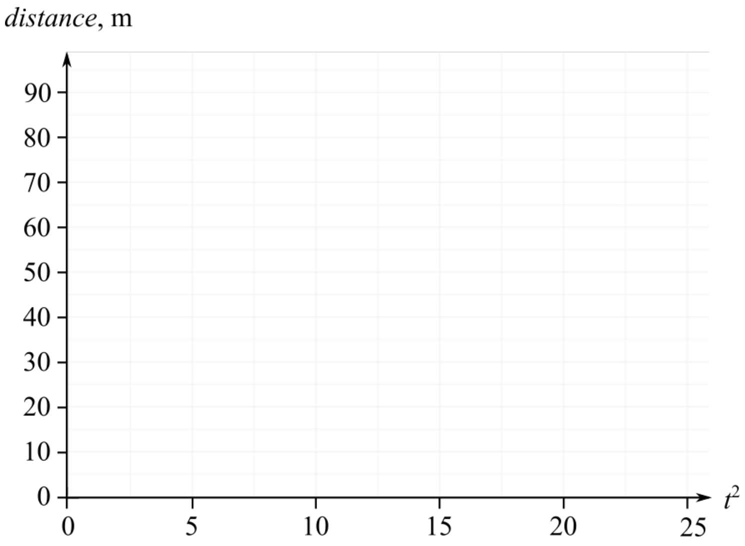
a. Calculate the value of k. 1 mark

b. What was the average speed of the snowboarder over this time interval? 1 mark

c. Calculate the time taken to travel 56 m. 1 mark

Ethan wants to linearise this data so he plots the graph of distance, d, against time, t^2.

d. On the set of axes below show this linearised plot for this relationship. 1 mark



Question 3 (4 marks)

The company manufactures two types of snowboards: *standard* and *delux*. The *standard* snowboards are sold for \$300 each and the *delux* snowboards are sold for \$500 each.

The company spends, per day, \$45000 to produce these snowboards plus an additional \$120 for each *standard* snowboard and an additional \$140 for each *delux* snowboard.

a. Show that, if the company sells 150 *standard* snowboards and 50 *delux* snowboards per day, the company breaks even. 1 mark

Let S be the number of *standard* snowboards manufactured each day, where $S > 150$ (Constraint 1).

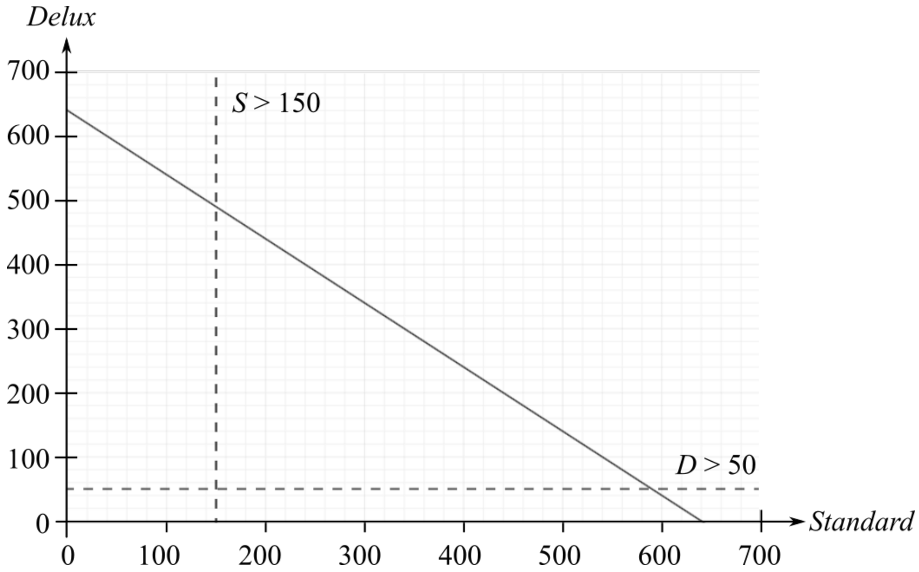
Let D be the number of *delux* snowboards manufactured each day, where $D > 50$ (Constraint 2).

The maximum number of snowboards that can be manufactured per day is 640 (Constraint 3).

For every *delux* snowboard, the company manufactures at least three *standard* snowboards (Constraint 4).

b. Write an equation corresponding to Constraint 4. 1 mark

Constraints 1, 2 and 3 have been plotted on the graph below.



c. On the same set of axes above plot constraint 4, clearly shade the feasible region and label all key features including intercepts. 2 marks

END OF QUESTION AND ANSWER BOOK